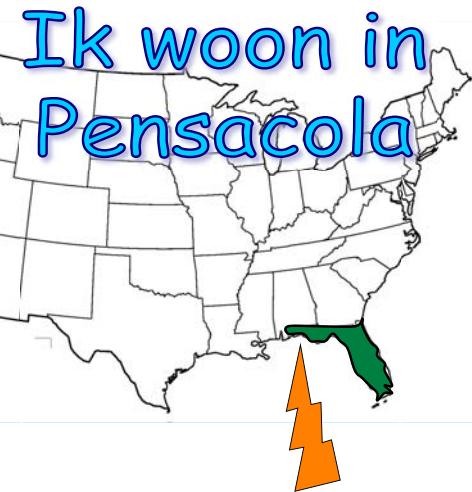


Mijn naam is Jim Bezdek



Hallo en welkom !



9/29/10

Some fuzzy models for Medical Applications

2



Kingfish Napier, NZ

Anomaly Detection in Wireless Sensor Networks

Introduction to Wireless Sensor Networks

DCAD: Data Capture Anomaly Detection

ESAD: Elliptical Summary Anomaly Detection

Four Measures of dissimilarity for Ellipsoids

Visual assessment : estimating c with iVAT

Anomaly detection with SL clustering

9/29/10

EAs in WSNs

3

WSNs

Sensors
Transceivers
Memory
Processor

* But ... Limited

Power (Battery)
Bandwidth
Memory
Processing capacity

✓ Cheap and small



**Use in unattended areas
(Environmental Monitoring)**



9/29/10

EAs in WSNs

4

Great Duck Island to monitor the sea bird Leach's Storm Petrel

43 Sensor nodes deployed at GDI, 15 km off coast of Maine, USA, summer, 2003



4D Data

Light intensity
Temperature
Humidity
Pressure



Sensors inside and outside underground nesting burrows



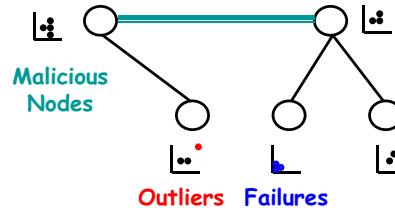
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EAs in WSNs

5

Anomalies : observations inconsistent with the bulk of the data

Anomalies in WSNs



Causes

Sudden environmental change (clouds obscure the sun)
Nodes lose calibration, power failure, physical damage
Malicious attacks (data injection, *angry seabirds !!*)

Detection

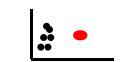
Analyze measurements/traffic in the network
Model normal behavior to classify anomalies

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EAs in WSNs

1

4 types of Anomalies

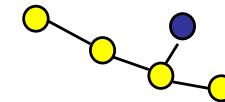


Isolated
Outliers

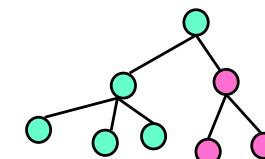


Outlier
Epoch

(1st Order)
Internal Node Anomalies



(2nd Order)
Whole Node Anomalies



(HO Order)
Anomalous Subtree

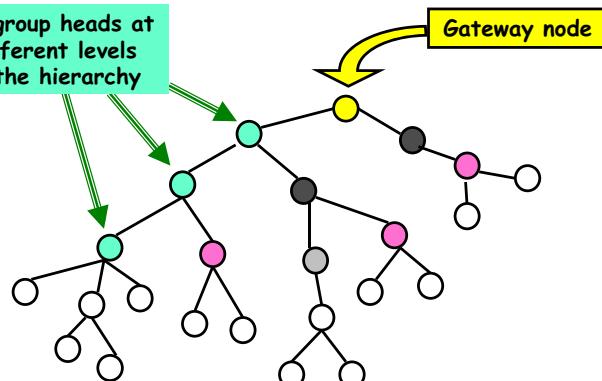
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EAs in WSNs

2

Hierarchical WSN Topology

Subgroup heads at different levels in the hierarchy

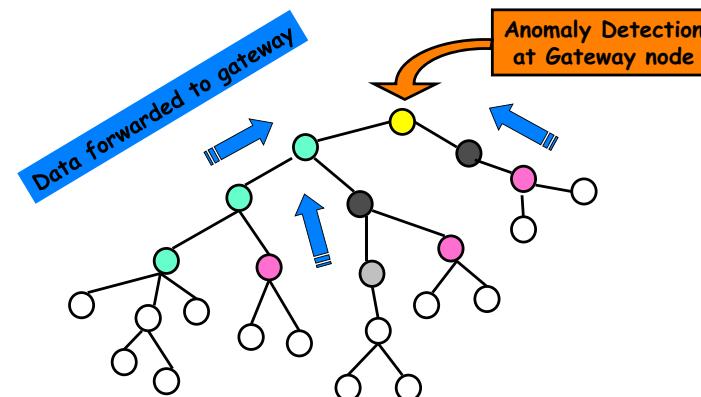


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EAs in WSNs

3

The *Centralized Approach* with sensor data routed to gateway



9/29/10

EAs in WSNs

4

Problems with the Centralized Data Approach

High energy consumption for data communication

Nodes near the gateway become a bottleneck

Imbalanced load distribution in the network

Reduced physical lifetime of the network

Scalability (*slows down* as nodes are added)

9/29/10

EAs in WSNs

5



DCAD: The Data Capture Anomaly Detection Model

Rajasegarar, Bezdek, Leckie, and Palaniswami (2009). Elliptical Anomalies in Wireless Sensor Networks", ACM TOSN, 6(1).

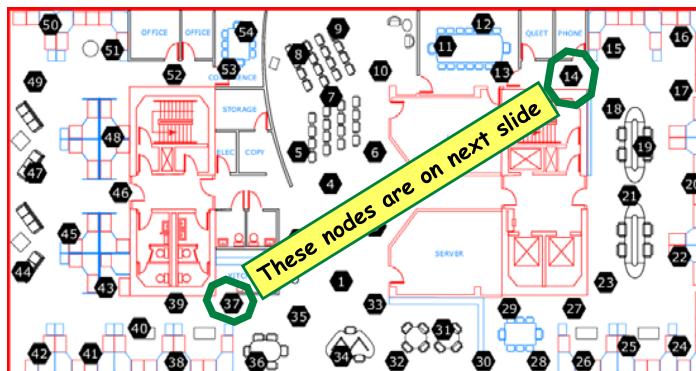


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EAs in WSNs

6

Example : Intel Berkeley Research Laboratory (IBRL) WSN



Period: March 1, 2004
2D Data collected
 $x = (\text{temp., humidity})$

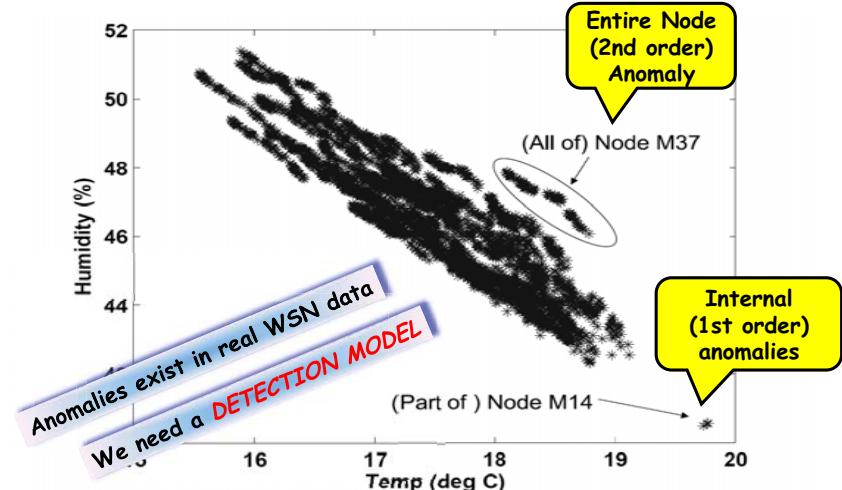
54 nodes deployed
Sampled every 30s
Time window = 4 hr

9/29/10

EAs in WSNs

7

Scatterplot of all data at the gateway node in the IBRL WSN



9/29/10

EAs in WSNs

8

A Flashback to ... (Hyper) Ellipsoids

Basic notation

$$x, m \in \mathbb{R}^p, S \in PD \subset \mathbb{R}^{p \times p}$$

Inner product norm induced by S

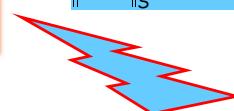
$$\|x - m\|_S = \sqrt{(x - m)^T S (x - m)}$$

Euclidean norm $S = p \times p$ identity I_p

$$\|x - m\|_{I_p} = \sqrt{(x - m)^T I_p (x - m)}$$

Mahalanobis norm $m = \text{mean}(X)$ $S = \text{cov}(X)$

$$\|x - m\|_{S^{-1}} = \sqrt{(x - m)^T S^{-1} (x - m)}$$



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EAs in WSNs

9

The geometry of Ellipsoids

Ellipsoid $E(S^{-1}, m; t)$ induced by S^{-1} , centered at m

Interior E_{int}

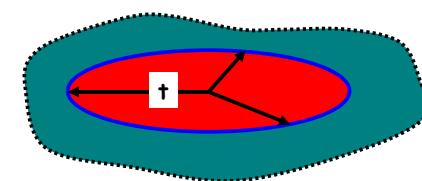
$$E(S^{-1}, m; t) = \{x \in \mathbb{R}^p \mid \|x - m\|_{S^{-1}}^2 < t^2\}$$

Surface E_{surf}

$$E(S^{-1}, m; t) = \{x \in \mathbb{R}^p \mid \|x - m\|_{S^{-1}}^2 = t^2\}$$

Exterior E_{ext}

$$E(S^{-1}, m; t) = \{x \in \mathbb{R}^p \mid \|x - m\|_{S^{-1}}^2 > t^2\}$$



"effective radius" t

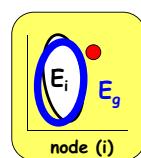
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EAs in WSNs

10

Data Capture Anomaly Detection (DCAD) Model

E_i = Ellipse at node i

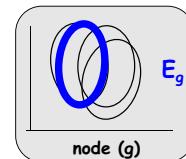


Detection of 1st order anomalies with level sets of E_i

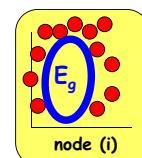
[or backpropagated] level sets of E_g on node data

$E_g = \Phi(E_i) = (\text{Compound})$ Ellipse at gateway node g

Transmit parameters of E_i 's - not DATA



Detection of 2nd/Higher order anomalies with backpropagated level sets of E_g on node data



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EAs in WSNs

11

Anomaly Detection by data capture with Ellipsoids

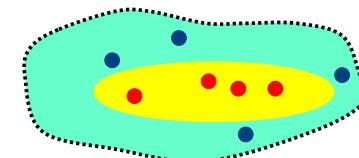
Sample $X = \{x_1, \dots, x_n\} \subset \mathbb{R}^p : m = \text{mean}(X); S = \text{cov}(X)$

"Normal" Points $NP_{X,t} = E_{int} \cup E_{surf}$

$$NP_{X,t} = \{x \in \mathbb{R}^p \mid \|x - m\|_{S^{-1}}^2 \leq t^2\}$$

Anomalous Points $AP_{X,t} = E_{ext}$

$$AP_{X,t} = \{x \in \mathbb{R}^p \mid \|x - m\|_{S^{-1}}^2 > t^2\}$$



$$\begin{aligned} NP_{X,t} \cap AP_{X,t} &= \emptyset \\ NP_{X,t} \cup AP_{X,t} &= \mathbb{R}^p \end{aligned}$$

$E(S^{-1}, m; t)$ has user-defined "effective radius" t

Three definitions of EAs depend on choice of t

EAs in WSNs

1

Elliptical Cardinality Anomalies (ECAs)

Compute and order n distances

$$\{d_k\} = \{\|x_k - m\|_{S^{-1}}^2\} \rightarrow d_{(1)} \leq \dots \leq d_{(n)}$$

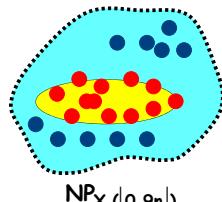
Choose capture % c, defines t

$$c \in \{0.01, \dots, 1.00\} \rightarrow t = d_{(\lfloor cn \rfloor)}$$

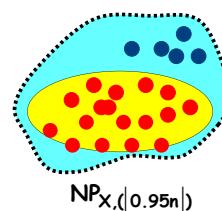
$NP_{X,\lfloor cn \rfloor}$ 100 c% "normal" data

$AP_{X,\lfloor cn \rfloor}$ exterior (anomalous) data

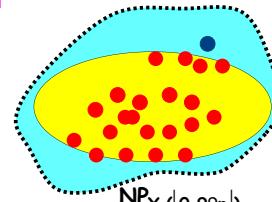
Example: n = 100



$NP_{X,[0.9n]}$



$NP_{X,[0.95n]}$



$NP_{X,[0.99n]}$

9/29/10

EAs in WSNs

2

Elliptical Number of Sigmas Anomalies (ENSAs)

Assume $X \sim n(\mu, \Sigma) \sim \text{Gaussian}$

$n_\sigma = \# \text{ of Std. Devs. from } m$

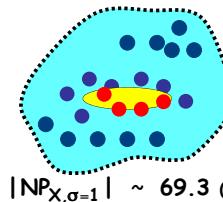
Choose prob(n_σ), define t = n_σ

$n_\sigma \in \{1, 2, \dots, k\} \rightarrow t = n_\sigma$

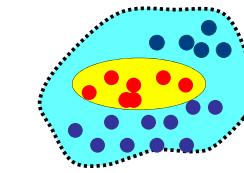
NP_{X,n_σ} 100 c% "normal" data

AP_{X,n_σ} exterior (anomalous) data

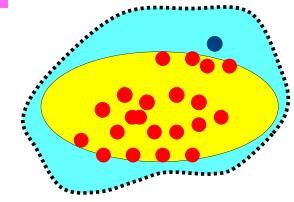
Example: n = 100



$|NP_{X,\sigma=1}| \sim 69.3 \text{ (%)}$



$|NP_{X,\sigma=2}| \sim 86.5 \text{ (%)}$



$|NP_{X,\sigma=3}| \sim 98.9 \text{ (%)}$

9/29/10

EAs in WSNs

3

Elliptical chi-squared Anomalies (ECSAs)

Assume $X \sim n(\mu, \Sigma)$

Pick $\alpha \in \{0.01, \dots, 1.00\}$

Choose % $(1-\alpha)$, defines t

$$t = \chi_p^2(\alpha)$$

$NP_{X,\chi_p^2(\alpha)}$

100(1- α) % "normal"

$AP_{X,\chi_p^2(\alpha)}$

exterior anomalies

$$\lim_{n \rightarrow \infty} \{ENSA - ECSA\} = 0$$

9/29/10

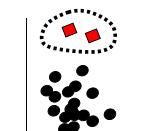
EAs in WSNs

4

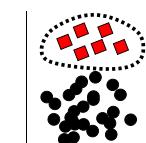
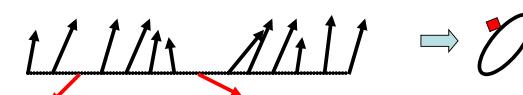
1st Order (or Type 1) Anomalies: Internal to a node



Normal node : all vectors similar



Data anomalies: odd vectors differ



Epoch anomalies: contiguous vectors differ

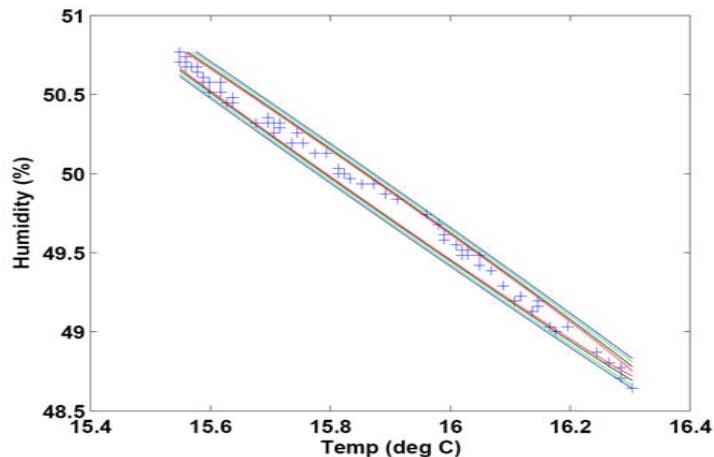


9/29/10

5

EAs in WSNs

IBRL Node 50 : Example of a normal node

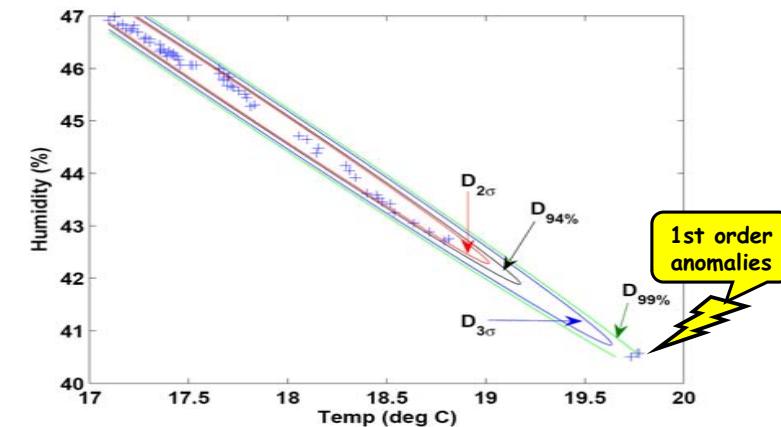


9/29/10

EAs in WSNs

6

1st order ECA and ENSA anomalies at IBRL Node 14

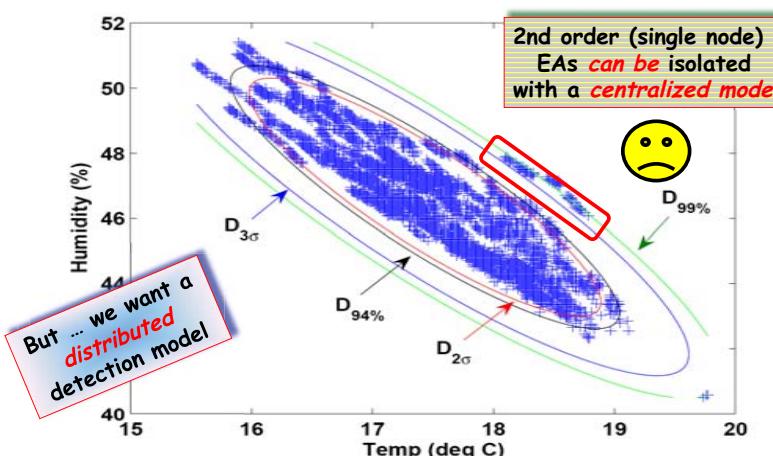


9/29/10

EAs in WSNs

7

ECA and ENSA ellipses based on *all data* at IBRL gateway Node

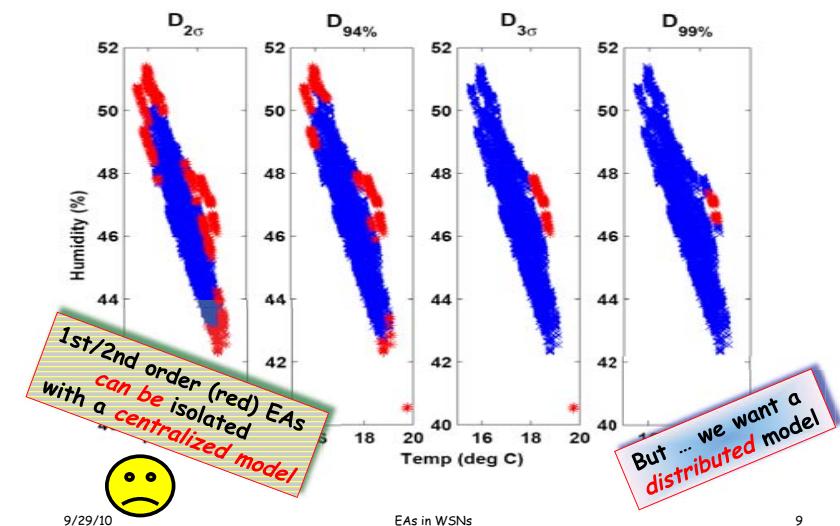


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EAs in WSNs

8

ECAs and ENSAs based on *all data* sent to the gateway node

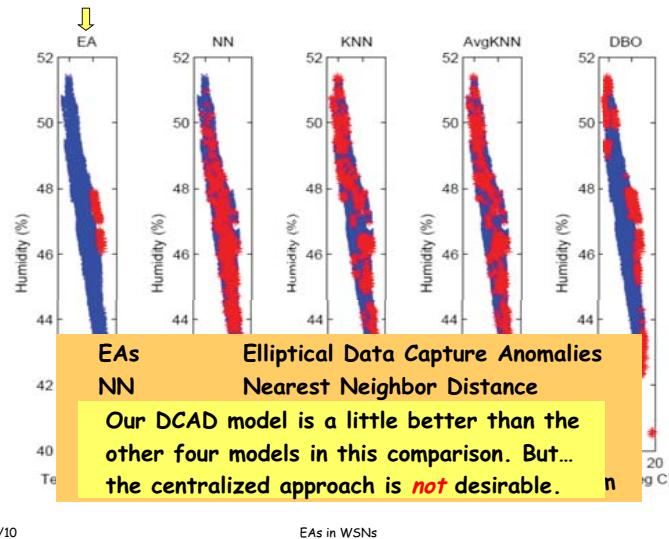


9/29/10

EAs in WSNs

9

5 centralized methods on all data at the gateway node



9/29/10

EAs in WSNs

10



ESAD: The Elliptical Summaries Anomaly Detection Model

Bezdek, Havens, Keller, Leckie, Park, Palaniswami, Rajasegarar,(2010). Clustering elliptical anomalies in sensor networks, Proc WCCI 2010.

Bezdek, Rajasegarar, Moshtaghi, Havens, Leckie, Palaniswami (2010). Anomaly Detection in Environmental Monitoring Networks, in press, IEEE CIM.

Moshtaghi, Havens, Park, Bezdek, Leckie, Rajasegarar, Keller, Palaniswami (2010). Clustering Ellipses for Anomaly Detection, in press, Patt. Recog.

Rajasegarar, Bezdek, Moshtaghi, Leckie, Palaniswami, Havens, (2010). Measures for Clustering and Anomaly Detection in Sets of 3D Ellipsoids, in review, CVGIP.

Our WSN Gang



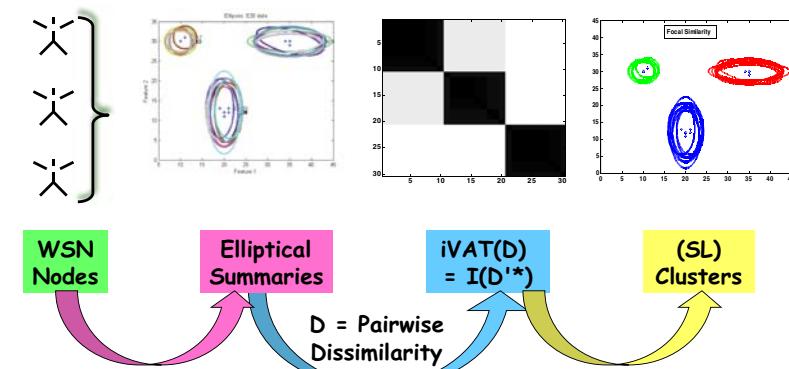
U. of Melbourne

EAs in WSNs

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U. of W. Sydney

Elliptical Summary Anomaly Detection (ESAD) Model



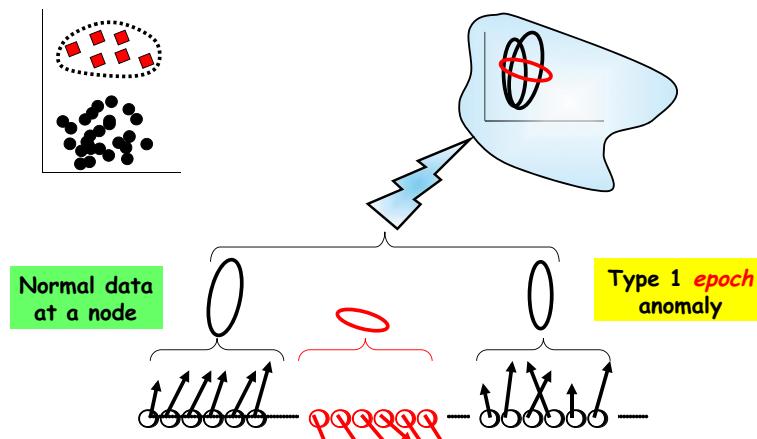
9/29/10

EAs in WSNs

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Detection form of 1st order epoch anomaly

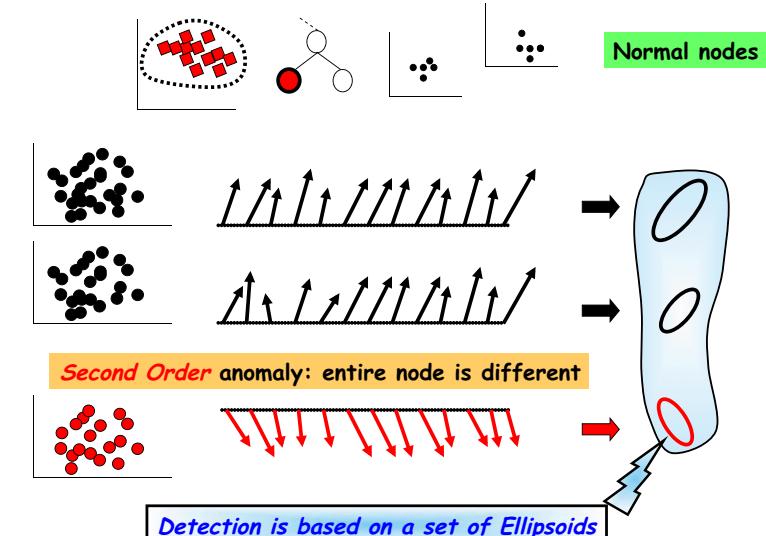
Detection space is a set of Ellipsoids



9/29/10

EAs in WSNs

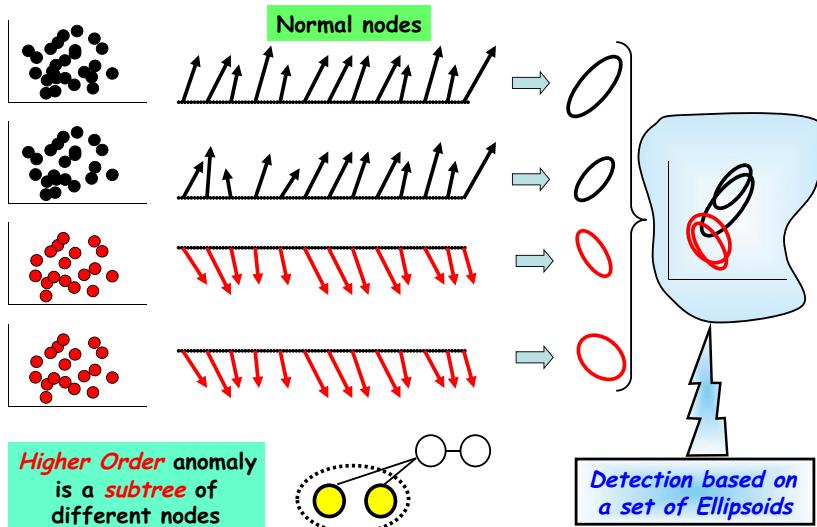
14



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EAs in WSNs

15



9/29/10

EAs in WSNs

16

Bezdek, Hathaway (2002). VAT: A tool for visually assessing (cluster) tendency, Proc. IJCNN, 2225-2230.



Visual Assessment with RDIs



Wang, Leckie, Kotagiri, Bezdek (2010). iVAT : Visual Analysis for cluster tendency assessment, in press, PAKDD.



Havens, Bezdek (2010). Recursive (iVAT), in review, TKDE.



EAs in WSNs

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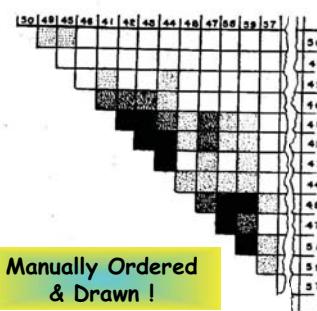
The First RDI

Cattell, R. (1944) *Psychometrika*, 9(3), 169-184

"With or without me lists he then lists the variables of the linkage diagonal or as possible. If the process is successful, the result striking"



An RDI in 1944... ???
Duhhh !



Manually Ordered & Drawn !

3 intensities
Black : $1 \geq r > 0.6$
Grey : $0.5 \leq r < 0.6$
Light : $0 \leq r < 0.5$

9/29/10

EAs in WSNs

1

Visual Assessment : VAT

Our "Claim" in 2002

The utility of $I(D)$ for visual assessment of clusters depends on the ordering in D

Our "Idea" in 2002



Reorder $D \rightarrow D^*$ so groups of similar objects are close in D^*

Nb: Cattell published both the claim and idea in 1944 !



"Man learns from history that man learns nothing from history" ... Hegel

9/29/10

EAs in WSNs

2

The (2002) VAT Algorithm ($n \leq \sim 5000$)

Input

Dissimilarity data D_{nxn} (if S , $D=[1]-S$)

Set

$K=\{1, \dots, n\} : I = J = \emptyset : i = j = 0$

Find

$(i, j) \in \operatorname{argmax}_{p \in K, q \in K} D_{pq} \Rightarrow P(1) = i$
 $I = \{i\}$
 $J = K - \{i\}$

Loop

$i, j = 2 \text{ to } n$

$(i, j) \in \operatorname{argmin}_{p \in K, q \in K} D_{pq}$

$P(k) = i : I = I \cup \{j\} : J = J - \{j\}$

VAT RDI

$[D^*]_{ij} = D_{P(i)P(j)}$ for $1 \leq i, j \leq n$

Display

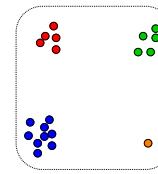
D^* as Image $I(D^*)$

9/29/10

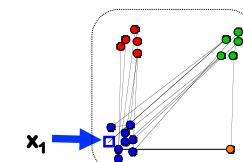
EAs in WSNs

3

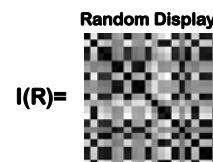
Example



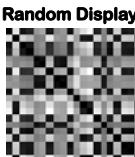
Data X



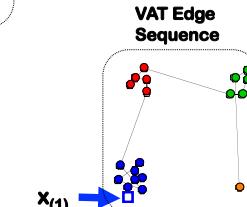
Random Edge Sequence



$I(R) =$



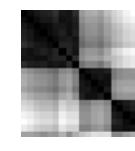
1 Largest
2 Medium
1 Singleton



VAT Edge Sequence



$I(R^*) =$



VAT Display

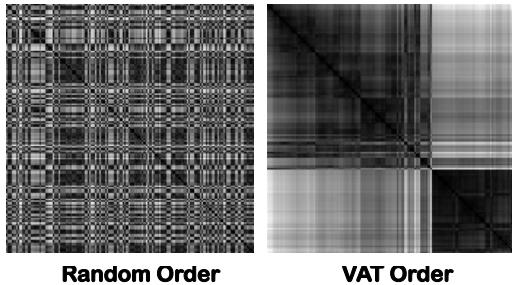
9/29/10

EAs in WSNs

4

Example : IRIS Data

$$\begin{array}{lll} n = 150 \text{ vectors} & c_{\text{physical}} = 3 \\ p = 4 \text{ features} & c_{\text{geometric}} = 2 \end{array}$$



Random Order

VAT Order

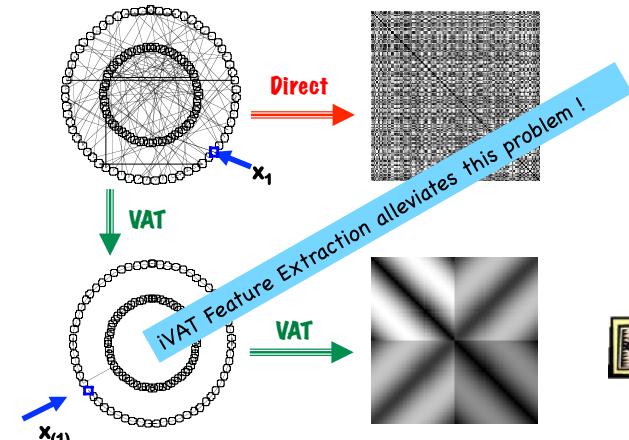
You can  2 clusters
Right Answer !

9/29/10

EAs in WSNs

四

Example : Two Rings Data



?

9/29/10

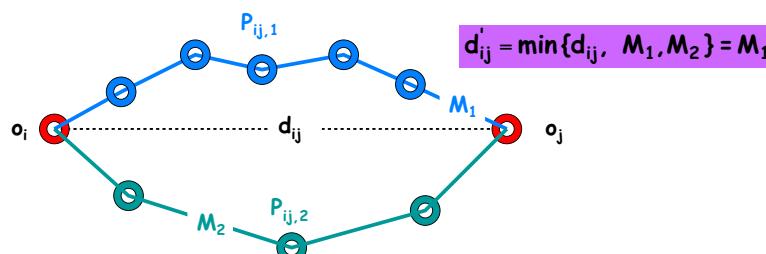
EAs in WSN

6

VAT image $I(D^*)$ may be blurry, or point to the "wrong" c

Transform D to D' before VAT by *Path-based distance* from i to j

$$P_{ij} = \text{All paths in } D \text{ from } i \text{ to } j \quad d'_{ij} = \min_{p \in P_{ij}} \{ \max_{1 \leq h \leq |p|} \{ d_{p[h]p[h+1]} \} \} \Rightarrow D \mapsto D'$$



This distance is well suited to chained (SL~VAT) clusters

9/29/10

FAs in WSNs

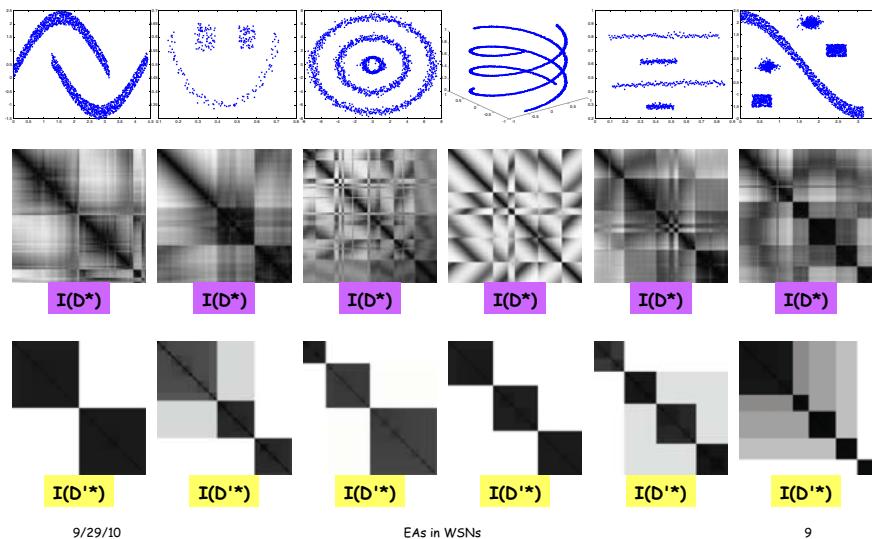
7

8

FAs in WSN

1

iVAT = VAT applied to D' instead of D : *Synthetic Data*



Conclusions : Why Visual Clustering ?

"The preliminary examination of most data is facilitated by the use of diagrams. **Diagrams prove nothing, but bring outstanding features readily to the eye**; they are therefore no substitute for critical tests as may be applied to the data, but are valuable in suggesting such tests, and in explaining conclusions founded upon them."

R. A. Fisher (1924) *Statistical Methods for Research Workers*

"[visual methods] should in no way be regarded as methods to be used to the **exclusion** of other types of multivariate analysis; indeed they will, in general, be **most** helpful when used alongside, and in association with, other forms of analysis."

Brian Everitt (1978) *Graphical Techniques for Multivariate Data*

'What is the use of a book', thought Alice, '**without pictures...**'

Lewis Carroll (1866), *Alice in Wonderland*

9/29/10

EAs in WSNs

10



**Measures of Dissimilarity
for Pairs of Ellipsoids**

9/29/10

EAs in WSNs

11

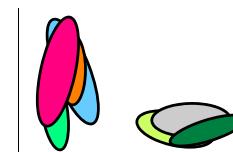
Similarity Measures for Ellipsoids $E_u = \{UE_j\}$

$s(E_i, E_j)$ is a
(strong) similarity
measure on $E_u \times E_u$

$$\begin{aligned} & 0 \leq s(E_i, E_j) \leq 1 \quad \forall i, j \\ \Leftrightarrow & s(E_i, E_j) = 1 \Leftrightarrow E_i = E_j \\ & s(E_i, E_j) = s(E_j, E_i) \quad \forall i \neq j \end{aligned}$$

s is a (weak) similarity measure if $s(E_i, E_j) = 1$ but $E_i \neq E_j$

Important properties for pairwise similarity of Ellipsoids



Location (centers of E_i and E_j)

Shape (eigenvalues of E_i and E_j)

Orientation (eigenvectors of E_i , E_j)

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EAs in WSNs

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Compound Normalized Similarity $s_{cn}(E_i, E_j)$

$$\text{evs of } E_i = (S_i^{-1}, m_i, 1) \quad \alpha = \{\alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_p\} \rightarrow \alpha^* = (1/\sqrt{\alpha_1}, \dots, 1/\sqrt{\alpha_p})^T$$

$$\text{evs of } E_j = (S_j^{-1}, m_j, 1) \quad \beta = \{\beta_1 \leq \beta_2 \leq \dots \leq \beta_p\} \rightarrow \beta^* = (1/\sqrt{\beta_1}, \dots, 1/\sqrt{\beta_p})^T$$

(vector) of angles between $\{\alpha_i, \beta_i\}$

$$\theta = \arccos(\text{diag}(R_i^T R_j))$$

$$s_{cn}(E_i, E_j) = e^{-\left(\underbrace{\|m_i - m_j\|_2^2}_{\text{Location}} + \underbrace{\|\alpha^* - \beta^*\|_2^2}_{\text{Shape}} + \underbrace{\|\sin \theta\|_2^2}_{\text{Orientation}} \right)}$$

Theorem

s_{cn} is a (strong) similarity measure on $E_u \times E_u$

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EAs in WSNs

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Transformation Energy Similarity $s_{te}(E_i, E_j)$

Basic Idea: Measure the "Energy" needed to transform E_i, E_j into a common space

$$E_k = E(R_k S_k^{-2} R_k^T, m_k, 1); k = i, j$$

$$x_j = f(x_i | E_i, E_j) = S_j R_j (R_i^{-1} S_i^{-1} x_i - m_i + m_j)$$

$$\|f(E_i, E_j)\|_2 = \max\{f(z | E_i, E_j) : z \in \mathbb{R}^p; \|z\|_2 = 1\}$$

$$s_{te}(E_i, E_j) = 1 / \max\{\|f(E_i, E_j)\|_2, \|f(E_j, E_i)\|_2\}$$

Theorem

s_{te} is a (strong) similarity measure on $E_u \times E_u$

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EAs in WSNs

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Bhattacharya "Distance" $d_{bc}(E_i, E_j)$

(1946) Bhattacharya distance between two p -variate Gaussians $X_1 \sim n(\mu_1, \Sigma_1)$ and $X_2 \sim n(\mu_2, \Sigma_2)$

$$d_{bc}(E_i, E_j) = e^{-\frac{1}{8}\|m_i - m_j\|_2^2 \left[(A_i^{-1} + A_j^{-1})/2 \right]^{-1}} \\ + \frac{1}{2} \ln \left[\det((A_i^{-1} + A_j^{-1})/2) / \sqrt{\det A_i^{-1} \det A_j^{-1}} \right]$$

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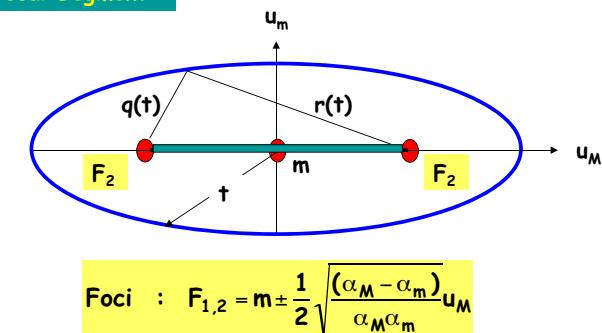
EAs in WSNs

15

Focal Distance $d_{fd}(E_i, E_j)$

A has eigenvalues $\{\alpha_m, \alpha_M\}$ with eigenvectors $\{u_m, u_M\}$

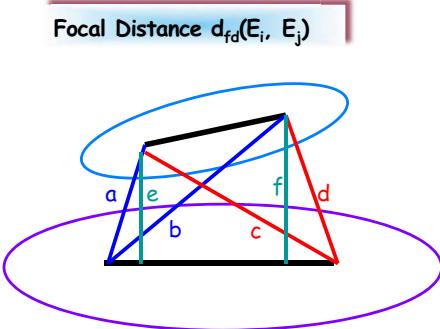
Focal Segment



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EAs in WSNs

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$$\delta_3 = \min\{a, c\} \mapsto e$$

$$\delta_4 = \min\{b, d\} \mapsto f$$

$$d_{fd}(E_1, E_2) = \frac{\delta_1 + \delta_2 + \delta_3 + \delta_4}{4}$$

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EAs in WSNs

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Compute default minimums

$$\begin{aligned}\delta_1 &= \min\{a, b\} \\ \delta_2 &= \min\{c, d\} \\ \delta_3 &= \min\{a, c\} \\ \delta_4 &= \min\{b, d\}\end{aligned}$$

If OG projection of a foci lands on opposite segment, replace default minimum with this OG distance

Focal Distance $d_{fd}(E_i, E_j)$

When $p > 2$

A_1 has eigenvalues $\{\alpha_1 = \alpha_m \leq \dots \leq \alpha_k \leq \alpha_{k+1} \dots \leq \alpha_p = \alpha_M\}$
 A_2 has eigenvalues $\{\beta_1 = \beta_m \leq \dots \leq \beta_k \leq \beta_{k+1} \dots \leq \beta_p = \beta_M\}$

$$d_{fd,(k,k+1)}(E_1, E_2)$$

$$d_{fd}(E_1, E_2) = \frac{\sum_{k=1}^{p-1} d_{fd(k,k+1)}(E_1, E_2)}{p-1}$$

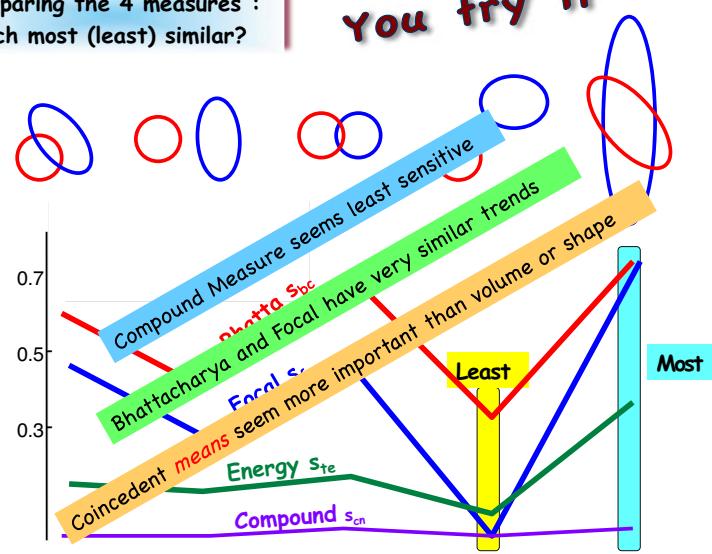
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EAs in WSNs

18

Comparing the 4 measures :
Which most (least) similar?

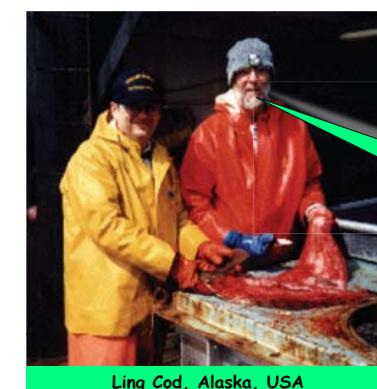
You try it



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EAs in WSNs

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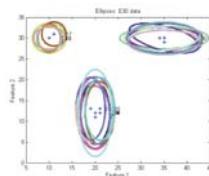


Ling Cod, Alaska, USA

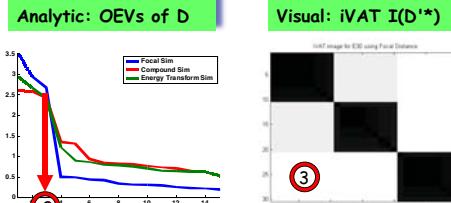
ESAD finds
WSN anomalies

1

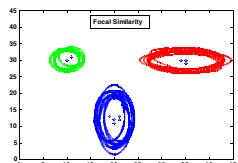
D_{focal} on Data E_{30}



Pre-clustering estimates of c

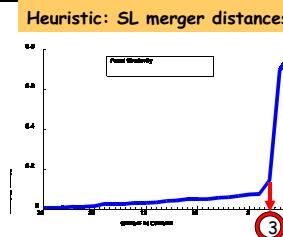


SL 3-partition



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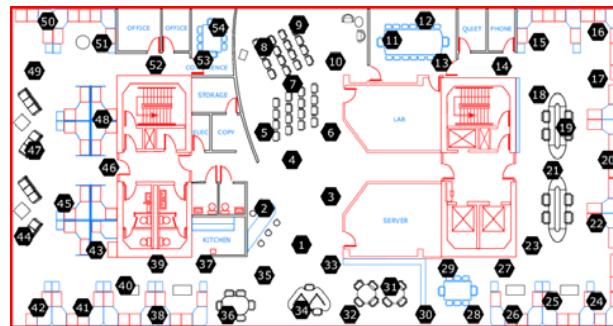
Post-clustering validation of U



EAs in WSNs

2

Intel Berkeley Research Lab (IBRL) monitors lab conditions



2D Data

Collection Period = 18 days, March 1-18, '09

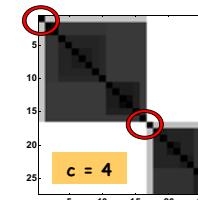
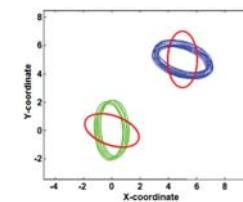
1 summary ellipse built for each IBRL node

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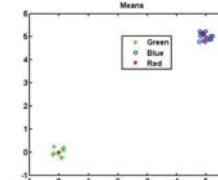
EAs in WSNs

4

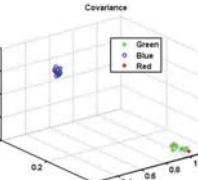
Data $E_{27} = \{10 \text{ green U1 red}\} \cup \{15 \text{ blue U1 red}\}$



Would numerical clustering, FCM, e.g., reveal structure in E_{27} ?



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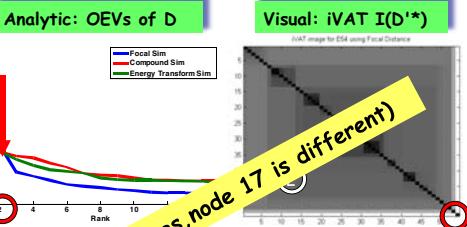
EAs in WSNs

3

No. In these feature spaces FCM would optimize at $c=2$!

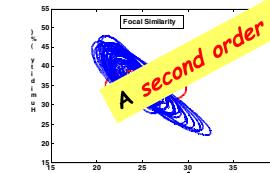
D_{focal} on IBRL Data E_{54}

Pre-clustering estimates of c



A second order anomaly (in the 54 ellipses, node 17 is different)

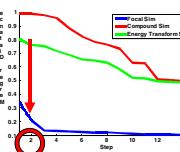
SL 2-partition



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Post-clustering validation of U

Heuristic: SL merger distances



5

Great Barrier Reef Ocean Observation System (GBROOS) to monitor reef environment/sea habitat

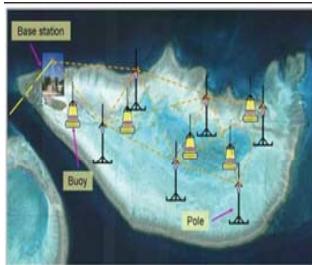


Figure1(a)

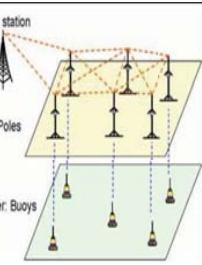


Figure1(b)



Figure1(c)

Land/Sea based Sensor nodes deployed at Heron Island, 2007

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EAs in WSNs

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Heron Island (GBROOS, Australia) Weather Station Data

3D Data

Temperature
Humidity
(Air) Pressure

Collection Period = 30 days,
Feb. 21 to March 22, 2009



Samples = 6 hrs x 6 per/hr
= 37 vectors \Rightarrow 1 ellipsoid

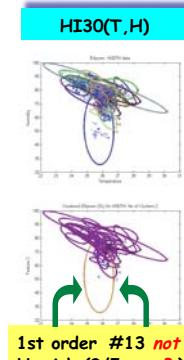
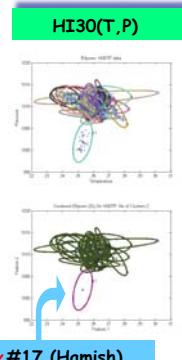
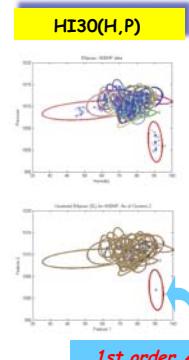
Cyclone Hamish occurred on
March 9, 2009, ellipse #17

A *first order anomaly* (in the
30 ellipses, one is different)



EAs in WSNs

7

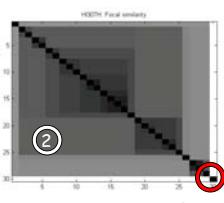
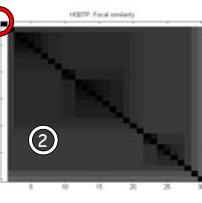
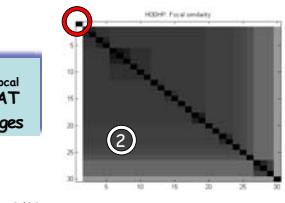


1st order Anomaly #17 (Hamish)

1st order #13 not
Hamish (3/5, no P)

SL
clusters

D_{focal}
iVAT
images

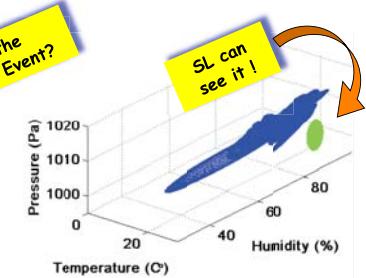
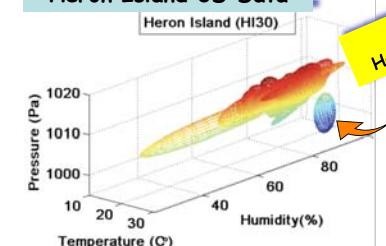


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EAs in WSNs

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Heron Island 3D Data

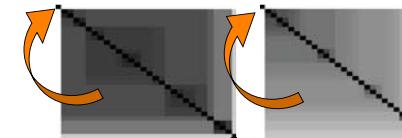


D_{focal} iVAT

D_{Bhatta} iVAT

D_{compound} iVAT

D_{Energy} iVAT



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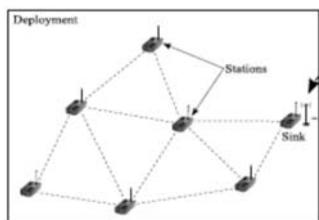
EAs in WSNs

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Focal and Bhatta
Can see it !

Compound and
Energy Cannot !

Grand St. Bernard GSB) WSN



WSN configuration



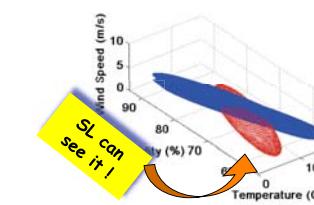
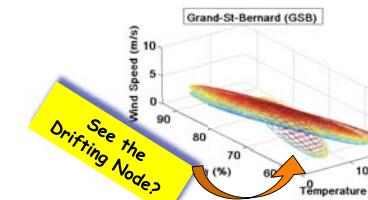
Sensor

17 stations in a mountain pass at Wannengrat, Switzerland.

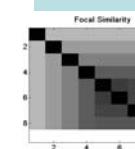
3D Data

{
Temperature
Humidity
Windspeed}

The GSB Data has a **2nd Order** Anomaly : (Drifting Node)

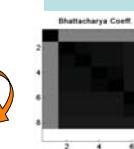


D_{focal} iVAT



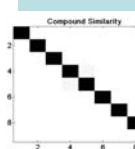
Focal and Bhatta can see it !

D_{Bhatta} iVAT



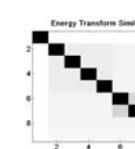
Focal and Bhatta can see it !

$D_{compound}$ iVAT



Compound and Energy cannot !

D_{Energy} iVAT



Conclusions

Three types of WSN anomalies

1st order ($1/E$) = internal node faults

2nd order = whole node faults

Higher order = subtree of nodes faults

Two models for detection that use data-based ellipsoids

DCAD model uses level sets of ellipsoids to capture data

ESAD model uses visual assessment of similar ellipsoids

Four measures of (dis)similarity for ellipsoids

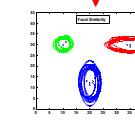
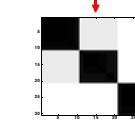
Compound normalized
Transformation energy

Bhattacharya Distance
Focal Distance



Comparing the DCAD and ESAD Models

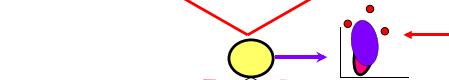
ESAD using
 $E_u = \cup E_{ij}$



$E_g = \Phi(E_{ij})$

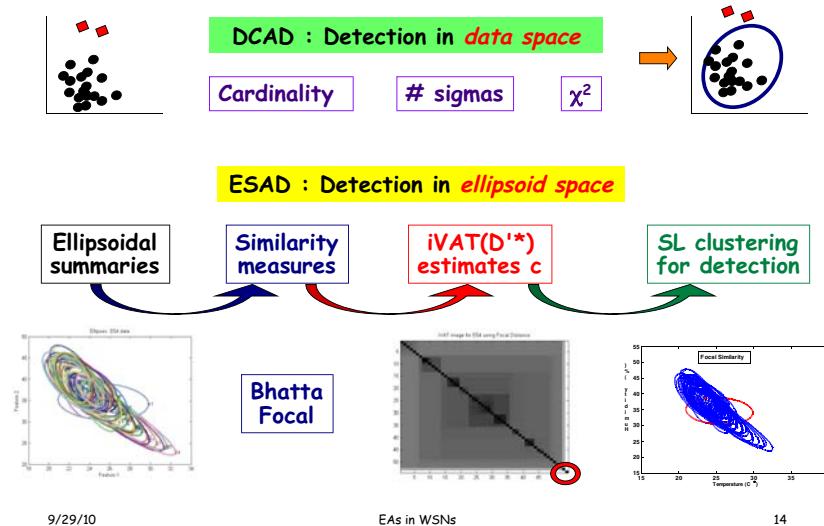


DCAD using
 $E_g = \Phi(E_{ij})$



Local ellipses $\{E_{ij}\}$

Comparing the DCAD and ESAD Models



**Wakker
worden**



**Het is
voorbij !**



Bedankt !

