The Global Price of Market Risk and Country Inflation

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Abstract

The advent of globalisation has meant greater access to foreign stocks for a US investor. The question of whether these are priced locally or globally is thus an important one. In this paper we examine the performance of international asset pricing models, both unconditional and conditional, for the size, book-to-market and momentum portfolios for the US, UK and Japan. We first consider a global asset pricing model where we augment the World CAPM with skewness and kurtosis factors, allowing for time-varying factor risk premiums that are functions of global variables. We then augment these global factors with two sets of local factors, first country-specific unexpected inflation and inflation skewness and then the country-specific Fama-French factors. This allows us to ascertain the global price of market risk factors as well as country-specific factors.

We find that a five factor model which augments the global three factor model with country-specific inflation and inflation skewness and has time-varying risk premiums that are functions of global variables is the best performing model overall. It outperforms the global three factor model augmented by country-specific size and book-to-market factors, even when the size and book-to-market factor premiums are allowed to be time-varying. Our findings suggest that the factor risk premiums for the World index, skewness and kurtosis factors are functions of lagged world market variables, while the inflation risk premiums are functions of term structure variables. We also find, somewhat surprisingly, that the factor risk premiums for the size and book-to-market factors are functions of lagged world market variables, rather than term structure variables, which casts doubt on whether these factors are a proxy for country-specific macro-economic risks.

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1 Introduction

The advent of globalization since the 1970s has meant that US investors now have access to both domestic as well as foreign stocks. This raises the important issue of whether these assets are priced globally or locally, that is can global factors alone price these assets, or is it necessary to introduce country-specific factors. Karolyi and Stulz (2003) state that there is clear evidence that national risk premiums are determined internationally, but less clear evidence that international factors affect the cross-section of expected returns. There is also considerable evidence that the factor risk premiums for both international and local factors are time-varying (Harvey (1991), Ferson and Harvey (1993), De Santis and Gerard (1998), Dahlquist and Sallstrom (2002) and Zhang (2005)).

In this paper we examine the performance of international asset pricing models, both unconditional and conditional, for the size, book-to-market and momentum portfolios for the US, UK and Japan. These assets display considerably greater cross-sectional variation than country indices, and thus pose a challenge to international asset pricing models. We first consider a global asset pricing model where we augment the World CAPM with skewness and kurtosis factors, allowing for time-varying factor risk premiums that are functions of global variables. This model is motivated by Bansal, Hsieh and Vishwathan (1993) who find that non-linear stochastic discount factors out-perform linear ones and is an extension of Harvey and Siddique (2000) and Dittmar (2002) to the context of integrated global markets. We then augment these global factors with two sets of local factors, first country-specific unexpected inflation and inflation skewness and then the country-specific Fama-French factors. The choice of country-specific unexpected inflation is
motivated by Chen, Roll and Ross (1996) and more recently by Errunza and Sy (2005) who also incorporate inflation skewness in the context of an international asset pricing model. The use of country-specific Fama-French factors is motivated by Griffin (2002) who shows that size and book-to-market are local rather than global factors. Our conditioning information is global in nature, motivated by the findings that country risk premiums are determined internationally, and consists of the lagged World index, which represents world market information, and the US 1-month Treasury Bill rate, the US term spread and a measure of convexity of the US yield curve all of which represent global term-structure information. Our analysis differs from Errunza and Sy (2005) in that we incorporate both global and country-specific factors while they focus on country-specific factors alone. We refer to the models with time-varying risk premiums as scaled, while those with constant risk premiums are referred to as unscaled.

We examine unconditional pricing which examines whether the factor model prices the base assets and is closely related to the Hansen-Jagannathan (1997) distance measure. We also examine conditional pricing with respect to the conditioning information, following Ferson and Siegel (2006) and Hansen and Richard (1987) which measures how well the factor models price ‘dynamically managed’ strategies that are functions of the conditioning information, in addition to pricing the base assets. We evaluate unconditional pricing by comparing the optimal factor Sharpe ratio in the presence of conditioning information to the fixed-weight asset Sharpe ratio, and conditional pricing by comparing the optimal factor Sharpe ratio in the presence of conditioning information to the optimal asset Sharpe ratio also in the presence of conditioning information. Our incorporation of time-varying factor risk-premiums
extends the analysis of Ferson and Harvey (1993) and Errunza and Sy (2005) in that we focus on the optimal use of the conditioning information, as opposed to the more ad-hoc modelling of factor risk premiums in those papers. Several studies (Ghysels (1998), Brandt and Chapman (2006)) have found that ad-hoc modeling of factor risk-premiums does not enhance the performance of conditional asset pricing models. In addition we also compute the average expected return error, an average of Jensen’s alpha across assets.

We find that a five factor model which augments the global three factor model with country-specific inflation and inflation skewness and has time-varying risk premiums that are functions of global variables is the best performing model overall. It achieves unconditional pricing for all sets of base assets and conditional pricing for the US and Japanese portfolios. It outperforms the global three factor model augmented by country-specific size and book-to-market factors, even when the size and book-to-market factor premiums are allowed to be time-varying. Our findings suggest that the factor risk premiums for the World index, skewness and kurtosis factors are functions of lagged world market variables, while the inflation risk premiums are functions of term structure variables. We also find, somewhat surprisingly, that the factor risk premiums for the size and book-to-market factors are functions of lagged world market variables, rather than term structure variables, which casts doubt on the assertion that these factors are a proxy for macro-economic risks.

We now analyze the results in more detail. The scaled global three factor model achieves unconditional but not conditional pricing for the US and Japanese portfolios, and achieves unconditional pricing for only the UK size portfolios. This indicates that there are country-specific effects particularly
for the UK that are not captured by our global model. We next consider the performance of the country-specific Fama-French model which augments the World index with country-specific size and book-to-market factors. We find that the unscaled version of this model achieves unconditional pricing on only the Japanese book-to-market portfolios. The scaled version of this model performs much better, achieving unconditional pricing on the UK portfolios as well as the US and Japan. It thus out-performs our global model and further confirms that country-specific effects are important and also that the size and book-to-market factor risk premiums exhibit time-variation which is very important for international asset pricing. However the model does not achieve conditional pricing for any of the base assets, suggesting that it does not fully capture all the country-specific effects.

We next augment the global three factor model with country-specific unexpected inflation and its square (inflation model), following Errunza and Sy (2005) who find that both country-specific inflation and inflation skewness are priced in international markets. The scaled version of this model achieves conditional pricing with respect to the conditioning information for the US and Japanese markets, and unconditional pricing but not conditional pricing for the UK market. It also has considerably lower expected return errors than the global model and thus performs considerably better than it. We also augment our global model with country-specific size and book-to-market factors and find that this model does not achieve conditional pricing for any of the base assets although it does achieve unconditional pricing in all cases. In terms of pricing performance, it is out-performed by the inflation model on all but the UK book-to-market portfolios. It achieves lower expected return errors than the inflation model on all the book-to-market portfolios, but has
higher return errors for all the size and momentum portfolios, except for the US.

We next consider the issue of the size, value and momentum premiums, which are all substantial except for the Japanese momentum premium, confirming the findings of Rouwenhorst (1999) and Chan, Hameed and Tong (2000). The scaled global three factor model achieves between 80% and 90% of the US premiums while the scaled inflation model captures the US size premium exactly, achieves 95% of the US value premium and over-estimates the US momentum premium by 3%. It performs slightly less well for the UK, over-estimating the value and momentum premiums by about 10% and 5% respectively and under-estimating the size premium by about 15%. The performance is better for the Japanese premiums as our scaled inflation model achieves 95% of the size premium, over-estimates the value premium by 5% and achieves 95% of the momentum premium.

We finally consider the issue of time-varying risk premiums and try to assess the importance of these as well as what variables they are correlated with. We first consider only the lagged World index as conditioning information and find that adding scaled skewness and kurtosis factors to the World market factor dramatically improves performance, while the addition of country-specific inflation factors does not lead to much improvement. This suggests that time-variation in skewness and kurtosis risk premiums is important for pricing and that this time-variation is strongly correlated with world market variables, while time-variation in inflation risk premiums is not. In contrast, when we use term-structure variables as conditioning information, we find that adding scaled skewness and kurtosis factors does not lead to much improvement, while adding inflation factors leads to a dramatic
improvement, suggesting that the inflation risk-premiums are functions of term-structure variables, while the skewness and kurtosis premiums are not. We also examine the time-variation in the size and book-to-market premiums and find that these appear to be functions of world market variables, rather than term-structure variables. If these factors were proxies for fundamental country-specific macroeconomic risks\footnote{Recent papers such as Petkova (2006) and Hahn and Lee (2006) suggest that these factors proxy for macroeconomic risk factors for the US} then we might expect that time-variation in their factor risk premiums would be more highly correlated with global term-structure variables rather than global market variables, and thus our findings seems to cast some doubt on whether this is the case.

The rest of the paper is organized as follows. The data and factors are described in Section 2 and the methodology is outlined in Section 3. The results are described in Section 4 and Section 5 concludes.

## 2 Data and Factors

### 2.1 Data

We use monthly equity data from Japan, the United Kingdom and the United States for the period between January 1981 and December 2004. For the U.S. equity data, we use all NYSE, AMEX and NASDAQ files from the Center for Research in Security Price (CRSP) and book value data from Compustat. For other countries, we use US dollar denominated monthly returns (including dividends and capital gains) and market capitalization data obtained from Datastream. We include both listed and delisted firms to mitigate the survivorship bias but exclude all non-common equities and companies listed outside of domestic exchanges. In December 2005 the sample covers non-U.S.
firms consisting of 1,441 in Japan and 1,745 in the United Kingdom. We use the Morgan Stanley Capital International (MSCI) World index as a proxy for the global market portfolio and the CRSP one-month Treasury bill rate as the risk-free rate.

We focus on the representative overlapping momentum strategies for each country that form equally-weighted portfolios by sorting stocks on their past 6-month compounded returns and hold portfolios for 6 months. We exclude all stocks with prices below $5 at portfolio formation as in Jegadeesh and Titman (1993). At the end of each month, the stocks within the top 10% of past returns comprise the ‘winner’ portfolio (M10) and stocks within the bottom 10% of past returns comprise the ‘loser’ portfolio (M01). Toward the end of each month, the overlapping momentum strategies thus consist of six strategies with each starting one month apart. We calculate average monthly portfolio returns of the six strategies as in Rouwenhorst (1998). For the size-sorted portfolios, we sort stocks by their market capitalizations at the time of portfolio formation. For each country, the small size portfolio (‘small’) and the big size portfolio (‘big’) contain stocks with the smallest and largest 10% of market capitalizations relative only to stocks from the same country, respectively. We re-construct size portfolios every 12 months, and do not non-overlap formation periods. We calculate monthly equally-weighted portfolio returns for each of the 12 months following portfolio formation. We also construct country value portfolios by sorting stocks into deciles on the basis of book-to-market equity ratios. For each country sample, the stocks within the top 10 percent of book-to-market equity relative only to stocks from the same country are assigned to the Value portfolio of the country, the bottom 10% of a country to the Growth portfolio. We re-construct value portfolios
every 12 months and calculate monthly equally-weighted portfolio returns for each of the twelve months following the formation of value portfolios.

2.2 Model

The global factors are the return on the World index, a skewness factor which is the square of the return on the World index, a kurtosis factor which is the cube of the return on the World index. The country-specific factors are country-specific unexpected inflation, which is the inflation rate minus its unconditional mean, and the square of country-specific unexpected inflation (inflation skewness) as well as a country-specific size factor and a country specific book-to-market factor (DETAILS HERE). The conditioning instruments are the lagged World index, the lagged return on the U.S 1 month Treasury Bill rate, the difference between the 10 year Treasury Bond and the one year Treasury Bill rate and the difference between the sum of the 1 year and the 10 year yield and twice the 5 year yield which represents the convexity of the yield curve. The scaled global three factor model has the World index, and its skewness and kurtosis as the factors and this model is augmented by both of the inflation factors (inflation model) in one case and by the country-specific Fama-French model in the other case. The model with time-varying risk premiums is referred to as the scaled model while that with constant risk premiums is referred to as the unscaled model.

3 Methodology

In this section we outline our empirical methodology as well as our method for constructing scaled factor models. Detailed formulas are given in the Appendix.
### 3.1 Conditional Moments

All our tests require the estimation of conditional moments of assets and factors and also cross-moments between assets and factors. We estimate these moments from a joint regression of assets and factors. Specifically given asset returns $R_t$, factor returns $F_t$ and a vector of predictive variables $y_{t-1}$, we construct the demeaned version $y_{t-1}^0$ and then run the regression

\[
R_t - r_f = \mu + \beta' y_{t-1}^0 + \epsilon_t \tag{1}
\]

\[
F_t = \nu + \gamma' y_{t-1}^0 + \eta_t
\]

The conditional asset mean $\mu_{t-1} = \mu + \beta y_{t-1}^0$, the conditional factor mean is $\nu_{t-1} = \nu + \gamma y_{t-1}^0$, the conditional second moment of asset returns is $\Lambda_{t-1} = \mu_{t-1}' \mu_{t-1} + \mathbb{E}_{t-1} (\epsilon_t \epsilon_t')$ and the cross-second moment of assets and factors is $Q_{t-1} = \mu_{t-1} \nu_{t-1}' + \mathbb{E}_{t-1} (\epsilon_t \eta_t')$

### 3.2 Factor Mimicking Portfolios

Since the factors need not be traded assets, we construct factor-mimicking portfolios within the space of managed returns.

We define an FMP via the concept of maximal correlation with the factor. In the literature, it is also common to characterize factor-mimicking portfolios by means of an orthogonal projection\(^2\). However, it can be shown that these characterizations are in fact equivalent. We now take the factor-mimicking portfolios themselves as base assets, and consider the space of pay-offs attainable by forming managed portfolios of FMPs. The explicit expressions for the factor-mimicking portfolios are given in Equation A-1.

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\(^2\)This is for example the approach taken in Ferson, Siegel and Xu (2005).
3.3 Unconditional Pricing

Given a set of factors and associated factor-mimicking portfolios as well as predictive instruments our candidate stochastic discount factor is the minimum second moment portfolio \( r^*_F \) of the factor-mimicking portfolios, following Hansen and Richard (1987) and we use the methodology of Ferson and Siegel (2001) to calculate the factor loadings. This leads to a stochastic discount factor of the form \( m_t = b_{t-1} + c'_{t-1}f_t \), where \( f_t \) denotes the set of factor mimicking portfolios and \( c_{t-1} \) denotes the vector of factor loadings, which are potentially nonlinear functions of the predictive variables. The term \( b_{t-1} \) is proportional to \( \phi^0_{t-1} \) in Equation A-3 while the vector of factor loadings is proportional to \( \phi_{t-1} \) in Equation A-4, which are both functions of the conditional moments and hence functions of the predictive variables.

We first evaluate how well the model prices the base assets unconditionally. This is done by comparing the optimal Sharpe ratio of the factors to the fixed-weight asset Sharpe ratios. The optimal factor Sharpe ratio is the optimal Sharpe ratio of the factor-mimicking portfolios, and is different for different sets of base assets. This compares the locations of the managed factor frontier to the fixed-weight efficient asset frontier in mean-standard deviation space. It is possible for the optimal factor Sharpe ratio to be higher than fixed-weight asset Sharpe ratio which indicates that (a portion of) the managed factor frontier is to the left of the fixed-weight asset frontier. In this case the unconditional projection of a dynamic combination of the factors lies on the fixed weight efficient asset frontier and thus from Roll (1977), this projection prices the base assets.

Finally we compute the (annualized) absolute value of the average difference in actual and model-implied expected return, which is our version of
Jensen’s alpha for conditional asset pricing models.

3.4 Conditional Pricing

We then evaluate how well the model prices the assets conditionally, with respect to the conditioning information\(^3\). We use a new measure of specification error for conditional factor models and the outline of the test is as follows.

For given factors \(F_t\), the model mis-specification error is defined as,

\[
\delta_F := \inf \sigma^2(r^*_t - r_t)
\]

(2)

where \(r_t\) spans over the entire factor or factor-mimicking return space. In other words, \(\delta_F\) measures the minimum variance distance between the efficient benchmark return \(r^*_t\) and the return space spanned by the factor-mimicking portfolios. \(\delta_F\) may be interpreted as a measure of model misspecification via the following two results. Specifically, (i) For given set of factors \(F_t\), the model admits a conditional factor structure if and only if \(\delta_F = 0\). In other words, our measure defines a necessary and sufficient condition for a given set of factors to constitute a viable conditional asset pricing model. (ii) Any return in the space of dynamic factor-mimicking portfolios (FMPs) that attains the minimum in also attains the maximum Sharpe ratio in the space spanned by the FMPs. Moreover, we can show that \(\delta_F\) is proportional to the difference in squared Sharpe ratios. In other words, \(\delta_F\) measures the distance between the efficient frontiers spanned by the base assets and by

\(^3\)It is important to note that even if the model prices the assets with respect to the conditioning information, it is not necessarily a true conditional asset pricing model as the true information set is not observable, the so called ‘Hansen-Richard’ critique (Cochrane (2001), Ferson and Siegel (2005)).
the FMPs, respectively\textsuperscript{4}. As a consequence of (i) and (ii), it follows that a given factor model is a true asset pricing model if and only if it is possible to construct a dynamic portfolio of the FMPs that is unconditionally mean-variance efficient in the asset return space. Thus, our condition is an extension of the Gibbons, Ross, and Shanken (1989) test to the case with conditioning information. In fact, the resulting test statistic is similar to a standard Wald test. This allows us to implement our test for a variety of factor models. We consider an extension of the Gibbons, Ross and Shanken (1989) test statistic to the case with conditioning information namely

\[ \Omega = \frac{\lambda^2 - \lambda_F^2}{1 + \lambda_F^2} \]  

(3)

where \( \lambda_s \) is the optimal asset Sharpe ratio in the presence of conditioning information and \( \lambda_F \) is the optimal factor Sharpe ratio in the presence of conditioning information\textsuperscript{5}. The explicit expressions for these Sharpe ratios in terms of the asset and factor moments is derived Equation A-5. Under the null hypothesis that the model prices the asset conditionally our test statistic \( T_\Omega \) is asymptotically distributed as \( \chi^2_{2N} \) where \( N \) is the number of assets. The extra \( N \) degrees of freedom are incorporated as we are asking the model to price managed strategies in addition to fixed weight strategies. This follows from the fact that conditional pricing i.e \( E_{t-1}(m_t R_t) = 1 \) is equivalent to \( E((\theta(z_{t-1})) R_t) = 1 \) for \( \sum \theta_i(z_{t-1}) = 1 \) and thus \( \theta \) may be interpreted as the weights of a dynamic or managed strategy (see Ferson and Siegel (2005)).

\textsuperscript{4}The proofs of these results are available from the authors.

\textsuperscript{5}A similar test statistic is also considered in Ferson and Siegel (2005)
4 Results

We first discuss the performance of the global three factor model where the factors are the return on the World index and the skewness and kurtosis factors, and compare it to the country-specific Fama-French model. We then consider the performance of two five factor country-specific models, namely the global three factor model augmented by a) country-specific size and book-to-market factors and b) unexpected inflation and inflation skewness.

4.1 Performance of the Global Three Factor Model

The performance of the global three factor model allows us among other things, to assess what role country-specific factors could play in the pricing of the country size, book-to-market and momentum portfolios. Table 1 shows the pricing results for both unconditional and conditional pricing, and we first focus on unconditional pricing. We see that for the US the scaled three factor model’s Sharpe ratio is higher than the fixed-weight asset Sharpe ratio for the size and momentum portfolios, and has a $p$-value of 4% based on the Gibbons, Ross and Shanken (1989) test for the book-to-market portfolios, and thus achieves unconditional pricing for all three sets of base assets. The situation is quite different for the UK where the scaled model achieves unconditional pricing only for the size portfolios. In the case of Japan, the scaled three factor model achieves unconditional pricing for all three sets of portfolios. In contrast the scaled one factor model with the return on the World index as the factor does not come close to achieving unconditional pricing for any of the base assets. This shows that skewness and kurtosis factors play an important role in pricing these assets and that the factor risk premiums are time-varying since the unscaled three factor model does not achieve unconditional pricing.
in any of the cases, and in fact under-performs the scaled one factor model in many cases.

For conditional pricing we use the complete set of predictive instruments which include both lagged world market and term structure variables. Our model does not achieve or come close to conditional pricing for any of the base assets indicating that it needs to be augmented with country-specific factors.

We now turn to the country-specific Fama-French model. From Table 2 we see that the unscaled Fama-French model does not achieve unconditional pricing on any of the assets except for the Japanese book-to-market portfolios. Following Griffin (2002) this suggests that a global version of the model would be unable to price the base assets unconditionally as well. The performance of the scaled country-specific model is much better indicating that the factor risk premiums on the size and book-to-market factors exhibit time-variation. The scaled model achieves unconditional pricing for all the US portfolios, and out-performs the scaled global three factor model on the book-to-market portfolios. It also achieves unconditional pricing on all UK portfolios, thus out-performing the global three factor model and showing that country-specific factors are important for pricing these portfolios. It also achieves unconditional pricing on the Japanese portfolios and slightly under-performs the global three factor model. The scaled Fama-French model does not come close to achieving conditional pricing relative to the conditioning information though, suggesting that these factors alone cannot completely price the base assets.

Overall, thus we see that our scaled global three factor model outperforms the unscaled country-specific Fama-French model and performs as well as
the scaled Fama-French model for the US and Japan. The scaled country-specific Fama-French model achieves unconditional pricing for the UK thus out-performing our global model, but none of the models are capable of conditional pricing for any of the base assets. We thus conclude that while country-specific factors are indeed important for pricing our base assets, the country-specific Fama-French factors fail to capture these country specific effects.

4.2 Performance of the Augmented Country-Specific Models

Table 1 shows the pricing results for both unconditional and conditional pricing. We first focus on the performance of the five factor model that augments the global model with country-specific unexpected inflation and its square (inflation model). As we see from Panel (A) for the US, the inflation model achieves conditional pricing, based on the test statistic $T\Omega$ in Section , (and hence unconditional pricing) at the 5% level for the size and book-to-market deciles and at the 1% level for the momentum deciles (see also Figure 1). The three factor model (World+Skewness+Kurtosis) achieves or comes close to unconditional pricing, so the inflation factors help in achieving conditional pricing. For the UK (Panel (B)) the results are not so strong, with conditional pricing not being achieved in any of the three cases. The five factor model does achieve unconditional pricing in all cases (see also Figure 2), but these results provide evidence that the level of country-specific idiosyncratic risk is higher in the UK and thus the portfolios are harder to price. The results for Japan are very similar to that for the US with the five factor model achieving conditional pricing and the three factor model
achieving unconditional pricing (as is evident from Figure 3).

In Table 2 we consider augmenting the global model with the country-specific Fama-French factors. Adding the country specific Fama-French factors to the global three factor model does not lead to a major increase in Sharpe ratios for the US and thus to the model achieving conditional pricing. However for the UK there is a substantial increase in Sharpe ratios indicating that country-specific effects are important, particularly for the UK book-to-market portfolios. In the case of Japan, the model does not achieve conditional pricing for any of the portfolios. Overall the model with country-specific Fama-French factors performs best on the book-to-market portfolios, particularly for the UK where it outperforms the inflation model. However for all the eight other sets of base assets our inflation model outperforms it in terms of pricing performance. This provides clear evidence that country-specific inflation factors are more important in pricing our base assets than the country-specific Fama-French factors. Griffin (2002) finds that the country-specific Fama-French model works better than the global Fama-French model for country-specific pricing and hence taken together we find that our scaled five factor model is the best international asset pricing model for pricing country-specific portfolios.

We next consider Table 3 shows the expected return errors, which is an average of the model alphas, for the global three factor model and the two five factor models. The two five factor models achieve the lowest expected return errors overall. The scaled five factor Fama-French model has lower expected return errors (0.3% and 3.5%) for the US size and book-to-market portfolios while our scaled five factor model has the lowest error for the US momentum portfolios (1.5%). For the UK our scaled five factor model achieves much
lower errors for the size portfolios (6.5% versus 13%). The five factor Fama-French model outperforms it on the book-to-market portfolios (6% versus 8%) and they have almost identical pricing errors for the momentum portfolios (6%). For the Japanese portfolios our scaled five factor model outperforms both the scaled and unscaled Fama-French models, with expected return errors around 7% for the size and book-to-market portfolios and 2% for the momentum portfolios. The scaled three factor model has higher errors in all cases except for the Japanese book-to-market portfolios, confirming the need for a country specific factor in pricing and explaining the average return of these country specific portfolios. As we discussed, this is due to the country-specific idiosyncratic risk in these portfolios which is probably diversified away in the G8 and country neutral portfolios.

We next consider the model-implied size, value and momentum premiums, which are reported in Table 4. These are all substantial except for the Japanese momentum premium, confirming the findings of Rouwenhorst (1999) and Chan, Hameed and Tong (2000). The global scaled three factor model captures between 80% and 90% of the size, value and momentum premiums for the US, which are 10.45%, 10.57% and 9.98% per annum respectively. The scaled five factor model captures the size premium exactly, achieves 95% of the value premium and slightly overestimates the momentum premium by about 3%. The story is very similar for the UK, where the size, value and momentum premiums are 8.27%, 12.59% and 8.84%, although the scaled five factor model over-estimates the value and momentum premiums by about 10% and 5% respectively and under-estimating the size premium by about 15%. The Japanese size and value premiums are comparable to those for the US and UK at 9.85% and 8.29%, but the Japanese
momentum premium is much lower at 2.11%. The scaled three factor model captures about 90% of the size premium, over-estimates the value premium by about 8% and the momentum premium by 2%. Our scaled five factor model achieves 95% of the size premium, over-estimates the value premium by 5% and achieves 95% of the momentum premium. The scaled five factor Fama-French model under-performs our scaled five factor model except for the US momentum premium where it captures 99% of the premium and the UK value premium where it achieves 95% of the premium. The unscaled five factor Fama-French model captures 95% of the US value premium and over-estimates the UK value premium by around 1% and is the best performing model in these two cases.

4.3 Factor Risk Premiums

We now analyze the issue of time-varying factor risk premiums for both the global and country-specific models. Our global predictive variables are of two types, global market variables (lagged World index) and term structure (short rate, term spread and convexity) and our goal is to ascertain how the various factor risk premiums are correlated with these two types of variables. To that end we report the performance of the various scaled models with only the lagged World index as conditioning information (Table 5) and only term structure variables as conditioning information (Table 6). From Table 5 we see that the optimal Sharpe ratio rises quite sharply when the skewness and kurtosis factors are added to the World index, while it rises very little when the inflation factors are added to the global three factor model, except for UK book-to-market portfolios. This shows that the factor risk premiums for the skewness and kurtosis factors are functions of the lagged World
index while evidence for the inflation risk premiums is not so clear. The scaled five factor model does not achieve conditional pricing with respect to the conditioning information for all the Japanese portfolios, but does not achieve unconditional pricing on any of the UK portfolios, suggesting again that time-variation for some of the country inflation risk premiums are not correlated with lagged world market variables. The situation is quite different for the country-specific Fama-French factors whence the optimal Sharpe ratio increases substantially in all cases. This scaled five factor model in fact achieves unconditional pricing on all the UK portfolios and conditional pricing on the UK book-to-market portfolios. The country-specific Fama-French factor premiums thus appear to be functions of world market variables and seem to be most effective in pricing the UK book-to-market portfolios.

In Table 6 the conditioning variables are the term structure variables and here we see the opposite effect. The optimal factor Sharpe ratio jumps dramatically when the country specific inflation factor is introduced in all cases, while only in some cases does the optimal Sharpe ratio increase sharply when the skewness and kurtosis factors are added. This provides strong evidence that the inflation risk premiums are functions of the term structure variables, while also suggesting that only in some cases are the skewness and kurtosis factor premiums correlated with these variables. It is also significant to note that the scaled five factor model does achieve conditional pricing for all the US and Japanese portfolios at the 5% level. This shows that inflation risk premiums are very important for country specific pricing. This also shows that it is relatively easier for our scaled five factor model to achieve conditional pricing with respect to the term structure variables and that adding the lagged World index as conditioning information makes conditional
pricing more difficult. Adding the Fama-French factors to the global three
factor model does not lead to such substantial increases in Sharpe ratios.
This model under-performs the scaled five factor model with country-specific
inflation for all base assets except the UK book-to-market portfolios, and does
not achieve unconditional pricing on the UK size portfolios. This indicates
that the factor risk premiums on the size and book-to-market factors are more
correlated with lagged world market variables than term structure variables.
This casts doubt on whether these factors are in fact proxies for country-
specific macroeconomic risk variables, as Petkova (2006) and Hahn and Lee
(2006), seem to suggest for the US, as if they were then we would expect
their factor risk premiums to be functions of term-structure variables which
capture macro-economic risks rather than world market variables.

5 Conclusion

The advent of globalization has meant that US investors now have greater
access to foreign stocks and the issue of whether these are priced locally and
globally is of importance. This paper examines the ability of international
asset pricing models that have nonlinear factors, both global and country
specific, together with time-varying factor risk premiums that are functions
of global predictive variables, to price size, value and momentum portfolios
in the US, UK and Japan. We first consider a global asset pricing model
where we augment the World CAPM with skewness and kurtosis factors,
which allows us to analyze the global price of market risk factors. We then
augment these global factors with two sets of local factors, first country-
specific unexpected inflation and inflation skewness and then the country-
specific Fama-French factors, to ascertain the global price of these sets of
factors.

We find that a five factor model which augments the global three factor model with country-specific inflation and inflation skewness and has time-varying risk premiums that are functions of global variables is the best performing model overall. It outperforms the global three factor model augmented by country-specific size and book-to-market factors, even when the size and book-to-market factor premiums are allowed to be time-varying. Our findings suggest that the factor risk premiums for the World index, skewness and kurtosis factors are functions of lagged world market variables, while the inflation risk premiums are functions of term structure variables. We also find, somewhat surprisingly, that the factor risk premiums for the size and book-to-market factors are functions of lagged world market variables, rather than term structure variables, which casts doubt on whether these factors are a proxy for country-specific macro-economic risks.

References


APPENDIX

Expressions for the Factor Mimicking Portfolios

For a given factor $F_i^t$ and a set of base assets with returns $R_t$, the factor mimicking portfolio (FMP) $f_i^t$ can be written as

$$ f_i^t = r_f + (R_t - r_f e)\theta_i^{t-1} $$  \hspace{1cm} (A-1)

$$ \theta_i^{t-1} = \Lambda_{t-1}^{-1}(q_i^{t-1} - \kappa_i \mu_t) $$

where $q_t$ is the column of $Q_{t-1}$ corresponding to factor $i$, and $\kappa_i$ is a constant, which is directly related to the unconditional mean of the FMP. In the case where a risk-free asset is present, this constant is not uniquely determined, since the first-order condition arising from maximizing the correlation is independent of that mean.

We now state the expressions for the first and second moments of the factor-mimicking portfolios, which we will need for the explicit characterization of the maximum Sharpe ratio spanned by the factors.

$$ E_{t-1}(f_i^t - r_f e) = Y_{t-1}' \Lambda_{t-1}^{-1} \mu_{t-1} $$  \hspace{1cm} (A-2)

$$ E_{t-1}((f_i^t - r_f e)(f_i^t - r_f e)'') = Y_{t-1}' \Lambda_{t-1}^{-1} Y_{t-1} $$

where $y_{t-1} = (q_i^{t-1} - \kappa_i \mu_{t-1})$ and $Y_{t-1}$ is the matrix whose columns are the $y_{t-1}$. 

27
Factor Loadings and Maximum Sharpe Ratios

We characterize the weights on the mimicking portfolios of the portfolio that attains the maximum Sharpe ratio. These weights are in fact proportional to the factor loadings in the optimal conditional factor model for given choice of factors. The weight on the risk free asset is given by

$$\phi_{t-1}^0 = \frac{1 + H_{F,t-1}^2}{1 + h_F^2}$$  \hspace{1cm} (A-3)

and the vector of weights on the factors is

$$\phi_{t-1} = -\frac{r_f}{1 + h_F^2} \left[ Y_{t-1}' \Lambda_{t-1}^{-1} \Sigma_{t-1} \Lambda_{t-1}^{-1} Y_{t-1} \right]^{-1} Y_{t-1}' \Lambda_{t-1}^{-1} \mu_{t-1}$$  \hspace{1cm} (A-4)

The conditional moments are defined in Section 3 and $h_F^2$ is the maximum unconditional squared factor Sharpe ratio which is the unconditional average of the squared conditional factor Sharpe ratio, $H_{F,t-1}^2$. The squared conditional factor Sharpe ratio $H_{F,t-1}^2$ is given by

$$H_{F,t-1}^2 = \mu_{t-1}' \Lambda_{t-1}^{-1} Y_{t-1} [ Y_{t-1}' \Lambda_{t-1}^{-1} \Sigma_{t-1} \Lambda_{t-1}^{-1} Y_{t-1} ]^{-1} Y_{t-1}' \Lambda_{t-1}^{-1} \mu_{t-1}$$  \hspace{1cm} (A-5)
Table 1: Performance of Unscaled and Scaled Models

In this table we provide the ex-post performance measures for our scaled and unscaled models on the ten size, book-to-market (BM) and momentum (MOM) portfolios for the US (Panel A), the UK (Panel B) and Japan (Panel C). The models considered are the one factor model where the factor is the return on the World index, the three factor model where the return on the World index and the square and the cube of the return on the Index are the factors (3F), the four factor model that adds country specific unexpected inflation (4F) and the five factor model which adds the square of country specific unexpected inflation. The conditioning variables for the scaled models are the lagged World index, the 1 month T bill rate, the term spread and the convexity of the yield curve. We report the
fixed weight and optimal Sharpe ratios for the assets (SRAF and SRAO respectively) and that for each of the unscaled and scaled factor models (FFSRF and FFSRO and xFSRF and xFSRO for x=3,4 and 5 respectively). A * denotes significance for conditional pricing, which is based on the test statistic in Section , at the 5% level and ** denotes significance at the 1% level.
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**Table 2: Performance of Unscaled and Scaled Country Specific Models**

In this table we provide the ex-post performance measures for our scaled and unscaled models on the ten size, book-to-market (BM) and momentum (MOM) portfolios for the US (Panel A), the UK (Panel B) and Japan (Panel C). The models considered are the country specific Fama-French three factor model where the return on the World index together with a country specific size factor and a country specific book to market factor (FF), the five factor inflation model (5FI) which adds country specific unexpected inflation and the square of country specific unexpected inflation, and the five factor model which has the three global factors together with a country specific size factor and a country specific book to market factor (5FF). The conditioning variables for the scaled models are the lagged World index, the 1 month T bill rate, the term spread and the convexity of the yield curve. We report the fixed weight and optimal Sharpe ratios for the assets (SRAF and SRAO respectively) and that for each of the unscaled and scaled factor models (FFSRF and FFSRO, 5FISRF and 5FISRO and 5FFSRF and 5FFSRO respectively). A * denotes significance for conditional pricing at the 5% level and ** denotes significance at the 1% level.
### Table 3: Expected Return Errors for the Scaled and Unscaled Models

In this table we provide the expected return errors (RE) which is the difference between realized and model-implied average return for our scaled and unscaled models in percent per year, on the ten size, book-to-market (BM) and momentum (Mom) portfolios for the US (Panel A), the UK (Panel B) and Japan (Panel C). The models considered are the global three factor model where the return on the World index and the square and the cube of the return on the Index are the factors (3F), the four factor model that adds country specific unexpected inflation (4FI), the five factor inflation model (5FI) which adds the square of country specific unexpected inflation, and the five factor model which has the three global factors together with a country specific size factor and a country specific book to market factor (5FF). The conditioning variables for the scaled models are the lagged World index, the 1 month T bill rate, the term spread and the convexity of the yield curve. We report average return across each set of base assets and the return errors for each of the unscaled and scaled factor models with URE denoting return errors for the unscaled factor models and SRE denoting return errors for the scaled factor models.

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Table 4: Model-Implied Premiums for the Scaled and Unscaled Models

In this table we provide the model-implied size, value and momentum premiums for our scaled and unscaled models in percent per year, for the US (Panel A), the UK (Panel B) and Japan (Panel C). The models considered are the global three factor model where the return on the World index and the square and the cube of the return on the Index are the factors (3F), the four factor model that adds country specific unexpected inflation (4FI), the five factor inflation model (5FI) which adds the square of country specific unexpected inflation, and the five factor model which has the three global factors together with a country specific size factor and a country specific book to market factor (5FF). The conditioning variables for the scaled models are the lagged World index, the 1 month T bill rate, the term spread and the convexity of the yield curve. For each country we report the realized size, value and momentum premiums and the model-implied premiums for each of the unscaled and scaled factor models with U denoting the premium for the unscaled model and S denoting premium for the scaled model.
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**Panel A: US**

| Size:| 1.47 | 2.00 | 0.48  | 1.19  | 1.25   | 1.25   | 1.48   |
| BM:  | 1.69 | 1.81 | 0.49  | 1.03  | 1.43   | 1.62   | 1.71** |
| Mom: | 1.77 | 2.17 | 1.19  | 1.42  | 1.50   | 1.53   | 1.80   |

**Panel C: UK**

| Size:| 0.83 | 1.09 | 0.35  | 0.67  | 0.74   | 0.94** | 0.77   |
| BM:  | 0.81 | 1.00 | 0.30  | 0.66  | 0.79   | 0.80*  | 0.83** |
| Mom: | 0.59 | 0.97 | 0.31  | 0.62  | 0.66   | 0.73** | 0.66   |

**Panel D: Japan**

Table 5: Performance of Scaled Models using the Lagged World Index as Conditioning Information

In this table we provide the ex-post performance measures for the scaled models on the ten size, book-to-market (BM) and momentum (MOM) portfolios for the US (Panel A), the UK (Panel B) and Japan (Panel C), when the factor risk premiums are assumed to be functions of the lagged World index alone. The models considered are the one factor model (1F) with the return on the World index as the factor, the global three factor model where the return on the World index and the square and the cube of the return on the Index are the factors (3F), the four factor model that adds country specific unexpected inflation (4FI), the five factor model which adds the square of country specific unexpected inflation (5FI) and the five factor model which has the three global factors together with a country specific size factor and a country specific book to market factor (5FF). We report the fixed weight and optimal Sharpe ratios for the assets (SRAF and SRAO respectively) and that for each of the scaled factor models (FSRO). A * denotes significance at the 5% level and ** denotes significance at the 1% level.
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**Table 6: Performance of Scaled Models using Term Structure Variables as Conditioning Information**

In this table we provide the ex-post performance measures for the scaled models on the ten size, book-to-market (BM) and momentum (MOM) portfolios for the US (Panel A), the UK (Panel B) and Japan (Panel C), when the factor risk premiums are assumed to be functions of term structure variables. The models considered are the one factor model (1F) with the return on the World index as the factor, the global three factor model where the return on the World index and the square and the cube of the return on the Index are the factors (3F), the four factor model that adds country specific unexpected inflation (4FI), the five factor model which adds the square of country specific unexpected inflation (5FI) and the five factor model which has the three global factors together with a country specific size factor and a country specific book to market factor (5FF). The conditioning variables for the scaled models are the 1 month T bill rate, the term spread and the convexity of the yield curve. We report the fixed weight and optimal Sharpe ratios for the assets (SRAF and SRAO respectively) and that for each of the scaled factor models (FSRO). A * denotes significance at the 5% level and ** denotes significance at the 1% level.
Figure 1: US Asset and Factor Frontiers

This figure shows the fixed weight and optimal asset and factor frontiers for the three sets of base assets. The heavy solid line is the optimal asset frontier, the circles denote the optimal factor frontier, the dashed line is the fixed weight asset frontier while the dotted line is the fixed weight factor frontier. The five factors are the return on the World index, the square and cube of the return on the World index, unexpected inflation for the US and its square. The conditioning variables for the scaled models are the lagged World index, the 1 month T bill rate, the term spread and the convexity of the US yield curve. Panel (A) considers the US size deciles, Panel (B) the US book-to-market deciles and Panel (C) the US momentum deciles.
Figure 2: UK Asset and Factor Frontiers

This figure shows the fixed weight and optimal asset and factor frontiers for the three sets of base assets. The heavy solid line is the optimal asset frontier, the circles denote the optimal factor frontier, the dashed line is the fixed weight asset frontier while the dotted line is the fixed weight factor frontier. The five factors are the return on the World index, the square and cube of the return on the World index, unexpected inflation for the UK and its square. The conditioning variables for the scaled models are the lagged World index, the 1 month T bill rate, the term spread and the convexity of the US yield curve. Panel (A) considers the UK size deciles, Panel (B) the UK book-to-market deciles and Panel (C) the UK momentum deciles.
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Figure 3: Japanese Asset and Factor Frontiers