Capital Asset Pricing with a Stochastic Horizon

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Abstract

In this paper we present empirical tests of an extended version of the Capital Asset Pricing Model that replaces the single period horizon with a probability distribution over different horizons. Adopting a simple parameterization of the probability distribution of the length of the horizon, we estimate the parameters of the distribution as well as the parameters of the CAPM. We find that the extended model is not rejected, and that the estimated stock turnover rate rises from 82.8% in the period 1926-62 to 266.4% in the period 1963-2009. We also find that long horizon betas are determined by identifiable firm characteristics as well as by short horizon betas

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1 Introduction

The capital asset pricing model of Sharpe (1964), Lintner (1965) and Mossin (1966) is the oldest and perhaps the most elegant and intuitive of the asset pricing paradigms. It is this intuitive elegance that has allowed the model to survive newer and more sophisticated paradigms in the classroom, the courtroom and the boardroom,[1] despite its perceived lack of empirical success and general rejection among academics. After early acceptance,[2] the model has come to be regarded by many academics as a useful classroom example that fails to capture the complexity of the real world relation between risk and return. Indeed rejection of the CAPM has become a touchstone for assessing the power of statistical tests of asset pricing models, and for empirical purposes the standard asset pricing paradigm has become the Fama-French three factor model which can be justified as a variant of either the ICAPM or Ross' Arbitrage Pricing Theory.[3] In this paper we show that the classical capital asset pricing model can be made empirically as well as theoretically relevant by a simple change in its assumptions. Instead of assuming a collection of mean-variance investors with a single horizon, we assume a single representative agent with mean variance preferences but with a stochastic horizon: this ensures that the agent is concerned with the distribution of wealth at several different future dates on which assets may be liquidated and returns realized. By parameterising the probability distribution of the liquidation date we are able to estimate the model parameters using a standard GMM approach. When the model is interpreted in terms of a stationary overlapping generations model of investors the probability distribution of liquidation dates yields an estimate of the implied stock turnover rate.

Since the CAPM is a single period model which does not specify the length of the time period or horizon for which the investor makes decisions, it is necessary to make some assumption about this for empirical purposes. The assumption of a stochastic horizon, which is critical to our analysis, contrasts with the usual assumption made in applying the single period CAPM, that the horizon is

[1] Graham and Harvey (2001) report that 73.5% of firms in their survey 'always or almost always use' the CAPM, and its use was much more common among large firms.

[2] Black, Jensen and Scholes (1972), Fama and MacBeth (1973).

[3] See Fama and French (1996).

fixed and exogenous. In by far the majority of asset pricing tests the horizon is taken as one month and returns are measured over monthly intervals. This appears to have been essentially an accident of history, occasioned by the monthly data that were made available in the first CRSP tapes. By the time that daily data became available it was realized that the theoretical advantages associated with the use of high frequency data are more than offset by the difficulties of measuring first and second moments with high frequency data due to non-synchronous trading, bid-ask spreads and other micro-structure related phenomena.

If security returns are distributed *iid* then the expected n period (one plus) rate of return will be equal to the n^{th} power of the expected 1 period rate of return. However, as Levhari and Levi (1977) showed, the relation between betas measured over one period and betas measured over longer intervals is more complex, and if returns are not serially independent then the length of the period becomes critical to the measurement of both returns and betas and therefore to the relation between them that is implied by the CAPM. In this paper, by parameterising the probability distribution of the length of the horizon, we allow the data to reveal the horizon or the distribution of horizons over which the model holds.

We are by no means the first to consider the effect of the time interval over which returns are measured on the fit of the CAPM. As long ago as 1977 Levhari and Levy complained that in various empirical tests of the CAPM the investment horizon (return interval) was selected arbitrarily. They showed that if returns are serially independent and homoscedastic then the beta coefficient computed using n period returns will differ systematically from that computed using m period returns and this will affect tests of the CAPM. [4] This result was generalised by Longstaff (1989) who also showed that the continuous time CAPM becomes a multi-factor model when discrete period returns are used if there is time variation in expected returns and risk. Lee (1976) considered a generalization of the CAPM to a single but unknown investment horizon under the assumption that periodic returns were serially independent, and showed how to estimate the horizon. Lee *et al.* (1990) develop a model in which returns are *iid* but investors have different horizons.

In a paper that is close in spirit to this, Handa *et al.* (1993) show that while the standard CAPM is rejected using monthly returns on size ranked portfolios, it is not rejected by a standard

[4] If $m > n$ and $\beta_n > 1(< 1)$, then $\beta_m > (<)\beta_n$. See also Hawawini (1983).

F-test when annual returns on the same portfolios are used.[5] Kothari *et al.* (1995) provide further evidence of a statistically significant risk premium associated with beta when annual data is used to compute beta. Finally, Jagannathan and Wang (2007) show that the Consumption CAPM explains the returns on 25 Fama-French size and book to market portfolios as well as does the FF three factor model, when returns are measured on a calendar year basis. They motivate the calendar year assumption by the observation that many decisions are made at the end of the year because of Christmas, bonuses, and end-of-year capital gains tax considerations. This paper differs from these papers by allowing the data themselves to determine the appropriate length of the period for the CAPM relation to hold, rather than specifying it *a priori*.

In recent unpublished work Beber *et al.* (2011) develop a model in which investors have mean variance utility functions and heterogeneous horizons and there are stochastic transaction costs, but the focus of their paper is on transaction costs and the clientele effects that develop when investors have different horizons. Kamara *et al.* (2012) argue that there are long and short horizon risk factors arising from the different autocorrelation properties of the factors. They find evidence that the long horizon factors (which include the market return) are priced when betas are estimated using annual returns, but not when using monthly returns.

The model in this paper is similar to that of Beber *et al.* (2011) in that it allows for heterogeneous investor liquidation dates, but unlike their model ours assumes that all investors are ex-ante identical and are subject to stochastic liquidation. Their model assumes that returns are *iid*, and ours does not impose this assumption. Finally, while these authors consider only investors with horizons of 1 month and 20 years, we are able to consider multiple horizons. When we interpret the model as an overlapping generations model with a continuum of investors we are able to calculate the share turnover rate implied by the model parameters.

In Section 2 we motivate our approach by showing how measured returns and betas vary systematically with the length of the return interval, and with the characteristics of the underlying firms. In Section 3 we show how the length of the return interval affects tests of the CAPM. In Section 4 we present our simple version of the CAPM with stochastic liquidation. In Section 5 we report the empirical results of tests of the model and estimate the distribution of liquidation dates. Section 6 concludes. For most of our results, we rely on the Fama-French size and book-to-market sorted portfolios as our test assets since these have become almost canonical in tests

[5] See also Handa, *et al.* (1989)

of asset pricing models. However, we also present results for the Fama-French industry portfolios both for robustness and because differences in costs of capital across industries are likely to be more important for capital allocation than differences across Size and Book-to-market sorted portfolios since small Size and high Book-to-Market are typically transient firm characteristics.

2 Returns, betas, and the measurement interval

In this section we show first how average returns and betas vary empirically with the length of the return interval, and then relate the variation of beta with the horizon to firm and portfolio characteristics.

2.1 Horizon effects for the Fama-French portfolios

We motivate our analysis by showing first how mean returns and betas on the Fama-French Size and Book-to-market sorted portfolios are both affected by the choice of time interval over which returns are measured. The τ month return starting in month t, R_t^τ, is defined by compounding the one month returns, R_t:

$$R_t^\tau \equiv \Pi_{k=0}^{k=\tau-1}(1 + R_{t+k}) - 1$$

For each of the 25 Size and Book-to-market sorted Fama-French portfolios, τ month betas, β_j^τ, $j = 1, \cdots, 25$, were estimated by Ordinary Least Squares for $\tau = 3, 6, 9, \cdots, 24$ months using overlapping monthly observations for the period January 1926 to December 2009. The estimation equation using τ month returns is:

$$R_{j,t}^\tau = \alpha^\tau + \beta_j^\tau R_{m,t}^\tau + e_{jt}^\tau$$

where $R_{m,t}^\tau$ is the τ month return on the market portfolio starting in month t.

Panel A of Table 1 reports for each of the 25 portfolios the 1 month betas and the ratio of the τ month beta to the 1 month beta for each τ. If the betas were independent of the return interval the entries after the 1-month beta would all be equal to unity. Instead, we see that there is a tendency for the betas to rise with the return interval. However, this effect is not the same for all portfolios. For the 3 and 12 month return intervals for example, there is a clear tendency for the betas of high book to market portfolios to be high relative to the 1 month betas, and this effect is concentrated in the small firm portfolios and to a lesser extent in the large firm portfolios; it is much less pronounced for the medium size firm portfolios.[6] In contrast, for the 24 month return interval there is a clear tendency for the betas of low book to market firm portfolios to be below their 1 month equivalents, and this is most pronounced for the small firm portfolios. In some cases these effects are very large. For example, the beta of the small low book to market firm portfolio declines by 10% (25%) relative to the 1 month value as the return interval is increased to 12 (24) months. In contrast, the beta of the small high book to market firm portfolio increases by 26% (18%) as the return interval is increased from 1 month to 12 (24) months.

However, it is possible that the variation in betas with the horizon is due mainly to sampling variation. To assess this, the distribution of the ratio of τ month betas to one month betas was bootstrapped from 10,000 samples generated by sampling at random from the joint vectors of monthly market and portfolio returns. By construction, these generated samples will be serially independent. For each sample, betas were estimated for all horizons and the ratios to the one month betas computed. Panel A of Table 2 reports the 10^{th}, 50^{th}, and 90^{th} percentiles of the bootstrapped distribution of the ratios of τ month betas to one month betas. The medians of the bootstrapped ratios tend to be close to unity for up to 6 months and almost all the 3-month empirical ratios and about half of the 6-month empirical ratios are outside the 10^{th} and 90^{th} percentiles of the bootstrapped distribution. For longer horizons the sampling variability disguises any systematic variation in betas with the length of the horizon. The fact that sampling variability alone can cause large variation in beta estimates with the horizon means that it is important in our empirical tests of the CAPM to allow for errors in the estimation of betas. We shall do this within a GMM framework.

It follows from the definition of the τ month return that, if 1 period returns are independent, then the expected one plus τ period return is equal to the expected one plus 1 period return raised

[6] Longstaff (1989) shows that the ranking of betas for different firms may reverse as a function of the interval over which returns are measured.

to the power τ:

$$E[1+R^\tau] \quad = \quad \{E[1+R^1]\}^\tau$$

Raising both sides of the equation to the power $12/\tau$, this implies that the annualized expected τ month return is given by:

$$\{E[1+R^\tau]\}^{12/\tau} = \{E[1+R^1]\}^{12} \tag{1}$$

so that the annualized expected return is independent of the return interval, τ, and the ratio of the annualized expected τ-month return to the annualized expected 1-month return is equal to unity.

It is natural to consider the sample equivalent of relation (1) by comparing the annualized average τ-month return to the annualized average 1-month return. The average τ month return is defined by $\bar{R}^\tau = \frac{1}{T-\tau}\Sigma_{t=1}^{T-\tau} R_t^\tau$. The *annualized* average τ month return, ${}^a\bar{R}^\tau$, is defined by $1 +^a \bar{R}^\tau \equiv (1+\bar{R}^\tau)^{(12/\tau)}$.

Panel B of Table 1 reports the annualized average of the 1-month returns for the 25 portfolios, and the ratios of the annualized average τ-month returns to the annualized averages of the 1-month returns. Panel B of Table 2 reports statistics from the bootstrapped distribution of this ratio. The medians of the bootstrapped ratios, which are calculated from serially independent returns, are almost all very close to unity. In contrast, the empirical ratios tend to be above unity for the 6 month horizon and even more so for the 3 month horizon. Almost all of the ratios for the 3-month horizon are greater than the 90^{th} percentile of the bootstrapped distribution which is consistent with positive short run auto-correlation in portfolio returns. For longer return intervals the ratios of the annualized average returns tend to be closer to unity and are well within the range of sampling variability.

2.2 Determinants of relative betas for different horizons

Betas that are calculated for the same security using different return intervals may differ for at least three reasons in addition to sampling variability. First, even if returns are independently and identically distributed over time, the multiplicative relation between *n*-period returns and *1*-period returns implies a complex relation between *1*-period betas, β^1, and *n*-period betas, β^n. Levhari and Levy (1977) show that if the CAPM holds over either *1* period or *n* periods, β^n will exceed (be less than) β^1 if and only if β^1 is greater (less) than unity.

The second reason, identified by Dimson (1979) and Scholes and Williams (1977), is purely empirical: betas that are estimated using short horizon data may be biased due to thin trading which causes the observed (closing) prices of the security and the market portfolio to be non-synchronous. Since the (value-weighted) market portfolio consists mainly of the frequently traded stocks of large companies, the effect is typically to bias down the betas of small stocks that are infrequently traded.

The third reason for differences between betas calculated using long and short period returns is that there are systematic differences across firms in the timing of information release and in the speed of adjustment of stock prices to market-wide information that give rise to complex patterns of cross-temporal covariances and thereby affect betas calculated over different return intervals. Lo and Mackinlay (1990) show that the returns on large firm stocks lead the returns on small firm stocks, and Brennan *et al.* (1993) show that the speed of adjustment to market-wide information depends also on the number of analysts following the stock holding firm size constant.[7]

To confirm that there are differences in betas at different horizons due to differences in the fundamental characteristics of stocks we estimate each year from 1969 to 2010 for all common stocks (firms with share code 10 or 11 and stock price higher than 5) on the CRSP tape with at least 36 monthly returns over the past 5 years, a *1 month* beta, a *6 month* beta, and a *12 month* beta using compounded overlapping returns over the past 60 months. For each firm the ratio of the two long horizon betas to the *1 month* beta is calculated and regressed on a set of regressors that are chosen to capture delays and time-variation in the rate of information release. The first regressor is firm *Size* which is the market value of the equity: we expect that the long horizon beta ratios will be declining in firm size because of the evidence cited above that large firms release more timely information.[8] The second regressor is the *Book-to-market* ratio: we hypothesize that the ratios of long to short return interval betas will be declining in the Book-to-market ratio because the news about value firms is primarily about current cash flows and we expect this to become available to

[7]Brennan and Copeland (1988) show that the betas of firms that split their stock increase by around 18% if daily returns are used, and Wiggins (1992) shows that the effect is attenuated if returns over longer time intervals are used to compute betas, which is consistent with stock splits increasing the speed of adjustment to information.

[8]Handa *et al.* (1989) have previously shown that betas of small firms increase and betas of large firms decrease with the measurement interval.

the market is a relatively continuous fashion, whereas the news about growth firms is primarily about the development of new growth opportunities and we expect this to be relatively lumpy. Both *Size* and *Book-to-market* ratio are measured at the end of the last fiscal year. The third variable is the *Seasonality* of earnings. We anticipate that firms whose earnings are more seasonal will have seasonal patterns in information release that will increase the ratio of long horizon to short horizon betas. *Seasonality* is measured by the average over the past 5 years of the ratio of the highest quarterly earnings per share for each year to the average quarterly earnings per share for that year. The fourth variable, following Brennan *et al.* (1993), is the number of analysts following the firm, which is given by the number of analysts issuing one year earnings forecasts in the previous year from IBES. These variables are subjected to a logarithmic transformation. To take account of the 'mechanical' tendency of *1 month* betas to increase (decrease) with the return horizon if the *1 month* is greater (smaller) than unity discussed by Levhari-Levi (1977), we include the estimated *1 month* beta. This requires two adjustments. First, the Levhari-Levi effect is non-linear since it predicts that long period betas will exceed (fall short of) short period betas if the beta is greater than unity. Therefore, in addition to including an estimate of β^1, we include a linear function of the estimated β^1 when this exceeds unity, $\mathbf{1}_{\beta^1} > 1 + max[\beta^1 - 1, 0]$. The Levhari-Levi analysis implies that the coefficient of the *1 month* beta will be negative, while the coefficient of the truncated β^1 variable will be positive. Secondly, it is necessary to take account of the fact that β^1 is measured with error since this will induce spurious correlation with the dependent variable, the ratio of β^n to β^1. Therefore we instrument the estimate of the β^1 regressor using firm *Size*, *Book-to-market* ratio, book leverage and monthly volatility estimated over the past 5 years.

Each year from 1969 to 2010,[9] the ratios of estimated betas, $\hat{\beta}^6/\hat{\beta}^1$ and $\hat{\beta}^{12}/\hat{\beta}^1$, are regressed on the independent variables described above. Following Fama and MacBeth (1973), the coefficients of the cross-section regressions are averaged over time, and standard errors are computed adjusting for serial correlation in the annual coefficient estimates. The results, which are reported in Table 3, show a strong *Size* effect which is consistent with the prior studies cited above: the betas of large firms tend to increase with the horizon at a slower rate than do the betas of small firms. The *Book-to-market* effect is negative in all the regressions but is not significant until *Analysts* is introduced: however, the introduction of this and other variables does not have a dramatic effect on the coefficient of *Book-to-market*. Considering these two variables alone, we see that they imply that *Large* size firms and *Value* firms will have relatively less systematic risk when their risk is

[9]When the number of analysts is included in the regression the sample starts in 1984.

measured at long horizons. Thus, if the 'true' horizon is, for example, one year, then measuring risk and return at the one month horizon will tend to understate the risk of small firms and growth firms since their *12-month* betas tend to be greater than their *1-month* betas: this is consistent with the apparent outperformance of small firms relative to standard CAPM benchmarks that measure risk at the *1-month* horizon, but is inconsistent with the apparent outperformance of value firms.[10]

The coefficients of the other variables are consistent with the discussion above. The betas of firms with *seasonal* earnings tend to increase as the horizon is extended, and the effect for the 12-month beta ratio is about twice as large as for the 6-month beta ratio. We also find that the more analysts that follow the firm, the smaller is the change in beta as the return horizon is extended.

Table 4 reports the results of similar regressions in which the dependent variable is the 12-month beta and the independent variables are the firm characteristics, *Size* and *Book-to-market* and the betas with respect to the Fama-French HML and SMB factors, as well as the one month beta. Panel A reports the results for regressions using individual securities.[11] The first regression shows that *Size* and *Book-to-market* are significant determinants of the 12-month beta after allowing for the effect of the 1-month beta, and the sign of the coefficients of these variables are consistent with the sign of the coefficients in Table 3: *Small* and High *Book-to-market* firms have low 12-month betas given their *1-month* betas. The second regression shows a modest increase in explanatory power when the SMB and HML betas are substituted for these firm characteristics, and the coefficients of these betas are consistent with the coefficients of the corresponding physical characteristics.

Panel B reports for the 25 Fama-French portfolios the results of regressing the 12-month beta on the Fama-French HML and SMB betas as well as $\hat{\beta}^1$. Note that while the positive coefficient of β_{SMB} is consistent with the negative coefficient of *Size* in Panel A and Table 3 (small firms have lower *12-month* betas), the positive coefficient of β_{HML} is inconsistent with the negative coefficient on *Book-to-market* in Panel A and Table 3. The third column shows that the SMB and HML betas alone explain over 90% of the variance of the 12-month betas. The fifth column shows that the coefficient of $\hat{\beta}^1$ is insignificant in the presence of the SMB and HML betas, so that the 12-month betas for these portfolios are close to being spanned by the HML and SMB betas, and inclusion of the 1-month beta does not improve the spanning. This implies that *if* the CAPM held with a 12-month horizon, then we would expect to find that 12-month average returns would also be well

[10]However, see the discussion of Panel B of Table 4 below.
[11]Similar results are obtained if the one month beta is instrumented.

explained by the Fama-French model loadings on SMB and HML estimated from monthly data. Moreover, we would expect the coefficients of both SMB and HML betas to be positive as in fact they are. Whether these betas would also explain the cross-section of 1-month average returns would depend on the relation between 1-month and 12-month average returns.

3 A preliminary analysis of the effects of the return interval on capital asset pricing

We have seen that the interval over which returns are measured has large effects on both the annualized return on a portfolio of stocks and on its measured beta coefficient, and that these effects vary across portfolios. In other words, the return interval affects both the left and right hand sides of standard CAPM regressions and therefore presumably affects the fit of the regression.

To illustrate this phenomenon and to assess its potential importance, Panel A of Table 5 reports the results of ordinary least squares cross section regressions of average portfolio excess returns on betas for the period January 1926 to December 2009 from the equation:

$$\overline{R_j - R_F} = a + \lambda\beta_j$$

and Figure 1 plots annualized monthly returns and annual returns against the predicted returns from regressions that use either annualized monthly returns or annual returns, and betas calculated from either monthly returns or annual returns.

In the first row of Panel A both average returns and betas are calculated from 1 month returns: the R^2 is less than 10%. The regression slope and the mean absolute error from the regression are annualized by multiplying by 12. The annualized slope is 5.2% and the mean absolute error is 2.6% per annum. The intercept is 4.0% *per annum.* These estimates are typical of CAPM estimates for similar sample periods. [12]

In the second row the dependent variable remains the average monthly return but the independent variable is now the annual beta, obtained by regressing overlapping 12 month compounded returns on the corresponding market return. Replacing the monthly beta by the

[12] Fama and French (1992), Kothari *et al.* (1995).

annual beta raises the R^2 of the cross section regression to 48.4% and the annualized mean absolute error falls to 1.5%. It is clear that the returns line up much more closely with the annual betas than they do with the monthly betas. Moreover the intercept is now only 0.3% *per annum*.[13]

In the third row average 12 month returns are regressed on the monthly betas. Now the R^2 is 13.0% and the mean absolute error is 2.7% which is similar to the results we obtained with monthly returns and monthly betas. Thus the monthly cross section regression in Panel A is greatly improved by substituting annual betas for monthly betas, while substituting annual returns for monthly returns makes relatively little difference. In the fourth row we report the results of regressing average annual returns on annual betas. Now the R^2 is 56% and the market price of risk is 10.2%. The intercept is only -1.7% *per annum*, and the mean absolute error is now only 1.7% *per annum*.

Figure 1 shows why the annual betas perform so much better than the monthly betas: there is insufficient variation across portfolios in the monthly betas and therefore the predicted returns. The standard deviation of the annual betas is 1.8 times as large as that of the monthly betas. In addition, the correlation between the actual and predicted returns is much higher when annual betas are used; for monthly (annual) returns the correlation increases from 0.30 to 0.72 (0.35 to 0.78) when the prediction is based on annual rather than monthly betas.

While these simple OLS regressions do not allow us to draw strong conclusions because the OLS assumptions are not satisfied, they do illustrate in dramatic fashion that the return interval for the purposes of both measuring mean returns and estimating betas makes a very large difference to the relation between realized average returns and measured betas.

We construct bootstrapped samples in order to assess the importance of the serial dependence structure of returns on the cross-sectional return-risk relation at different horizons. Specifically, we generate 10,000 data samples from the joint time series of market and portfolio returns by sampling randomly from the vector of joint returns: this approach maintains the cross sectional covariance properties of the data while ensuring that there is no time series dependence within the generated samples. For each generated sample we estimate the parameters (a, λ) and calculate the resulting R^2. The p-values which are reported in brackets are the proportion of the generated samples in which the estimated parameter exceeds that calculated from the sample data. A low p-value suggests that the parameter estimates are inconsistent with the assumption of serially

[13] Kothari *et al.* (1995) also regress monthly returns on annual betas and report significant slope coefficients.

independent returns, given the means and the cross-sectional moments of the joint distribution of monthly returns. Not surprisingly the p-values for the first regression which uses only monthly data are close to 50%, since for this regression the temporal ordering of the sample data makes no difference to the parameter estimates. For the other regressions the temporal ordering of the monthly returns will be important if the returns are serially dependent. The p-values for the R^2 statistic are below 0.5 for the other regressions which involve either annual returns or annual betas. When annual returns are regressed on monthly betas the p-value drops to 0.43, less than 0.5 but not greatly less. However, when the independent variable is the annual beta the p-values are only 0.11 and 0.07, so that it is quite unlikely that the observed risk-return relations could have been generated by monthly returns with the same joint distribution if the returns were serially independent. Therefore the serial dependence of monthly returns must be taken into account in assessing risk-return relations, and this can only be done by considering returns measured over longer horizons than one month.

As a more formal illustration of the effect of the return interval, we test the first order condition of a representative agent who has a quadratic utility function and a τ month horizon, and does not rebalance his portfolio. Let $U_\tau(W)$ be his utility of wealth if the horizon is τ months. Denote his initial wealth by W_0, and suppose that he can invest in N risky assets as well as a risk free asset. Let R_j^τ denote the return on risky asset j if held for τ periods, and similarly let R_F^τ denote the return on the risk free asset if held for τ periods. Then the investor's objective function may be written as:

$$\max_{x_j} E_\tau \left\{ U_\tau[W_0(R_F^\tau + \Sigma_{j=1}^N x_j(R_j^\tau - R_F^\tau)] \right\} \tag{1}$$

where $x_j, (j = 1, \cdots, N)$ is the fraction of wealth allocated to security j, and E_τ denotes expectations over the τ period distribution of returns. The first order condition for an optimum in (1) is:

$$E_\tau \left\{ U'_\tau[W^\tau](R_j^\tau - R_F^\tau) \right\} = 0, \qquad j = 1, \cdots, N \tag{2}$$

where $W^\tau = W_0[R_F^\tau + \Sigma_{j=1}^N x_j(R_j^\tau - R_F^\tau)]$. If the utility function $U_\tau()$ is quadratic, then the marginal utility will be a linear function of the return on the investor's portfolio which, since he is a representative investor, will be the market portfolio, so that the FOC (2) may be written as:

$$E_\tau\left\{(1 - b_\tau R_m^\tau)(R_j^\tau - R_F^\tau)\right\} = 0, \qquad j = 1, \cdots, N \tag{3}$$

where $b_\tau > 0$. We estimate equation (3) by GMM for different return intervals using the returns on the 25 FF portfolios for the period January 1926 to December 2009, and for two subperiods. The market portfolio is the CRSP value weighted portfolio and the risk free rate is the (compounded) one-month T-bill rate. In Panel A of Table 6 we report the parameter estimates and tests of the over-identifying restrictions for selected return intervals of 1 to 24 months. The over-identifying restrictions imposed by the CAPM are rejected (accepted) at the 5% level for all horizons less (more) than 10 months for the whole sample period and for less (more) than 4 and 6 months for the 1926-62 and 1963-2009 subsamples respectively. The (annualized) mean absolute pricing error is greatest for the one month horizon, except for the period 1926-62 when the maximum is at 3 months. However, the mean absolute pricing error is only an indicative measure of goodness of fit since it does not take account of covariances between the returns on the portfolios.

Motivated by the finding of Jagannathan and Wang (2007) that December to December returns are particularly important for the Consumption CAPM, we report in Panel B estimates of the model that use annual December to December returns only. It can be seen that the results are not significantly different from the results that use all overlapping 12 month returns, although the estimated risk aversion coefficient, b, is larger when only the December to December returns are used in the estimation: 3.0 as compared with 1.6 for the overlapping estimate for the whole sample period.

Table 7 reports the annualized pricing errors for the individual portfolios using the CAPM pricing kernel for the period January 1926 to December 2009 with different assumed horizons. There is a clear tendency for the pricing errors to decrease with the horizon - the root mean square pricing error is 3% at the one month horizon and achieves a minimum of 2.2% at the twelve month horizon. Generally the pricing errors of the low book-to-market portfolios increase (in magnitude) as the horizon increases while those of the high book-to-market portfolios decrease.

We have seen that our conclusions about the empirical validity of the simple CAPM rest heavily on what we assume about the appropriate interval over which returns are measured. In the following section we introduce a simple model that allows for different investment horizons.

4 Asset Pricing with Stochastic Liquidation

Consider a representative investor who trades only to meet stochastic liquidation needs. We assume for simplicity that if any of the portfolio is to be liquidated, then the whole portfolio will be liquidated. Let π_τ be the probability that he is forced to liquidate at the end of τ periods, and let $U_\tau(W)$ be his utility of wealth if the liquidation takes place at the end of period τ. Then, assuming that the latest liquidation date is τ^*, the investor's objective function may be written:

$$\max_{x_j} \Sigma_{\tau=1}^{\tau^*} \pi_\tau E\left\{U_\tau[W_0(R_F^\tau + \Sigma_j x_j(R_j^\tau - R_F^\tau)]\right\} \tag{4}$$

where $x_j, (j = 1, \cdots, N)$ is the fraction of wealth allocated to security j. The first order condition for an optimum in (4) is:

$$\Sigma_{\tau=1}^{\tau^*} \pi_\tau E\left\{U'_\tau[W^\tau](R_j^\tau - R_F^\tau)\right\} = 0, \qquad j = 1, \cdots, N \tag{5}$$

where $W^\tau = W_0[R_F^\tau + \Sigma_{j=1}^N x_j(R_j^\tau - R_F^\tau)]$.

Such a representative agent economy can be sustained by a stationary overlapping generations exchange economy with a continuum of identical investors, each of whom is subject to stochastic liquidation. If there are n generations of investors in the market in the steady state, $(1/n)^{th}$ of the market will be liquidated each period and will be purchased by the new generation of investors, each of whom will maximize an objective function (4).

If the utility functions $U_\tau()$ are quadratic, then the marginal utility will be a linear function of the return on the investor's portfolio which, since he is a representative investor, will be the market portfolio, so that the FOC (5) can be written as:

$$\Sigma_{\tau=1}^{\tau^*} \pi_\tau E\left\{(1 - b_\tau R_m^\tau)(R_j^\tau - R_F^\tau)\right\} = 0, , \qquad j = 1, \cdots, N \tag{6}$$

$b_\tau > 0$. Equation (6) nests as special cases discrete time versions of the CAPM with different trading intervals. For example, $\pi_{\hat{\tau}} = 1, \pi_\tau = 0$, for $\tau \neq \hat{\tau}$ implies that the CAPM holds with a $\hat{\tau}$ period trading interval. Equation (6) suggests that if investors trade only infrequently then the simple CAPM will hold for a long holding interval, τ. In general, equation (6) represents a weighted of average of different versions of the CAPM which differ only in the definition of the holding period. Of course it is possible that the distribution of holding periods will differ for different securities; for example, Amihud and Mendelson (1986) suggest that securities with higher transactions costs will be held by investors with longer trading horizons. However, the assumption of different liquidation probabilities for different securities would complicate our model considerably. For example, if the whole of the portfolio is not liquidated at time τ then the notion of the marginal utility of wealth at time τ becomes problematic. For this reason we stick with the simple formulation (6).

Using the definition of covariance, equation (6) may be written as:

$$\Sigma_{\tau}^{\tau^*}\pi_{\tau}\left\{(E[R_j^{\tau}] - R_F^{\tau}) - b_{\tau}E[R_m^{\tau}(R_j^{\tau} - R_F^{\tau})]\right\} = 0 \tag{7}$$

Or,

$$\Sigma_{\tau}^{\tau^*}\pi_{\tau}\left\{(E[R_j^{\tau}] - R_F^{\tau})(1 - b_{\tau}E[R_m^{\tau}]) - b_{\tau}cov(R_m^{\tau}, R_j^{\tau})\right\} = 0 \tag{8}$$

Define $\pi_{\tau}^* = \pi_{\tau}(1 - b_{\tau}E[R_m^{\tau}])/\Sigma_s\pi_s(1 - b_sE[R_m^s])$. Since $1 - b_{\tau}E[R_m^{\tau}]$ is the marginal utility of wealth in the event of liquidation at time τ, $\pi_{\tau}^* > 0$ is the marginal utility of wealth weighted probability of liquidation at time τ. Then

$$\Sigma_{\tau}^{\tau^*}\pi_{\tau}^*\left\{(E[R_j^{\tau}] - R_F^{\tau}) - \lambda_{\tau}\frac{cov(R_m^{\tau}, R_j^{\tau})}{var(R_m^{\tau})}\right\} = 0 \tag{9}$$

where $\lambda_{\tau} = b_{\tau}var(R_m^{\tau})/(1 - b_{\tau}E[R_m^{\tau}])$. Finally, defining $\beta_j^{\tau} \equiv cov(R_j^{\tau}, R_m^{\tau})/var(R_m^{\tau})$,

$$\Sigma_{\tau}^{\tau^*}\pi_{\tau}^*\left\{E[R_j^{\tau}] - R_F^{\tau}\right\} = \Sigma_{\tau}\pi_{\tau}^*\lambda_{\tau}\beta_j^{\tau} \tag{10}$$

Thus, the CAPM with stochastic liquidation leads to an equilibrium condition that expresses the sum of expected excess returns for different horizons weighted by the marginal utility of wealth weighted probability as the same probability weighted sum of betas times the market risk premium for each horizon. In what follows we shall assume that the expected marginal utility of wealth is the same for each possible liquidation date. Then $\pi_t^* \equiv \pi_t$, and the model also has implications for share turnover rates if we interpret it in terms of an overlapping generations economy with a continuum of investors: we shall explore this in the following section.

4.1 Turnover in an overlapping generations economy

Consider a steady state overlapping generations economy with a continuum of identical investors, each of whom maximizes an objective function of the form (4). Denote by $\mathbf{x}$ the $(\tau^* x1)$ vector of the mass of investors liquidating τ periods in the future, $\tau = 1, \cdots, \tau^*$, by μ the mass of investors liquidating and entering the market each period, and let π denote the $(\tau^* x1)$ vector with typical element π_τ. Then the steady state vector of horizon proportions, $\mathbf{x}$ must satisfy:

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mu\pi \tag{11}$$

where the $(\tau^* x \tau^*)$ matrix $\mathbf{A}$ is given by:

$$A = \begin{bmatrix} 01000\cdots\cdots \\ 00100\cdots\cdots \\ 00010\cdots\cdots \\ \cdots\cdots \\ 00\cdots\cdots 1 \\ 00\cdots\cdots 0 \end{bmatrix}$$

Then

$$\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mu\pi \tag{12}$$

Carrying out the inversion, the i^{th} element of the vector $\mathbf{x}$ is given by $x_i = \Sigma_{k=i}^{\tau^*}\pi_k$. Then the *Turnover* per period, TO, is given by the proportion of investors who liquidate in the current period, which is the ratio of $x_1 = \Sigma_{k=1}^{\tau^*}\pi_k = 1$ to $\mathbf{j}'\mathbf{x}$:

$$TO = \frac{1}{\Sigma_{i=1}^{\tau^*}[\Sigma_{k=i}^{\tau^*}\pi_k]} \tag{13}$$

We shall use equation (13) to calculate turnover rates from our estimates of the π vector.

5 Estimation and test of an asset pricing model with stochastic liquidation

The moment condition corresponding to equation (6) can be written as:

$$E\left\{\Sigma_{\tau=1}^{\tau^*}\phi_\tau(1/\tau)(1-b_\tau R_m^\tau)(R_j^\tau - R_F^\tau)\right\} = 0 \tag{14}$$

where $\phi_\tau = \tau\pi_\tau$ and τ^* is the longest possible liquidation horizon. We write the moment condition in the form (14) so that the returns are all expressed on a monthly basis after the division by τ. We refer to ϕ_τ as the *horizon* weight to distinguish it from the *probability* weight, π_τ. In order to specify the horizon weights, ϕ_τ, flexibly and parsimoniously, we follow Ghysels, Santa-Clara and Valkanov (2006) and define,

$$\phi(\tau,\gamma) \equiv \frac{f(\frac{\tau}{\tau^*},\gamma_1;\gamma_2)}{\sum_{\tau=1}^{\tau^*} f(\frac{\tau}{\tau^*},\gamma_1;\gamma_2)} \tag{15}$$

where $f(z,a,b) \equiv z^{a-1}(1-z)^{b-1}/\beta(a,b)$ and $\beta(a,b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$ where $\Gamma()$ is the Gamma function. This specification allows the weights, which sum to one by construction, to take various patterns: for example, gradually decaying over time or hump-shaped. At the same time the weighting function requires only two free parameters $\gamma = (\gamma_1, \gamma_2)$.

In order to limit the number of parameters to be estimated, we set $\tau^* = 24$, allow for only 7 possible liquidation dates, 1, 4, 8, 12, 16, 20 and 24 months, and set the market risk aversion parameters, b_τ, equal to a constant, b, for all horizons.

Define $h(\mathbf{x_t},\theta)$ as an $Nx1$ vector, with j^{th} element:

$$\left\{\Sigma_{\tau=1}^{\tau^*}\phi(\tau,\gamma)(1/\tau)(1-bR_{m,t}^\tau)(R_{j,t}^\tau - R_{F,t}^\tau)\right\}$$

where θ is a vector of parameters (γ_1,γ_2,b), and $\mathbf{x}_t$ is the vector of market, portfolio, and risk free returns starting at time t: $(R_{j,t}^1, R_{j,t}^\tau, j = 1,..N, R_{m,t}^1, R_{m,t}^\tau, R_{F,t}^1, R_{F,t}^\tau)$.

Define the $N \times 1$ vector:

$$g_T(\theta) \equiv \frac{1}{T-\tau} \Sigma_{t=1}^{T-\tau} h(\mathbf{x_t}, \theta)$$

The parameters θ are chosen to minimize the weighted sum of squared pricing errors given by the quadratic form $g_T(\theta)' W g_T(\theta)$ where W is a $N \times N$ weighting matrix: the identity matrix in the first step, and inverse of the variance-covariance matrix in the second step.

Define $d = \frac{\partial g_T(\theta)}{\partial \theta}$. Then

$$d = \frac{1}{T-\tau} \begin{bmatrix} \Sigma_{\tau=1}^{\tau^*} \phi_{\gamma_1}(\tau, \gamma)(1/\tau)(1 - bR_{m,t}^{\tau})(R_{j,t}^{\tau} - R_{F,t}^{\tau}) \\ \Sigma_{\tau=1}^{\tau^*} \phi_{\gamma_2}(\tau, \gamma)(1/\tau)(1 - bR_{m,t}^{\tau})(R_{j,t}^{\tau} - R_{F,t}^{\tau}) \\ -\Sigma_{\tau=1}^{\tau^*} \phi_(\tau, \gamma)(1/\tau) R_{m,t}^{\tau}(R_{j,t}^{\tau} - R_{F,t}^{\tau}) \end{bmatrix}$$

each row of d is a $1 \times N$ vector. If the matrix d is of full rank, then

$$\sqrt{T}(\widehat{\theta}_T - \theta) \sim N(0, V), \tag{16}$$

where $V = (dW^{-1}d')^{-1}$.

The sample variance-covariance ($\widehat{S}$) is calculated following Newey-West to take account of autocorrelations in long horizon returns. It is

$$\widehat{S} = \sum_{j=-k}^{k} (\frac{k - |j|}{k}) \frac{1}{T} \sum_{t=1}^{T} (u_t u_{t-j}') \tag{17}$$

where $u_t \equiv h(\mathbf{x_t}, \theta)$ is the vector of pricing errors. We set the number of lags, k, equal to 30.[14] To test the over-identifying restrictions, we compute the J-statistic,

$$J = T g_T(\hat{b})' \widehat{S}^{-1} g_T(\hat{b}) \tag{18}$$

which follows a χ^2 distribution with degrees of freedom equal of the number of over-identifying restrictions, $(N-3)$.

The estimation results are presented in Panel A of Table 8. Panel B reports the horizon weight, ϕ_τ, for each liquidation date implied by the parameters reported in Panel A; Panel C

[14]Similar results were obtained using 24 to 36 lags.

reports the probability weights, π_τ, which are shown graphically in Panel A of Figure 2, as well as the implied average horizon and turnover rate calculated from equation (13). We note first that the model, which has only three free parameters, is not rejected either for the full sample period or for either of the two subperiods by the standard J-statistic, although the p-value for the second half of the sample period is only 0.10. Secondly, a likelihood ratio test is unable to reject the hypothesis of equality of the risk aversion parameter, b_τ across horizons for the whole sample period, and for the first half: the test statistic for the second half of the sample period is at the margin of significance. The risk aversion parameter, b, which is highly significant, increases from 1.69 in the first half of the sample period to 2.06 in the second half: the full sample estimate is 1.57.[15]

Both parameters of the weighting function are highly significant for all three sample periods. For all three sample periods the estimated probability of liquidation after 24 months is zero, providing assurance that our arbitrary 24 month cutoff is not binding. Most significantly, as seen in Panel C, the estimated probability weighting system assigns *zero* probability to a liquidation at the one month horizon for either the whole sample or for the first half, and the estimated probability of a liquidation at the 4 month horizon is less than 4% in the full sample. In the first half of the sample period the probability of liquidation is zero for 1,4, and 8 month horizons. In the second half of the sample period the probability of a liquidation after one month rises to 68.0% and after 4 months to 23.3%. In the first half of the sample period the modal liquidation horizon is 16 months; this falls to 1 month in the second half, and for the whole sample period the modal liquidation horizon is 12 months. The probability weighted number of months to liquidation is 16.7 in the first half of the sample, 2.4 in the second half and 12.1 for the whole period. The implied annual turnover rate rises from 58.0% in the first half of the sample period to 209.6% in the second half,, and for the whole sample is 74.3%. This reduction in the expected holding period and increase in turnover rate is consistent with casual empiricism which shows that turnover rates have increased, particularly since the abandonment of fixed commissions in May 1975. Table 9 reports the estimated average time to liquidation from the model fit for different 20 year subperiods along the implied turnover rate and the corresponding estimate of the average turnover rate of NYSE stocks, which is obtained by averaging the annual turnover rate during the subperiods. The estimated turnover rate is around 3.5 times as great as the estimated average turnover rate of NYSE stocks, and the correlation between the two series is 0.95.

[15] Fama and French (1992, p 433) report that there is no 'obvious relation' between β and average returns over the period 1963-1990.

Panel A of Figure 3 plots the estimated horizon weighted average excess returns, $\Sigma_{\tau=1}^{\tau^*}\phi_\tau(1/\tau)\overline{(R_j^\tau - R_F^\tau)}$, against the estimated horizon weighted expected returns from the model, $-\Sigma_{\tau=1}^{\tau^*}\phi_\tau(1/\tau)(bR_m^\tau)(\bar{R}_j^\tau - R_F^\tau)$. With the exception of the smallest and second smallest firm low book-to-market portfolios, the points cluster fairly tightly about the line implied by the theoretical relation. The correlation between the horizon weighted average excess returns and the horizon weighted predicted excess returns is 0.81. This rises to 0.90 when the small firm low book-to-market portfolio is excluded.[16]

To assess the importance of the serial dependence structure of returns for these results, the parameters were estimated from 10,000 samples obtained by sampling randomly from the joint vector of market and portfolio returns so as to preserve the cross sectional dependence of returns, while eliminating any serial dependence. Statistics from these bootstrapped full sample regressions are reported in Panel D of Table 8. The sample estimates of the 1 and 4 month horizon weights for the full sample period fall far below the 1^{st} percentile of the bootstrapped regression estimates while the estimated weights for the 8-24 month horizons fall far above the 99^{th} percentile of the bootstrapped estimates. Moreover, the sample $J-statistic$ of 28.1 is close to the 2.5 percentile of the bootstrapped statistics. That is, the success of the model in pricing the 25 portfolios is highly dependent on the serial dependence structure of returns, which of course is not taken into account in standard asset pricing tests that use only monthly returns.

6 Further tests

Kothari *et al.* (1995) have suggested that COMPUSTAT selection biases[17] lead to an upward bias to the measured returns on firms with high book-to-market ratios. Furthermore, firms in the high book-to-market category are more likely to be past 'losers' and to have unusually high returns because of the reversal effect.[18] To mitigate these problems the Size and Book-to-market portfolios

[16] As Campbell and Vuolteenaho (2003) remark, 'this small growth portfolio is well known to present a particular challenge to asset pricing models, for example the 3 factor model of Fama and French (1993) which does not fit this portfolio well.'

[17] In particular, in 1978 the database was more than doubled in size, and up to five years of past data were added for these firms, creating a survivorship bias which is likely to lead to an overstatement of mean returns on portfolios of firms with relatively low probabilities of survival and high book-to-market ratios. In addition, firms that become financially distressed are likely to be deleted from the database before they are delisted and, in the event they recover, the past data are filled in, creating an upward bias for measured returns on distressed firms. (See also Alford *et al.* (1994).)

[18] Jegadeesh and Titman (1993).

were re-formed using lagged balance sheet data to determine the portfolio allocations. This should eliminate the short run reversal effect and, when longer lags are used, eliminate also the back-filling bias. Five lags from one to five years relative to the Fama-French lag of 6 months were used. Table 10 reports the pricing errors for the 25 portfolios for the different lags, as well as the J-statistic for the model. As anticipated, the pricing errors tend to get smaller as the lag used for the portfolio formation increases; for example, for the small low book-to-market portfolio the annualized pricing error shrinks from 4.8% for the original portfolios to 2.9% when the portfolio formation lag is increased to 5 years. Similarly, the root mean square pricing error shrinks from 3.3% to 1.9%. The J-statistic also decreases from 29.53 to 10.46 and the p-value increases as the lag is increased, suggesting that model deviations for these portfolios are related to transient rather than permanent firm characteristics, which is consistent with the literature on investor over-reaction.

Table 11 reports the parameters estimated by fitting the model to the 30 Fama-French industry portfolios, and Panel B of Figure 2 shows the estimated probability distribution of liquidation horizons. The risk aversion coefficient and the parameters of the weighting function are highly significant in all three sample periods, the J-statistics are well within the acceptance region, and the estimated average liquidation horizon decreases from 10.0 to 7.5 months between the two sample subperiods. However, the magnitude of the risk aversion coefficient is in excess of 4 for both the whole sample period and the second half, which is well above the value of 1.6 obtained using the FF25 portfolios.

Table 12 reports parameter estimates from fitting the model to the combined set of 55 industry and size and book-to-market sorted portfolios, and Panel C of Figure 2 shows the implied probability distribution over horizons. The risk aversion coefficient is significant for all three subperiods and is in the range $1.3-2.3$; the J-statistic is far from the rejection region and the average horizon declines from 12.1 months in the first subperiod to only 1.4 months in the second subperiod. The turnover rate estimated for the first half of the sample period is 82.8% and this rises in the second half to 266.4%; the full sample estimate is 74.4% An F-test fails to reject the null hypothesis that the model parameters are the same across the two sets of portfolios, the size and book-to-market sorted and industry portfolios. Panel B of Figure 3 shows the relation between (horizon weighted) actual and predicted returns. The industry portfolios have less dispersion in predicted returns than the Size and Book-to-market portfolios, reflecting their reduced dispersion in beta, and their returns exhibit greater dispersion about the predicted return reflecting the lower level of diversification achieved by industry portfolios. It is interesting to note that the two industry portfolios whose returns depart furthest from the 45^o line are the two 'polluting' industries, Coal and Smoke.

7 Conclusion

In this paper we have shown that the monthly returns on the 25 Fama-French portfolios are not serially independent and that this has major consequences for tests of the CAPM. Measured average returns and portfolio betas, and the relation between them, are all strongly affected by the length of the period over which returns are measured. We show that the ratio of long-horizon (12-month) betas to short-horizon (1-month) betas depends on such firm characteristics as the number of analysts following the firm and the seasonality of earnings, as well as firm size and book-to-market ratio. We also find for the Fama-French Size and Boo-to-Market portfolios that the 12 month beta is significantly related to the SMB and HML betas but is not significantly related to the one month market beta, given the other two betas. This raises the possibility that the empirical success of the Fama-French 3-factor model is due to its ability to capture the long-horizon risk characteristics of the portfolios.

Using returns on the Fama-French *Size* and *Book-to-Market* sorted portfolios from 1926 to 2009, we show the simple CAPM is rejected at the 5% level for all return horizons less than 11 months, but is not rejected at longer horizons. We develop an extended version of the CAPM with a representative investor who has a stochastic horizon or liquidation date. This gives rise to a relation between a probability weighted sum of expected returns over different horizons, and the same probability weighted sum of betas times market risk premia for the corresponding horizons. When we parameterize the probability weighting function we find that the modified asset pricing model, which has only three parameters, is not rejected at conventional significance levels for either the whole sample period or for the two halves of the period. The correlation between the horizon weighted average excess returns and the horizon weighted predicted excess return is 0.81, and the correlation rises to 0.90 when the small firm low book-to-market portfolio is excluded. We also find that the pricing errors of the model are reduced when the Fama-French portfolios are replaced by portfolios formed on the basis of lagged firm characteristics: in particular, the pricing error of the small firm low book-to-market portfolio falls from 4.8% *p.a.* to only 1.3% *p.a.* when the portfolios are formed using data lagged by 3 years.

The model is also not rejected when it is fit to returns on 30 industry portfolios, or to the 55 portfolios that include both the *Size* and *Book-to-market* portfolios and we cannot reject the null that the model parameters are the same across the two sets of portfolios.

Using returns on the 25 *Size* and *Book-to-market* portfolios, the estimated probability of liquidation function implies that for the whole sample period as well as for the first half of the sample there is a zero probability of liquidation after one month. This contrasts with the implicit assumption made in the majority of conventional tests of asset pricing models that the probability of liquidation after one month is 100%. In the second half of the sample the probability of a liquidation after one month rises to 68%. The probability weighted number of months to liquidation is 12.1 in the first half of the sample, 2.4 in the second half and 12.1 for the whole sample period. The reduction in the average liquidation horizon is consistent with the reduction in trading commissions and the increase in turnover that has followed the abandonment of fixed brokerage commissions in 1975 and the further development of competition in trading securities.

The model that we have developed and tested is extremely simple. It assumes that all the assets in the representative investor's portfolio are liquidated at the same time time. A more realistic model would allow for the fact that, as Amihud and Mendelson (1986) have pointed out, investors with short horizons (high probabilities of liquidation) will be likely to hold more liquid securities, while investors with longer horizons will be likely to hold the more illiquid securities. It is beyond the scope of this paper to explore these interesting extensions which are left for future work. Some progress has been made in the paper by Beber *et al.* cited above, but at the expense of assuming that returns are *iid*. The focus of this paper has been on exploring the implications of non-*iid* returns.

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Table 1: The effect of the return interval on estimates of betas and mean returns for the Fama-French 25 portfolios

Panel A shows the ratio of the betas estimated using τ month returns to those estimated using 1 month returns where the τ month beta, β_j^τ, is estimated using monthly overlapping observations from the equation:

$$R_{j,t}^\tau = \alpha^\tau + \beta_j^\tau R_{m,t}^\tau + e_{jt}^\tau$$

where $R_{j,t}^\tau = \Pi_{k=0}^{k=\tau-1}(1 + R_{j,t+k}) - 1$ and the τ month market return, $R_{m,t}^\tau$ is similarly defined. Panel B shows the ratio of the mean annualized returns using a τ month return interval to the mean annualized returns using a 1 month return interval. The annualized return using a τ month return interval, ${}^a\bar{R}_j^\tau$, is defined by ${}^a\bar{R}_j^\tau \equiv (1 + \bar{R}_j^\tau)^{(12/\tau)} - 1$, where the (non-annualized) average τ month return, $\bar{R}_j^\tau$, is defined by $\bar{R}_j^\tau = \Sigma_{t=1}^{T-\tau}\Pi_{k=0}^{\tau-1}(1 + R_{j,t+k}^1)$.
The sample period is January 1926 to December 2009.

Panel A: 1 month β's, β_j^1, and ratio of beta estimated using τ month returns, β_j^τ, to beta estimated using 1 month returns,β_j^1.

			Ratio of τ month beta to 1 month beta							
Size	*BM ratio*	β_j^1	$\tau = 3$	6	9	12	15	18	21	24
Small	**Low**	1.654	1.151	0.988	0.915	0.900	0.899	0.815	0.792	0.750
	2	1.478	1.175	1.070	0.961	0.995	0.910	0.857	0.846	0.801
	3	1.396	1.181	1.149	1.056	1.085	1.088	1.058	1.061	1.037
	4	1.309	1.264	1.223	1.139	1.245	1.234	1.178	1.218	1.174
	High	1.393	1.248	1.214	1.130	1.258	1.246	1.199	1.220	1.177
2	**Low**	1.250	1.062	1.112	1.074	1.046	1.027	0.996	0.981	0.967
	2	1.276	1.148	1.065	1.005	1.041	1.019	1.007	1.021	1.005
	3	1.187	1.184	1.135	1.054	1.121	1.091	1.047	1.052	1.014
	4	1.228	1.208	1.146	1.050	1.120	1.106	1.051	1.053	1.008
	High	1.362	1.176	1.112	1.012	1.083	1.078	1.012	1.030	0.985
3	**Low**	1.280	1.117	1.070	1.005	1.056	1.033	0.987	1.002	0.962
	2	1.126	1.055	1.074	1.050	1.045	1.033	1.014	1.008	0.988
	3	1.149	1.107	1.055	1.005	1.048	1.033	1.018	1.043	1.029
	4	1.128	1.141	1.094	1.020	1.062	1.046	1.002	0.999	0.983
	High	1.394	1.145	1.013	0.933	1.001	0.969	0.922	0.927	0.880
4	**Low**	1.073	0.993	1.024	1.019	0.997	0.985	0.971	0.949	0.934
	2	1.097	1.058	1.040	1.002	1.015	1.000	0.980	0.982	0.967
	3	1.089	1.054	1.052	1.029	1.068	1.057	1.028	1.035	1.012
	4	1.174	1.105	1.017	0.960	1.016	1.016	0.978	0.975	0.935
	High	1.443	1.150	1.027	0.942	1.011	0.990	0.916	0.913	0.855
Big	**Low**	0.972	0.975	0.988	0.994	0.985	0.991	1.008	1.022	1.038
	2	0.924	0.957	0.975	0.994	0.979	0.979	0.988	0.986	0.986
	3	0.979	1.033	0.993	0.988	1.024	1.024	1.013	1.016	1.018
	4	1.132	1.116	0.988	0.931	1.008	0.993	0.955	0.955	0.925
	High	1.152	1.065	1.048	1.026	1.054	1.077	1.058	1.071	1.052

Table continued on next page ...

Panel B: Annualized average 1 month returns and ratio of annualized average τ month returns to annualized average 1 month returns.

Size	*BM ratio*	1 month return	Ratio of annualized τ month return to annualized 1 month return							
			$\tau = 3$	6	9	12	15	18	21	24
Small	**Low**	0.091	1.121	0.954	0.899	0.866	0.870	0.843	0.832	0.803
	2	0.139	1.083	1.005	0.961	0.983	0.956	0.943	0.944	0.926
	3	0.168	1.073	1.047	1.010	1.021	1.029	1.026	1.025	1.010
	4	0.190	1.089	1.069	1.041	1.065	1.069	1.061	1.063	1.049
	High	0.222	1.076	1.053	1.019	1.032	1.036	1.031	1.029	1.013
2	**Low**	0.107	1.059	1.049	1.034	1.042	1.037	1.035	1.032	1.021
	2	0.157	1.055	1.020	0.999	1.009	1.006	1.003	1.003	0.989
	3	0.171	1.056	1.033	1.007	1.019	1.014	1.009	1.007	0.994
	4	0.177	1.065	1.046	1.018	1.031	1.033	1.027	1.026	1.012
	High	0.196	1.061	1.032	0.990	0.994	0.996	0.984	0.982	0.966
3	**Low**	0.120	1.073	1.042	1.020	1.040	1.035	1.025	1.026	1.008
	2	0.147	1.029	1.023	1.015	1.016	1.014	1.013	1.013	1.003
	3	0.163	1.041	1.020	1.002	1.010	1.005	1.001	1.002	0.990
	4	0.165	1.044	1.025	0.999	1.002	1.000	0.995	0.994	0.986
	High	0.185	1.054	1.011	0.984	0.995	0.989	0.977	0.973	0.953
4	**Low**	0.122	1.023	1.014	1.007	1.005	0.996	0.990	0.984	0.970
	2	0.130	1.035	1.019	1.004	1.005	0.998	0.996	0.996	0.986
	3	0.144	1.032	1.027	1.020	1.029	1.027	1.025	1.028	1.020
	4	0.158	1.041	1.021	1.001	1.005	1.007	1.002	0.999	0.984
	High	0.174	1.064	1.018	0.983	0.987	0.979	0.962	0.957	0.938
Big	**Low**	0.111	1.016	1.009	1.008	1.010	1.008	1.010	1.014	1.008
	2	0.112	1.005	0.997	0.997	0.997	0.996	0.998	0.998	0.992
	3	0.119	1.027	1.017	1.020	1.032	1.036	1.037	1.039	1.034
	4	0.124	1.054	1.014	0.999	1.018	1.016	1.009	1.007	0.993
	Low	0.155	1.027	1.013	0.997	0.996	0.995	0.990	0.985	0.967

Table continued from previous page.

Table 2: Bootstrapped distribution of betas and mean returns

Panel A reports the 10^{th}, 50^{th} and 90^{th} percentiles of the bootstrapped distribution of the ratio of the betas estimated using τ month returns to those estimated using 1 month returns. The bootstrapped distribution is generated from 10,000 samples constructed by sampling at random with replacement from each monthly cross-sectional vector of historical returns from 1926 to 2009. Panel B reports percentiles of the bootstrapped distribution of the ratio of the annualized average returns using a τ month return interval to the annualized average returns using a 1 month return interval.

Panel A: Long horizon betas

		τ							
Size	**BM**	3	6	9	12	15	18	21	24
Small	**Low**	[0.946 0.995 1.05]	[0.906 0.989 1.09]	[0.874 0.983 1.11]	[0.845 0.975 1.14]	[0.82 0.969 1.16]	[0.796 0.959 1.18]	[0.772 0.95 1.2]	[0.75 0.942 1.22]
	2	[0.956 1 1.05]	[0.927 1.01 1.1]	[0.909 1.01 1.13]	[0.891 1.01 1.16]	[0.878 1.01 1.19]	[0.865 1.02 1.22]	[0.85 1.02 1.24]	[0.838 1.02 1.27]
	3	[0.971 1.01 1.05]	[0.957 1.02 1.09]	[0.949 1.03 1.12]	[0.941 1.04 1.16]	[0.936 1.05 1.19]	[0.931 1.06 1.22]	[0.926 1.07 1.26]	[0.922 1.08 1.29]
	4	[0.975 1.01 1.05]	[0.965 1.03 1.09]	[0.961 1.04 1.13]	[0.959 1.06 1.17]	[0.959 1.07 1.21]	[0.959 1.09 1.25]	[0.96 1.1 1.29]	[0.96 1.12 1.32]
	High	[0.975 1.01 1.06]	[0.967 1.04 1.11]	[0.968 1.06 1.17]	[0.97 1.08 1.22]	[0.972 1.1 1.27]	[0.977 1.13 1.32]	[0.982 1.15 1.37]	[0.988 1.17 1.42]
2	**Low**	[0.969 0.999 1.03]	[0.947 0.999 1.05]	[0.93 0.997 1.07]	[0.913 0.996 1.08]	[0.9 0.995 1.09]	[0.887 0.993 1.11]	[0.874 0.992 1.12]	[0.861 0.989 1.13]
	2	[0.978 1.01 1.03]	[0.967 1.01 1.07]	[0.962 1.02 1.09]	[0.958 1.03 1.12]	[0.955 1.04 1.14]	[0.953 1.05 1.17]	[0.95 1.06 1.19]	[0.949 1.07 1.21]
	3	[0.981 1.01 1.04]	[0.973 1.02 1.07]	[0.971 1.03 1.1]	[0.97 1.04 1.13]	[0.969 1.05 1.15]	[0.969 1.07 1.18]	[0.97 1.08 1.21]	[0.972 1.09 1.23]
	4	[0.982 1.01 1.04]	[0.975 1.02 1.07]	[0.973 1.03 1.1]	[0.974 1.05 1.13]	[0.974 1.06 1.16]	[0.975 1.07 1.19]	[0.978 1.09 1.22]	[0.98 1.1 1.25]
	High	[0.98 1.01 1.05]	[0.975 1.03 1.09]	[0.973 1.05 1.13]	[0.974 1.06 1.17]	[0.977 1.08 1.21]	[0.979 1.1 1.25]	[0.982 1.12 1.28]	[0.986 1.13 1.32]
3	**Low**	[0.977 1 1.02]	[0.963 1 1.04]	[0.952 1 1.06]	[0.941 1 1.07]	[0.933 1 1.09]	[0.926 1.01 1.1]	[0.918 1.01 1.11]	[0.91 1.01 1.12]
	2	[0.986 1 1.02]	[0.98 1.01 1.04]	[0.977 1.02 1.06]	[0.975 1.03 1.08]	[0.975 1.03 1.1]	[0.974 1.04 1.11]	[0.974 1.05 1.13]	[0.973 1.06 1.15]
	3	[0.987 1.01 1.03]	[0.983 1.02 1.05]	[0.982 1.03 1.08]	[0.983 1.04 1.1]	[0.985 1.05 1.12]	[0.986 1.06 1.14]	[0.989 1.07 1.16]	[0.991 1.08 1.18]
	4	[0.984 1.01 1.03]	[0.978 1.02 1.06]	[0.976 1.03 1.09]	[0.975 1.04 1.11]	[0.976 1.05 1.13]	[0.976 1.06 1.16]	[0.978 1.07 1.18]	[0.978 1.08 1.2]
	High	[0.981 1.01 1.04]	[0.975 1.02 1.08]	[0.973 1.04 1.12]	[0.973 1.06 1.15]	[0.974 1.07 1.19]	[0.977 1.09 1.22]	[0.979 1.1 1.26]	[0.981 1.12 1.29]
4	**Low**	[0.983 1 1.02]	[0.973 1 1.03]	[0.966 1 1.04]	[0.96 1.01 1.05]	[0.954 1.01 1.06]	[0.948 1.01 1.07]	[0.943 1.01 1.08]	[0.938 1.01 1.09]
	2	[0.986 1 1.02]	[0.979 1.01 1.03]	[0.973 1.01 1.05]	[0.969 1.01 1.06]	[0.966 1.01 1.07]	[0.963 1.02 1.08]	[0.961 1.02 1.09]	[0.958 1.02 1.1]
	3	[0.986 1 1.02]	[0.979 1.01 1.04]	[0.975 1.02 1.06]	[0.972 1.02 1.08]	[0.971 1.03 1.09]	[0.969 1.03 1.11]	[0.968 1.04 1.12]	[0.968 1.05 1.14]
	4	[0.983 1.01 1.03]	[0.975 1.01 1.06]	[0.972 1.02 1.08]	[0.971 1.03 1.1]	[0.97 1.04 1.12]	[0.97 1.05 1.15]	[0.97 1.06 1.17]	[0.97 1.07 1.19]
	High	[0.978 1.01 1.04]	[0.97 1.02 1.08]	[0.966 1.03 1.11]	[0.964 1.05 1.15]	[0.962 1.06 1.17]	[0.961 1.07 1.21]	[0.961 1.08 1.24]	[0.961 1.1 1.27]
Big	**Low**	[0.986 0.999 1.01]	[0.975 0.998 1.02]	[0.967 0.997 1.03]	[0.96 0.996 1.03]	[0.953 0.995 1.04]	[0.946 0.994 1.04]	[0.94 0.992 1.05]	[0.934 0.991 1.05]
	2	[0.985 0.999 1.01]	[0.973 0.998 1.02]	[0.964 0.997 1.03]	[0.956 0.996 1.03]	[0.95 0.995 1.04]	[0.943 0.994 1.04]	[0.937 0.993 1.05]	[0.931 0.992 1.05]
	3	[0.981 1 1.02]	[0.969 1 1.03]	[0.96 1 1.04]	[0.952 1 1.05]	[0.944 1 1.06]	[0.937 1 1.07]	[0.93 1 1.08]	[0.925 1 1.09]
	4	[0.976 1 1.03]	[0.96 1 1.05]	[0.947 1 1.06]	[0.937 1 1.08]	[0.928 1.01 1.09]	[0.919 1.01 1.11]	[0.911 1.01 1.12]	[0.904 1.01 1.13]
	Low	[0.974 1.01 1.04]	[0.96 1.01 1.07]	[0.95 1.02 1.1]	[0.942 1.03 1.13]	[0.936 1.04 1.15]	[0.931 1.05 1.18]	[0.928 1.06 1.2]	[0.923 1.07 1.23]

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Panel B: Long horizon returns

		τ							
Size	*BM*	3	6	9	12	15	18	21	24
Small	**Low**	[0.915 0.997 1.08]	[0.861 0.993 1.14]	[0.823 0.987 1.17]	[0.785 0.982 1.2]	[0.755 0.976 1.23]	[0.725 0.971 1.25]	[0.701 0.965 1.27]	[0.676 0.958 1.28]
	2	[0.964 0.999 1.03]	[0.939 0.996 1.06]	[0.921 0.994 1.07]	[0.905 0.991 1.09]	[0.893 0.988 1.09]	[0.882 0.985 1.1]	[0.87 0.982 1.11]	[0.86 0.979 1.12]
	3	[0.976 0.999 1.02]	[0.96 0.998 1.04]	[0.949 0.997 1.05]	[0.938 0.995 1.06]	[0.93 0.993 1.07]	[0.922 0.991 1.07]	[0.915 0.989 1.08]	[0.907 0.987 1.08]
	4	[0.982 0.999 1.02]	[0.969 0.998 1.03]	[0.96 0.997 1.04]	[0.952 0.996 1.04]	[0.945 0.995 1.05]	[0.938 0.993 1.06]	[0.933 0.992 1.06]	[0.928 0.99 1.06]
	High	[0.981 0.999 1.02]	[0.968 0.998 1.03]	[0.958 0.997 1.04]	[0.95 0.995 1.05]	[0.942 0.994 1.05]	[0.936 0.992 1.06]	[0.93 0.991 1.06]	[0.925 0.989 1.07]
2	**Low**	[0.97 0.999 1.03]	[0.951 0.998 1.05]	[0.937 0.996 1.06]	[0.925 0.994 1.07]	[0.914 0.992 1.08]	[0.904 0.99 1.09]	[0.895 0.989 1.09]	[0.886 0.988 1.1]
	2	[0.981 1 1.02]	[0.968 0.999 1.03]	[0.959 0.998 1.04]	[0.951 0.996 1.05]	[0.944 0.995 1.05]	[0.938 0.994 1.06]	[0.931 0.992 1.06]	[0.926 0.991 1.07]
	3	[0.985 1 1.01]	[0.974 0.999 1.02]	[0.966 0.998 1.03]	[0.96 0.997 1.04]	[0.954 0.996 1.04]	[0.949 0.995 1.05]	[0.945 0.994 1.05]	[0.941 0.993 1.05]
	4	[0.984 1 1.02]	[0.974 0.999 1.03]	[0.965 0.998 1.03]	[0.958 0.997 1.04]	[0.953 0.996 1.04]	[0.947 0.995 1.05]	[0.942 0.993 1.05]	[0.938 0.992 1.06]
	High	[0.981 0.999 1.02]	[0.969 0.998 1.03]	[0.96 0.997 1.04]	[0.952 0.996 1.05]	[0.945 0.994 1.05]	[0.939 0.993 1.06]	[0.933 0.991 1.06]	[0.928 0.99 1.06]
3	**Low**	[0.976 1 1.02]	[0.96 0.998 1.04]	[0.948 0.997 1.05]	[0.938 0.996 1.06]	[0.929 0.994 1.07]	[0.921 0.992 1.07]	[0.914 0.991 1.08]	[0.908 0.989 1.08]
	2	[0.985 1 1.01]	[0.975 0.999 1.02]	[0.967 0.998 1.03]	[0.961 0.997 1.04]	[0.956 0.996 1.04]	[0.951 0.995 1.05]	[0.946 0.994 1.05]	[0.942 0.994 1.05]
	3	[0.986 1 1.01]	[0.976 0.999 1.02]	[0.97 0.998 1.03]	[0.964 0.998 1.03]	[0.958 0.996 1.04]	[0.954 0.996 1.04]	[0.95 0.995 1.05]	[0.946 0.994 1.05]
	4	[0.986 1 1.01]	[0.976 0.999 1.02]	[0.969 0.999 1.03]	[0.964 0.998 1.03]	[0.959 0.996 1.04]	[0.954 0.995 1.04]	[0.95 0.994 1.05]	[0.945 0.994 1.05]
	High	[0.981 0.999 1.02]	[0.968 0.998 1.03]	[0.958 0.998 1.04]	[0.95 0.996 1.05]	[0.943 0.995 1.05]	[0.937 0.993 1.06]	[0.931 0.992 1.06]	[0.926 0.99 1.07]
4	**Low**	[0.983 1 1.02]	[0.973 0.999 1.03]	[0.965 0.998 1.03]	[0.958 0.997 1.04]	[0.952 0.996 1.04]	[0.946 0.995 1.05]	[0.941 0.995 1.05]	[0.936 0.994 1.06]
	2	[0.985 1 1.02]	[0.974 0.999 1.02]	[0.967 0.998 1.03]	[0.96 0.998 1.04]	[0.954 0.997 1.04]	[0.949 0.995 1.05]	[0.944 0.994 1.05]	[0.939 0.994 1.05]
	3	[0.986 1 1.01]	[0.976 0.999 1.02]	[0.969 0.998 1.03]	[0.963 0.998 1.03]	[0.958 0.997 1.04]	[0.953 0.995 1.04]	[0.949 0.994 1.05]	[0.944 0.993 1.05]
	4	[0.985 1 1.01]	[0.975 0.999 1.02]	[0.967 0.999 1.03]	[0.961 0.998 1.04]	[0.955 0.996 1.04]	[0.95 0.995 1.05]	[0.946 0.994 1.05]	[0.941 0.993 1.05]
	High	[0.978 0.999 1.02]	[0.964 0.998 1.03]	[0.953 0.997 1.04]	[0.943 0.995 1.05]	[0.934 0.994 1.06]	[0.928 0.992 1.07]	[0.921 0.991 1.07]	[0.915 0.988 1.08]
Big	**Low**	[0.986 1 1.01]	[0.976 0.999 1.02]	[0.969 0.998 1.03]	[0.963 0.998 1.04]	[0.957 0.997 1.04]	[0.952 0.996 1.04]	[0.948 0.995 1.05]	[0.944 0.994 1.05]
	2	[0.987 1 1.01]	[0.978 0.999 1.02]	[0.971 0.999 1.03]	[0.965 0.998 1.03]	[0.961 0.997 1.04]	[0.956 0.996 1.04]	[0.952 0.996 1.04]	[0.948 0.995 1.05]
	3	[0.986 1 1.01]	[0.976 0.999 1.02]	[0.969 0.998 1.03]	[0.963 0.998 1.03]	[0.958 0.997 1.04]	[0.953 0.996 1.04]	[0.948 0.995 1.05]	[0.945 0.994 1.05]
	4	[0.981 1 1.02]	[0.969 0.999 1.03]	[0.96 0.998 1.04]	[0.951 0.997 1.05]	[0.944 0.996 1.05]	[0.938 0.994 1.06]	[0.932 0.993 1.06]	[0.926 0.991 1.07]
	Low	[0.982 1 1.02]	[0.97 0.999 1.03]	[0.961 0.998 1.04]	[0.953 0.996 1.04]	[0.947 0.995 1.05]	[0.941 0.994 1.05]	[0.935 0.993 1.06]	[0.93 0.991 1.06]

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Table 3: Long Horizon Market βs and Firm Characteristics

Panel A presents Fama-MacBeth regression estimates of:

$$log(\frac{\beta_{6,i,t}}{\beta_{1,i,t}}) = a+b_1 log(Size_{i,t})+b_2 log(BM_{i,t})+b_3 log(Seasonality_{i,t})+b_4 log(1+Analysts_{i,t})+b_5\hat{\beta}_1+b_6\mathbf{1}_{\hat{\beta}_1>1}+b_7\hat{\beta}_1\mathbf{1}_{\hat{\beta}_1>1}+\varepsilon_{i,t}$$

$\beta_{1,i,t}$ ($\beta_{6,i,t}$) is estimated using monthly (6-month compounded) returns over the last 5 years (at least 36 observations required). $Size_{i,t}$, $BM_{i,t}$, and $Analysts_{i,t}$ are all normalized by their cross-sectional average at time t. $Seasonality_{i,t}$ is measured by the average over the past five years of the ratio of the highest quarterly EPS over the average EPS for the year. $\hat{\beta}_1$ is the predicted value of β_1 using size, book-to-market, book leverage and past 5-year return volatility. $\mathbf{1}_{\hat{\beta}_1>1}$ is a dummy variable which is equal to 1 if $\hat{\beta}_1$ is larger than 1. In Panel B the dependent variable is $log(\frac{\beta_{12,i,t}}{\beta_{1,i,t}})$ where $\beta_{12,i,t}$ is estimated using 12-month compounded) returns over the last 5 years (at least 36 observations required). All standard errors are adjusted for serial correlation using Newey-West with 6 lags. The sample period is 1983 to 2009 for regressions controlling for analyst coverage and 1968 to 2009 for other specifications.

Panel A: 6-month β over 1-month β

$log(Size)$	-0.059	-0.061	-0.059	-0.055	-0.057	-0.053	-0.050
	(-6.87)	(-7.02)	(-11.74)	(-9.93)	(-9.32)	(-7.78)	(-7.25)
$log(BM)$	-0.031	-0.031	-0.052	-0.050	-0.050	-0.047	-0.047
	(-1.50)	(-1.44)	(-4.25)	(-3.45)	(-3.64)	(-3.15)	(-2.92)
$log(Seasonality)$		0.009	0.010	0.011	0.011	0.011	0.011
		(2.63)	(1.89)	(2.44)	(2.51)	(2.38)	(2.23)
$log(1+Analysts)$			-0.044	-0.047	-0.048	-0.048	-0.041
			(-7.37)	(-5.51)	(-5.46)	(-5.32)	(-4.55)
$\hat{\beta}_1$				0.037		-0.081	-0.306
				(0.70)		(-1.13)	(-3.04)
$\mathbf{1}_{\hat{\beta}_1>1}$					0.044		-0.374
					(1.31)		(-4.07)
$\hat{\beta}_1\mathbf{1}_{\hat{\beta}_1>1}$						0.084	0.447
						(2.12)	(4.48)
Ave. Adj. R^2	0.039	0.040	0.042	0.047	0.046	0.049	0.053
T	43	43	27	27	27	27	27
Ave N	2959	2959	3582	3554	3554	3554	3554

Panel B: 12-month β over 1-month β

$log(Size)$	-0.055	-0.059	-0.053	-0.051	-0.053	-0.048	-0.044
	(-5.29)	(-5.48)	(-6.22)	(-5.63)	(-5.74)	(-4.67)	(-4.26)
$log(BM)$	-0.035	-0.035	-0.061	-0.060	-0.060	-0.056	-0.056
	(-1.56)	(-1.51)	(-4.31)	(-4.11)	(-4.29)	(-3.86)	(-3.60)
$log(Seasonality)$		0.022	0.027	0.021	0.022	0.021	0.020
		(4.46)	(4.4)	(4.94)	(5.60)	(4.85)	(4.50)
$log(1+Analysts)$			-0.049	-0.048	-0.052	-0.050	-0.042
			(-5.91)	(-5.11)	(-5.22)	(-4.93)	(-3.89)
$\hat{\beta}_1$				0.001		-0.204	-0.452
				(0.01)		(-1.62)	(-2.44)
$\mathbf{1}_{\hat{\beta}_1>1}$					0.03		-0.460
					(0.57)		(-3.11)
$\hat{\beta}_1\mathbf{1}_{\hat{\beta}_1>1}$						0.145	0.578
						(2.55)	(3.50)
Ave. Adj. R^2	0.032	0.033	0.031	0.042	0.038	0.045	0.050
T	43	43	27	27	27	27	27
Ave N	2760	2760	3351	3324	3324	3324	3324

Table 4: Long Horizon Market βs and HML SMB βs

Panel A reports Fama-MacBeth estimates of:

$$\beta_{12,i,t} = a + b_1 log(Size_{i,t}) + b_2 log(BM_{i,t}) + b_3 \beta_{1,i,t} + b_4 \beta_{smb,i,t} + b_4 \beta_{hml,i,t} + \varepsilon_{i,t}$$

using firm level data for the period 1963-2009. $\beta_{1,i,t}$ ($\beta_{12,i,t}$) is estimated using monthly (12-month compounded) returns over the last 5 years (at least 36 observations required). $Size_{i,t}$ and $BM_{i,t}$ are normalized by their cross-sectional average at time t. $\mathbf{1}_{\beta_1>1}$ is a dummy variable which is equal to 1 if β_1 is larger than 1. Panel B reports Fama-MacBeth estimates for the 25 Fama-French Size and Book-to-market portfolios of:

$$\beta_{12,i} = a + b_1 \beta_{1,i} + b_2 \beta_{rmrf,i} + b_3 \beta_{smb,i} + b_4 \beta_{hml,i} + \varepsilon_i,$$

where β_{12} (β_1) is the 12-month (1-month) CAPM β, and β_{rmrf}, β_{smb}, and β_{hml} are the Fama-French 3-factor βs). The sample period is 1927 to 2009 in Panel B.Standard errors are adjusted for serial correlation using Newey-West with 6 lags.

Panel A: Firm-level Results

β_1	0.952***	0.903***	0.907***	0.714***
	(11.77)	(12.08)	(12.21)	(14.56)
$\mathbf{1}_{\hat{\beta}_1>1}$				-0.558***
				(-3.77)
$\beta_1 \mathbf{1}_{\beta_1>1}$	0.113***	0.116***	0.103***	0.529***
	(2.72)	(2.89)	(3.06)	(4.77)
$log(Size)$	-0.051***		-0.020	-0.011
	(-2.80)		(-1.64)	(-0.99)
$log(BM)$	-0.098***		-0.073***	-0.068***
	(-3.45)		(-3.01)	(-2.87)
$\beta_{smb,1}$		0.135**	0.123**	0.124**
		(2.52)	(2.44)	(2.47)
$\beta_{hml,1}$		-0.101**	-0.092**	-0.087**
		(-2.60)	(-2.34)	(-2.15)
Ave. Adj. R^2	0.241	0.278	0.285	0.291
T	43	43	43	43
Ave N	3428	3428	3428	3428

Panel B: Portfolio Results

β_1	1.045				0.192
	(8.634)				(1.227)
β_{rmrf}		0.726		0.194	
		(1.498)		(1.243)	
β_{smb}			0.293	0.287	0.248
			(12.186)	(11.787)	(5.683)
β_{hml}			0.242	0.238	0.210
			(7.281)	(7.250)	(5.004)
Const	0.005	0.523	1.022	0.824	0.826
	(0.035)	(1.028)	(47.489)	(5.110)	(5.116)
Adj. R^2	0.754	0.049	0.904	0.906	0.906

Table 5: Cross-sectional CAPM regressions using monthly and annual data

This table reports the results of ordinary least squares cross-sectional regressions of mean excess returns on betas for the 25 Fama-French portfolios for the period January 1926 to December 2009.

$$\overline{R_j - R_F} = a + \lambda\beta_j$$

t-statistics, which were calculated using the Shanken (1992) correction, are in parentheses. For regressions that use monthly returns the returns are annualized by multiplying by 12. p-values in brackets are computed from bootstrapped samples that ensure that returns are serially independent.

	a	λ	*Adjusted* R^2	*Mean Absolute Error*
Monthly Returns and βs	0.040	0.052	0.099	0.026
	(0.106)	(1.185)		
		[0.51]	[0.51]	
Monthly Returns and Annual βs	0.003	0.072	0.484	0.015
	(0.023)	(1.679)		
		[0.43]	[0.11]	
Annual Returns and Monthly βs	0.031	0.078	0.130	0.027
	(0.081)	(1.654)		
		[0.37]	[0.43]	
Annual Returns and βs	-0.017	0.102	0.564	0.017
	(0.142)	(2.235)		
		[0.29]	[0.07]	

Table 6: Estimation and test of CAPM pricing kernel for different horizons

This table reports GMM estimates of the parameter b_τ from the equation:

$$E\left\{(1 - b_\tau R_m^\tau)(R_j^\tau - R_F^\tau)\right\} = 0, j = 1, 25$$

where R_m^τ is the τ month return on the market portfolio and $R_j^\tau, j = 1, \cdots, 25$ are the τ month returns on the Fama-French 25 Size and Book-to-market portfolios, and R_F^τ is the τ month risk free rate. In Panel A the returns are overlapping and, to take account of this the weighting matrix, **S**, is calculated following Newey-West with number of lags equal to horizon plus 12 months. In Panel B the returns are non-overlapping December to December returns. J which is distributed $\chi^2(24)$ is a test of the over-identifying restrictions. t-statistics are shown in parentheses and p-values in brackets.

Panel A: Overlapping returns

	1926-2009			1926-1962			1963-2009		
τ (months)	b_τ	J	*Mean Abs. Error*	b_τ	J	*Mean Abs. Error*	b_τ	J	*Mean Abs. Error*
1	2.156	78.121	0.026	1.827	40.615	0.023	2.030	72.115	0.041
	(4.329)	[0.000]		(2.852)	[0.018]		(2.082)	[0.000]	
2	2.006	75.833	0.018	1.421	39.774	0.028	3.322	56.326	0.027
	(5.043)	[0.000]		(4.746)	[0.023]		(3.976)	[0.000]	
3	1.863	68.218	0.017	1.312	38.638	0.029	3.194	46.551	0.026
	(5.383)	[0.000]		(6.414)	[0.030]		(4.358)	[0.004]	
4	1.918	61.329	0.018	1.382	35.774	0.028	3.101	40.349	0.026
	(5.731)	[0.000]		(6.069)	[0.058]		(4.884)	[0.020]	
5	1.972	55.665	0.018	1.609	32.790	0.019	3.045	36.520	0.026
	(5.362)	[0.000]		(5.616)	[0.109]		(5.481)	[0.049]	
6	1.956	50.070	0.018	1.778	29.639	0.018	2.940	33.454	0.025
	(5.612)	[0.001]		(6.452)	[0.197]		(6.265)	[0.095]	
7	1.960	45.671	0.018	1.808	26.859	0.018	2.857	31.401	0.025
	(6.037)	[0.005]		(8.208)	[0.311]		(7.151)	[0.143]	
8	1.947	41.958	0.018	1.809	24.743	0.018	2.536	30.079	0.025
	(6.308)	[0.013]		(9.755)	[0.420]		(6.942)	[0.182]	
9	1.906	38.934	0.017	1.766	23.203	0.018	2.490	28.881	0.025
	(6.539)	[0.028]		(10.706)	[0.508]		(7.756)	[0.225]	
10	1.815	36.514	0.017	1.513	22.231	0.016	2.431	28.472	0.024
	(6.924)	[0.049]		(11.629)	[0.565]		(8.620)	[0.241]	
11	1.692	34.509	0.016	1.248	21.316	0.025	2.129	28.217	0.026
	(7.201)	[0.076]		(12.990)	[0.620]		(7.890)	[0.251]	
12	1.602	32.791	0.016	1.167	23.131	0.026	2.082	30.190	0.025
	(7.409)	[0.109]		(15.568)	[0.512]		(8.796)	[0.179]	
18	1.490	25.437	0.015	1.362	24.785	0.018	2.062	28.789	0.027
	(15.052)	[0.382]		(37.753)	[1.000]		(12.695)	[0.231]	
24	1.405	23.141	0.015	1.237	25.424	0.016	1.815	37.316	0.021
	(19.939)	[0.511]		(29.183)	[0.383]		(42.520)	[0.041]	

Panel B: Non-overlapping December to December returns

1926-2009		**1926 to 1962**		**1963 to 2009**	
b	J	b	J	b	J
2.985	33.596	2.364	26.191	3.846	26.927
(11.99)	[0.092]	(12.25)	[0.344]	(9.69)	[0.308]

Table 7: CAPM pricing errors at different horizons

This table shows the pricing errors for the 25 Fama-French Size and Book-to-market portfolios from the CAPM pricing kernel whose estimates are given in Table 6 for the period January 1926 to December 2009. All pricing errors are annualized by multiplying by $12/\tau$. $RMSE$ is the root mean square pricing error.

				τ		
Size	**BM**	1	6	12	18	24
Small	Low	-0.071	-0.065	-0.074	-0.058	-0.054
	2	-0.017	-0.016	-0.026	-0.011	-0.010
	3	0.016	0.014	0.001	0.011	0.008
	4	0.041	0.038	0.019	0.031	0.027
	High	0.061	0.056	0.032	0.047	0.043
2	Low	-0.029	-0.026	-0.033	-0.024	-0.023
	2	0.014	0.017	0.004	0.012	0.008
	3	0.031	0.030	0.016	0.026	0.024
	4	0.037	0.036	0.020	0.031	0.028
	High	0.040	0.041	0.024	0.035	0.031
3	Low	-0.018	-0.013	-0.026	-0.015	-0.016
	2	0.018	0.019	0.007	0.014	0.012
	3	0.027	0.031	0.017	0.024	0.019
	4	0.032	0.033	0.019	0.027	0.024
	High	0.026	0.033	0.018	0.029	0.027
4	Low	-0.002	0.003	-0.008	-0.003	-0.005
	2	0.005	0.009	-0.003	0.003	0.001
	3	0.020	0.023	0.008	0.015	0.012
	4	0.024	0.030	0.015	0.022	0.019
	High	0.018	0.024	0.006	0.018	0.017
Big	Low	-0.004	0.002	-0.008	-0.006	-0.010
	2	0.002	0.007	-0.004	-0.002	-0.005
	3	0.006	0.011	-0.001	0.003	0.000
	4	0.000	0.006	-0.008	-0.001	-0.003
	High	0.023	0.025	0.009	0.013	0.008
$RMSE$		0.030	0.029	0.022	0.024	0.022

Table 8: Asset Pricing with Stochastic Liquidation

The table reports GMM estimates of the moment conditions (14). The system is estimated assuming the probability of liquidation takes a specific form as explained below using monthly returns on the Fama-French 25 Size and Book-to-market sorted portfolios for the period 1926-2009. The moment condition is $E[\sum_{\tau=1}^{K} w_\tau X_\tau] = 0$ where $w_\tau = w(\tau, \gamma) = \frac{f(\frac{\tau}{K}, \gamma_1; \gamma_2)}{\sum_{i=1}^{K} f(\frac{i}{K}, \gamma_1; \gamma_2)}$ for $\tau = 1$ to K, where $f(z, a, b) = z^{a-1}(1-z)^{b-1}/\beta(a, b)$ and $\beta(a, b)$ is based on the Gamma function, or $\beta(a, b) = \Gamma(a)\Gamma(b)/\Gamma(a+b)$, and $X_\tau \equiv (1 - bR_m^\tau)(R_j^\tau - R_F^\tau)$. A likelihood ratio test is performed against the unrestricted model where each horizon is allowed to have a different risk premium, b_τ. The likelihood ratio, LR, is reported and the p-values are based on χ^2 distribution with 6 *d.f.* t-statistics for b are reported in parentheses; standard errors of the γ coefficients are reported in italics, and p-values for the J-statistic and likelihood ratio are in square brackets. Panel A presents the estimation results. Panel B reports the horizon weights, $\phi(\tau_i) \equiv w_i$ and Panel C reports the probability of liquidation at each horizon date, $\pi(\tau_i) = (\phi(\tau_i)/\tau_i) / \Sigma_{j=1}^{K} \phi(\tau_j)/\tau_j$, the estimated Mean Horizon and the estimated Turnover Rate. Panel D reports the distribution of bootstrapped horizon weights and J-statistics.

Panel A: Parameters

	Full Sample	**1926 to 1962**	**1963 to 2009**
γ_1	6.000	37.110	8.465
	0.24	*0.35*	*2.17*
γ_2	5.291	16.880	15.274
	0.26	*0.40*	*3.18*
b	1.572	1.687	2.062
	(3.55)	(7.82)	(4.35)
J	28.066	30.483	29.525
	[0.17]	[0.11]	[0.10]
LR	5.989	10.583	11.697
	[0.424]	[0.102]	[0.07]

Panel B: Horizon Weights (ϕ_τ)

Horizon (months)	1	4	8	12	16	20	24
Full Sample	0.000	0.013	0.164	0.395	0.344	0.084	0.000
1926 to 1962	0.000	0.000	0.000	0.008	0.775	0.216	0.000
1963 to 2009	0.283	0.388	0.227	0.083	0.017	0.001	0.000

Panel C: Probability Weights (π_τ) , Mean Horizon, and Turnover Rate

Horizon (months)	1	4	8	12	16	20	24	Mean Horizon (months)	Turnover (%)
Full Sample	0.000	0.039	0.250	0.400	0.261	0.051	0.000	12.1	74.3
1926 to 1962	0.000	0.000	0.000	0.011	0.808	0.180	0.000	16.7	58.0
1963 to 2009	0.680	0.233	0.068	0.017	0.003	0.000	0.000	2.4	209.6

Panel D: Bootstrapped Horizon Weights and Test Statistics

Horizon (months)	1	4	8	12	16	20	24	*J-stat*
2.5 **percentile**	0.067	0.079	0.002	0.000	0.000	0.000	0.000	28.843
5 **percentile**	0.445	0.086	0.002	0.000	0.000	0.000	0.000	29.648
25 **percentile**	0.871	0.099	0.003	0.000	0.000	0.000	0.000	31.967
50 **percentile**	0.888	0.109	0.004	0.000	0.000	0.000	0.000	33.266
75 **percentile**	0.898	0.123	0.006	0.001	0.000	0.000	0.000	34.387
95 **percentile**	0.912	0.151	0.043	0.065	0.024	0.008	0.005	36.022
97.5 **percentile**	0.915	0.210	0.158	0.110	0.108	0.099	0.050	36.520

Table 9: Probability weights, Estimated Average Horizon and Turnover Rate, and Reported Turnover on NYSE stocks

This table reports the probability weights for different horizons estimated for different sample periods using the Fama-French 25 Size and Book-to-market portfolios. The Turnover Rate is calculated using equation (13). The Reported Turnover Rate for New York Stock Exchange stocks is taken from the New York Stock Exchange Factbook.

Horizon (months)	1	4	8	12	16	20	24	Estimated Mean Horizon	Estimated Turnover	Reported NYSE Turnover
1927 to 1946	0.002	0.156	0.794	0.049	0.000	0.000	0.000	7.6	103.7%	40.8%
1947 to 1966	0.000	0.000	0.000	0.005	0.183	0.797	0.015	19.3	51.5	15.2
1967 to 1986	0.099	0.562	0.300	0.027	0.005	0.004	0.003	5.3	130.3	30.8
1987 to 2006	0.948	0.042	0.004	0.002	0.002	0.001	0.001	1.2	279.1	68.3

Table 10: Pricing errors of portfolios sorted by lagged BM ratio

This table reports the the pricing errors from the CAPM with stochastic liquidation of the 25 Fama-French Size and Book-to-market sorted portfolios (FF25) along with portfolios sorted by Size and (1- to 5- year) lagged Book-to-market ratios (BM Lag1 - BM Lag5). The sample period from 1963 to 2009. The model predicted return is based on the probability weights estimated in Table 8. All pricing errors are annualized.

Size	**BM**	**FF25**	**BM Lag1**	**BM Lag2**	**BM Lag3**	**BM Lag4**	**BM Lag5**
Small	Low	-0.048	-0.029	-0.023	-0.013	-0.024	-0.029
	2	0.022	0.016	0.024	0.025	0.011	0.018
	3	0.030	0.048	0.039	0.032	0.028	0.012
	4	0.058	0.043	0.039	0.041	0.043	0.027
	High	0.066	0.054	0.048	0.048	0.037	0.035
2	Low	-0.028	-0.018	-0.017	-0.016	-0.017	-0.017
	2	0.013	0.018	0.013	0.021	0.013	0.008
	3	0.044	0.033	0.028	0.030	0.031	0.019
	4	0.049	0.046	0.032	0.040	0.040	0.038
	High	0.053	0.039	0.040	0.042	0.028	0.026
3	Low	-0.023	-0.026	-0.027	-0.026	-0.025	-0.024
	2	0.019	0.024	0.009	0.013	0.017	0.018
	3	0.031	0.019	0.024	0.030	0.023	0.021
	4	0.041	0.036	0.037	0.035	0.036	0.030
	High	0.065	0.050	0.042	0.036	0.025	0.015
4	Low	-0.005	-0.003	-0.005	-0.008	-0.005	-0.003
	2	0.000	0.009	0.006	0.007	0.004	-0.001
	3	0.020	0.016	0.013	0.010	0.007	0.013
	4	0.036	0.018	0.017	0.015	0.018	0.018
	High	0.035	0.034	0.025	0.030	0.019	-0.007
Big	Low	-0.011	-0.011	-0.012	-0.012	-0.018	-0.016
	2	-0.001	-0.006	-0.006	-0.007	-0.006	-0.013
	3	-0.001	0.003	-0.007	0.003	0.004	0.002
	4	0.010	0.012	0.011	0.010	0.005	0.004
	High	0.016	0.016	0.001	-0.001	0.010	-0.004
$RMSE$		0.033	0.029	0.026	0.026	0.023	0.019
J		29.525	28.650	28.850	20.039	17.581	10.459
p		[0.10]	[0.12]	[0.12]	[0.52]	[0.68]	[0.97]

Table 11: Pricing Industry Portfolios

The table reports GMM estimates of the moment conditions (14). The system is estimated using monthly returns on the Fama-French 30 Industry portfolios for the period 1926-2009. See note to Table 8.

Panel A: Parameters

	Full Sample	**1926 to 1962**	**1963 to 2009**
γ_1	13.184	28.094	2.602
	0.15	*0.67*	*1.32*
γ_2	35.040	7.914	2.971
	0.13	*0.50*	*1.15*
b	4.577	1.901	4.705
	(7.459)	(19.160)	(15.972)
J	28.895	13.100	20.109
	[0.32]	[0.98]	[0.79]

Panel B: Horizon Weights

Horizon (months)	1	4	8	12	16	20	24
Full Sample	0.058	0.189	0.515	0.065	0.058	0.058	0.058
1926 to 1962	0.039	0.039	0.039	0.040	0.127	0.675	0.041
1963 to 2009	0.028	0.127	0.242	0.272	0.212	0.102	0.017

Panel C: Probability Weights (π_τ), Mean Horizon and Turnover Rate

Horizon (months)	1	4	8	12	16	20	24	Mean Horizon (months)	Turnover (%)
Full Sample	0.315	0.257	0.351	0.029	0.020	0.016	0.013	5.5	131.4
1926 to 1962	0.390	0.097	0.049	0.033	0.079	0.336	0.017	10.0	88.5
1963 to 2009	0.215	0.240	0.229	0.171	0.100	0.038	0.005	7.5	105.8

Table 12: Pricing 30 Industry Portfolios and 25 Size and Book-to-market sorted Portfolios

The table reports GMM estimates of the moment conditions (14). The system is estimated using monthly returns on the Fama-French 25 Size and Book-to-market portfolios and 30 Industry portfolios for the period 1926-2009. See note to Table 8.

Panel A: Parameters

	Full Sample	**1926 to 1962**	**1963 to 2009**
γ_1	6.303	9.486	1.004
	0.50	*0.16*	*0.40*
γ_2	5.624	8.013	11.673
	0.65	*0.17*	*0.50*
b	1.662	1.279	2.304
	(3.876)	(5.455)	(2.787)
J	32.183	12.267	17.786
	[0.98]	[1.00]	[1.00]

Panel B: Horizon Weights

Horizon (months)	1	4	8	12	16	20	24
Full Sample	0.000	0.011	0.164	0.408	0.343	0.074	0.000
1926 to 1962	0.015	0.016	0.101	0.426	0.373	0.054	0.015
1963 to 2009	0.645	0.180	0.047	0.033	0.032	0.032	0.032

Panel C: Probability Weights (π_τ), Mean Horizon and Turnover Rate

Horizon (months)	1	4	8	12	16	20	24	Mean Horizon (months)	Turnover (%)
Full Sample	0.000	0.034	0.249	0.412	0.260	0.045	0.000	12.1	74.4
1926 to 1962	0.161	0.043	0.135	0.378	0.248	0.029	0.007	10.6	82.8
1963 to 2009	0.917	0.064	0.008	0.004	0.003	0.002	0.002	1.4	266.4

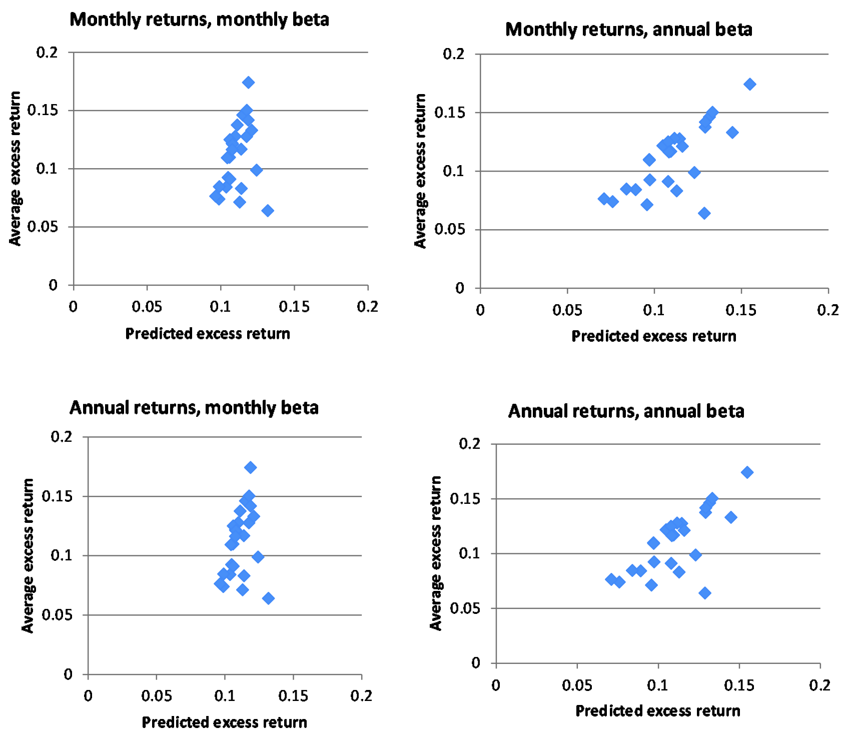
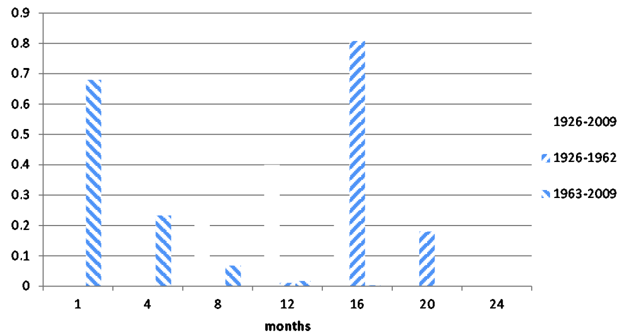
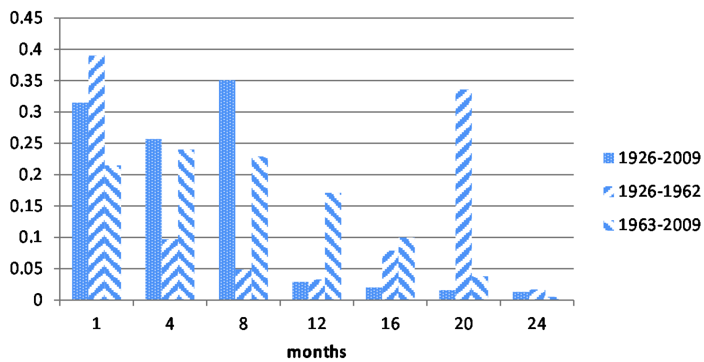


Figure 1: Actual and Predicted Returns

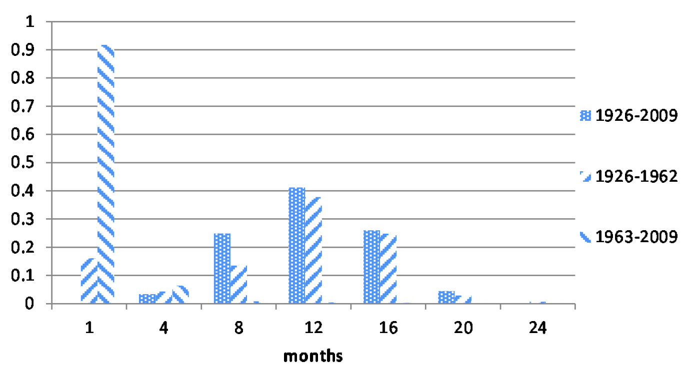
The plots show different measures of average excess return against CAPM predicted returns for the Fama-French 25 Size and Book-to-market sorted portfolios for the period 1926-2009. The average excess returns are either average annual excess returns (Annual returns) or annualized average monthly excess returns (Monthly returns). The CAPM predicted return is provided by the CAPM cross-sectional regressions using annual or monthly returns for the period 1926-2009 whose parameters are reported in Table 5.



A. 25 Size and Book-to-market sorted portfolios



B. 30 Fama-French industry portfolios



C. 55 Fama-French Size and Book-to-market and industry portfolios

Figure 2: Estimated probability distributions of liquidation dates for Fama-French Size and Book-to-market and industry portfolios 1926-2009

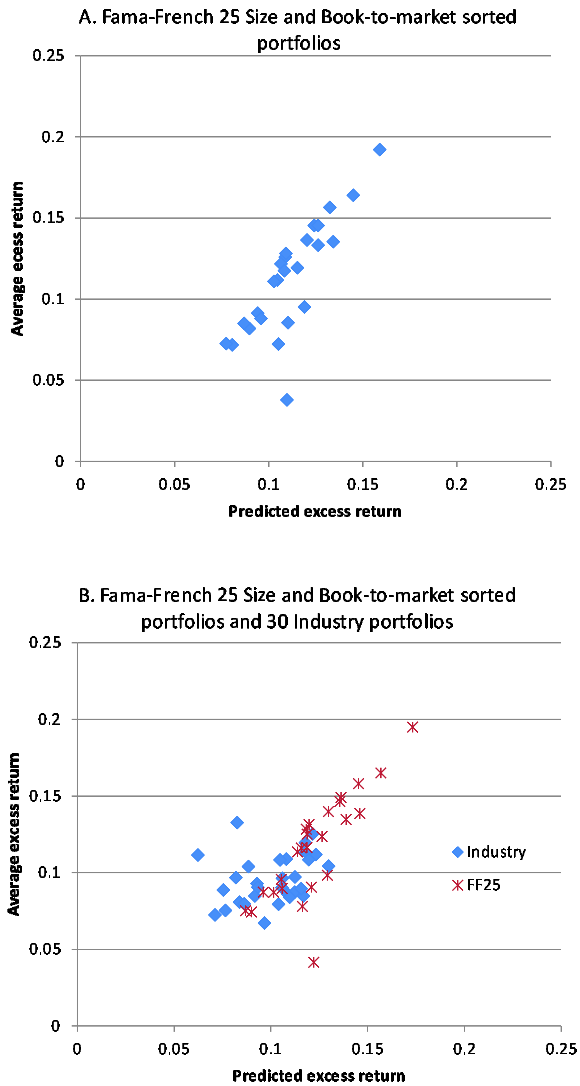


Figure 3: Actual and Stochastic Liquidation CAPM Predicted Returns
Horizon weighted annualized average excess returns and horizon weighted annualized predicted returns from the stochastic liquidation CAPM for the Fama-French Size and Book-to-market portfolios and industry portfolios 1926-2009