Interbank network and bank bailouts: Insurance mechanism for non-insured creditors?[☆]

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Abstract

This paper presents a theory that explains why it is beneficial for banks to be highly interconnected on the interbank market. Using a simple network structure, it shows that, if there is a non-zero bailout probability, banks can significantly increase the expected repayment of uninsured creditors by entering into cyclical liabilities on the interbank market before investing in loan portfolios. Therefore, banks are better able to attract funds from uninsured creditors. Our results show that implicit government guarantees incentivize banks to have large interbank exposures, to be highly interconnected, and to invest in highly correlated, risky portfolios.

Keywords: bailout, cycle flows, interconnectedness, interbank network, leverage
JEL: G01, G21, G28

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1. Introduction

The 2008-2009 financial crisis has prompted many questions about the resilience of the interbank market. Strong growth in the size and density of the interbank network has made concerns such as "too big to fail" and "too interconnected to fail" widespread. This raises the question of why market solutions did not emerge to an extent that would have avoided concerns about this high interconnectedness? However, there is only scarce knowledge of why banks enter into such a high degree of connectivity in the first place, especially since these connections often include cyclical liabilities that could potentially be netted out.[1]

Such cyclical liabilities can be observed both, bilaterally between banks as well as in the form of large structural cycles throughout the financial system. For example, after the removal of explicit public guarantees for German Landesbanken had been announced in 2001, these banks started to issue longterm debt and invest the proceeds in bonds of other Landesbanken (Fitch, 2006). In a broader perspective, Heijmans, Pröpper, and van Lelyveld (2008) show the existence of large circular interbank net flows (up to EUR 90 billion) domestically and across the entire TARGET system.

The goal of this paper is to provide a theoretical underpinning for the high bank interconnectedness and the existence of these large circular net flows. We claim that the interbank network serves as an insurance mechanism for bank creditors if they are not already covered by deposit insurance (e.g., the FDIC). If a bank failure occurs and there is a nonzero probability that banks will be bailed out by the government, connections to other banks (e.g., exposures arising from credit default swap (CDS) contracts, bonds, and interbank lending) increase the expected repayment of uninsured creditors.

The mechanism presented in this paper differs from the effects of government bailouts on bank behavior considered in the literature so far. It is well known that the possibility of a government bailout increases the potential for moral hazard at the individual bank level. Moreover, it has been argued that banks try to increase the probability of a bailout by becoming very large and/or highly interconnected (e.g., Freixas, 1999). We show that, even if we abstract from these two moral hazard channels, there is still an incentive for banks to be highly interconnected since this increases the value of government bailouts for individual banks by transferring wealth from the government to the private sector.

[1] Note that high interconnectedness implies many cyclical liabilities (e.g., Takács, 1988).

Even with a constant, exogenously given bailout probability (i.e., a bailout probability that is not increasing in either balance sheet size, interconnectedness, or the number of failing banks), we show that the wealth transfer from the government to the private sector increases with the degree of interconnectedness. In a nutshell, cyclical interbank connections increase banks' liabilities and thus increase the amount of cash governments have to inject to bail out banks. As a result of the banks' interconnectedness, this extra cash trickles down to other banks in the network, benefiting them and their creditors. This result holds even if we allow the interbank market to exist for a different reason (e.g., liquidity coinsurance). Due to the resulting high interconnectedness, banks lend large amounts among themselves, leading to increased leverage for each bank and high systemic risk.

Given that a high degree of interconnectedness and the resulting cycle flows create an additional transfer from the government to the private sector, in a second step we analyze how banks can optimally exploit these transfers. By creating high interbank exposure and by investing in risky, correlated assets, banks can maximize the government subsidy per invested unit of capital. Furthermore, we show that this investment behavior does not rely on the conjecture that the individual bailout probability is potentially increasing with the number of failing banks (Acharya and Yorulmazer, 2007); that is, such behavior is still prevalent for a constant, exogenously given bailout probability.

Understanding the interdependence between investment behavior and interbank connections is crucial, since systemic risk not only arises from bank interconnectedness but also results from a "joint failure risk arising from the correlation of returns on the asset side of bank balance sheets" (Acharya, 2009, p. 225). In essence, the mechanism presented in this paper provides an incentive for banks to increase both types of systemic risk. Moreover, we show that these types of risk cannot be considered individually, since the benefits from high interconnectedness are maximized by investing in correlated loan portfolios. Therefore, our model helps explain why banks invested in risky correlated investments (e.g., US subprime loans) in the run-up to the financial crisis.

The rest of the paper is organized as follows. Section 2 provides an overview of the related literature. Using a simple example, Section 3 shows how cycle flows create an additional wealth transfer from the government to the private sector in case there is a positive bailout probability. Section 4 develops our main model and determines how banks can maximize the value of government bailouts. Section 5 provides two extensions of our main model. First, we introduce asymmetric bank bailout probabilities and analyze the impact on the optimal level of interbank exposure. Second, we show that, given that banks are interconnected, they have an incentive to engage in risk shifting. Section 6 discusses policy implications, while Section 7 concludes.

2. Related literature

Our paper is related to several strands of the theoretical literature. First, it adds to the literature on liquidity and interbank markets. Pioneering work in this area has been accomplished by Allen and Gale (2000), who show that banks can coinsure each other through an interbank market against liquidity shocks as long as these shocks are not perfectly correlated. This theme has been taken on by many other papers. For example, Freixas and Holthausen (2005) analyze the scope for international interbank market integration when cross-border information about banks is less precise than home country information. Here, banks can cope with these shocks by investing in a storage technology or can use the interbank market to channel liquidity. Allen, Carletti, and Gale (2009) show that the interbank market is characterized by excessive price volatility if there is a lack of opportunities for banks to hedge aggregate and idiosyncratic liquidity shocks.

Furthermore, our paper relates to the literature on bank bailouts. Acharya and Yorulmazer (2007) focus on whether governments have an incentive to bail out banks ex post if they engaged in herding behavior ex ante. Diamond and Rajan (2002) show that bailouts alter available liquidity in the economy and distinguish between well targeted bailouts (which can be beneficial) and poorly targeted ones that can lead to a systemic crisis. Gorton and Huang (2004) argue that there is a potential role for governments to provide liquidity through, for example, bank bailouts to reduce the problem of agents hoarding liquidity inefficiently. In contrast to these studies, we use a constant exogenously given bailout probability to avoid mingling the mechanism presented in this paper with the incentive to become interconnected that results from an increase in the individual bailout probability. Leitner (2005) and David and Lehar (2011) show that interbank linkages can be optimal ex ante because they act as a commitment device to facilitate mutual private sector bailouts. In contrast, we investigate the effect of government bailouts on the incentives of banks to create such liabilities.

Our paper also provides a theoretical underpinning for several empirical findings. There is ample evidence that the global banking network has a very high density and a high degree of concentration. Using locational statistics from the Bank for International Settlements (BIS) on exchange-rate adjusted changes in cross-border bank claims, Minoiu and Reyes (2011) analyze the global banking network and find that, besides a high network density, there exists a positive correlation between network density and the circularity of liabilities (measured by the network's clustering coefficient). For the overnight market in the United Kingdom, Soramäki, Wetherlit, and Zimmermann (2010) find that the net lending/borrowing amounts are much lower than the gross trades, implying many cyclical liabilities in this market. Kubelec and Sá (2010) show that the interconnectivity of the global financial network has increased significantly over the past two decades. In line with our results, they find that the global financial network is characterized by a large number of small links and a small number of large links and that the network has become more clustered. Similar evidence can be found for national interbank markets (Wells, 2004; Mueller, 2006; Arnold, Bech, Beyeler, Glass, and Soramäki, 2006). Furthermore, there is also a very high interconnectedness in other interbank markets besides the traditional interbank lending market. For example, a 2011 report by the Bank for International Settlements shows that banks also have very high cross-exposures due to derivative contracts (mainly CDSs), since banks that sell CDSs in turn also purchase them to hedge their risk.

3. Main idea

We use a very simple framework to illustrate how cycle flows create an additional wealth transfer from the government to the private sector. The main model then analyzes how banks can optimally exploit this mechanism to maximize the expected value of government bailouts. We assume that the interbank market consists of a few banks and some uninsured creditors (e.g., mutual funds, bondholders, smaller banks). One of the banks has an investment project that costs one unit in the first period and generates a return $R > 1$ in the second period with probability λ and a return of zero otherwise. The only source of capital to fund this project is to borrow from the uninsured creditors. In return for the initial funding, the bank must repay R_D to its uninsured creditor. All parties are risk-neutral.

We develop the intuition of our mechanism in two steps. First, we discuss a situation without network connections to other banks. At $t = 0$ the bank (B_A) borrows one unit from the uninsured creditor (C) and invests in a project (P). In the second period, the cash flow from the project is realized. If the project is successful, the bank receives an amount R and is able to fully repay its uninsured creditor. If the project fails and the bank is not bailed out, the uninsured creditor receives no repayment. Conversely, if the government bails out the bank (i.e., takes over the bank and settles all its liabilities), the creditor again receives his full repayment (see Fig. 1).

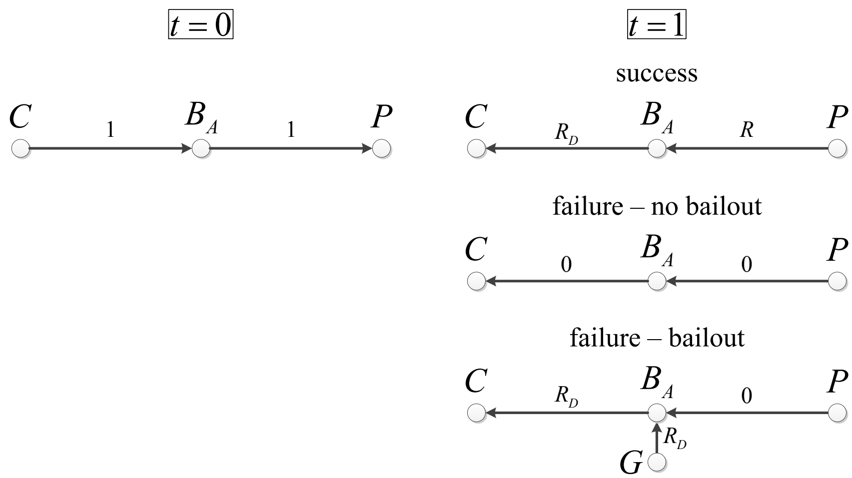

Figure 1: Capital flows without interbank market

In a second step, we allow the bank to establish an interbank network at $t = 0$ by lending funds in a circular way before investing into the project. To be precise, bank B_A lends the funds it receives from its creditor to bank B_B, which in turn lends it to bank B_C, from which the capital flows back to B_A and is then invested into the project. For entering into an interbank exposure of K, this circular lending procedure has to be repeated K times. For now, we assume that banks B_B and B_C do not have any other investments. We relax this assumption in the next Sections. If the project is successful, B_A receives the project return R and uses it to settle its liabilities with B_C.[2] After receiving the payment from B_B, B_A repays its uninsured creditor. If the project fails, bank B_A defaults since it cannot repay its creditors. If the government steps in and bails out bank B_A, both the uninsured creditor of B_A and bank B_C receive their full repayment R_D, implying that all claims are settled in

[2]Throughout the paper we assume that, as soon as there exists a clearing payment vector, the banks use this vector to settle all liabilities in the network. If the sequence of payments is chosen in a less sophisticated manner, banks can still default, even though there is enough liquidity in the system to settle all claims. However, an unsophisticated settlement process would only reinforce our mechanism, since it would increase the value of the government's implicit guarantee.

this case. If the government refuses to bail out B_A, B_C defaults as well. Now it depends on whether the government (not necessarily the same one as in the case of B_A, since B_C could be established in another country) bails out B_C. If it does, it takes over B_C and settles its liabilities. Therefore, B_B receives K from B_C and hence B_B can pay back its debt to B_A. However, B_A has total liabilities of $R_D + K$ and is therefore still unable to meet all its obligations. Consequently, the funds B_A received from B_B must be divided among the creditors of B_A, that is, the uninsured creditor of B_A, on the one hand, and B_C, on the other hand.

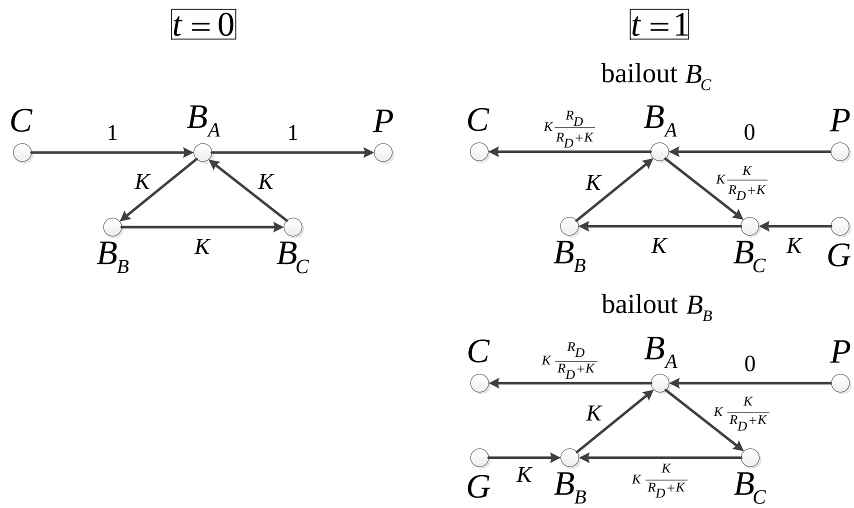

Figure 2: Capital flows with interbank market

The common procedure in bankruptcy proceedings is for debt to be paid back on a pro rata basis once a default occurs. Therefore, the uninsured creditor of B_A receives $K \cdot R_D/(R_D + K)$ and bank B_C receives $K \cdot K/(R_D + K)$. Hence, even though the uninsured creditor's own bank fails and is not bailed out, he receives a positive repayment due to the existence of the interbank network. Furthermore, this repayment increases with the level of interbank exposure. Since the government takes over B_C, it receives the $K \cdot K/(R_D+K)$ from B_C. However, it has to pay R_D+K to bail out the bank and hence records a loss. The case in which B_C is not bailed out but B_B is can be described analogously. The corresponding cash flows are presented in Fig. 2. Hence, in case there is a positive probability of a government bailout if a bank defaults, the bank can considerably increase the expected repayment of its uninsured creditor by first channeling funds through the interbank market and only investing them into the project afterwards. The reason is that, with an interbank market in place, the uninsured creditor receives a positive repayment as soon as at least one of the banks is bailed out. Furthermore, it is important to note that this mechanism works with any other sharing rule during bankruptcy proceedings and becomes even stronger if interbank funding has a lower seniority than the liabilities of uninsured creditors. The reason is that C's share $R_D/(R_D + sK)$ of the bailout funds received from B_B is higher the lower the interbank funding seniority s.

If the bank has the bargaining power, creditors will demand a lower interest rate (risk premium) given the existence of an interbank network (the participation constraint of uninsured creditors is already binding for lower values of R_D), which considerably reduces the bank's borrowing cost. This reduction in turn leads to higher profits for the bank, which can help explain the comparatively high return-on-equity ratios of banks. If, on the other hand, the uninsured creditor has the bargaining power, he will increase his expected repayment by increasing R_D until the participation constraint of the owners of the bank is just binding. Furthermore, creditors will only deposit money in banks that are part of a highly connected interbank network, since the expected repayment in this case is higher than when the bank is not connected to others via an interbank market.

4. The main model

Having described how a high degree of interconnectedness and the resulting cycle flows create an additional wealth transfer from the government to the private sector, we now investigate how banks can use cycle flows to optimally exploit implicit bailout guarantees. We consider an economy that consists of two dates $t = 0$ and $t = 1$ and two different regions, A and B (which can be interpreted as, e.g., two different countries). Each region is comprised of a continuum of identical banks. We assume that, due to competition, all banks adopt the same behavior and can thus be described by a representative bank (protected by limited liability). The representative bank in region A (B) is denoted by B_A (B_B).

Furthermore, we assume that there exists a risk neutral uninsured creditor and one investor who provides equity financing to the bank in each region. Creditors are denoted C_A and C_B in regions A and B, respectively. The contract between the uninsured creditor and the bank takes the form of a standard debt contract; that is, it specifies the interest payment R_D and it cannot be made contingent on either the realization of the investment or the realization of the state of nature. However, the parties can contractually specify the bank's interbank exposure and the structure of its loan portfolio. We abstract from deposit financing, since such funds are explicitly protected by a deposit insurance scheme and thus depositors are not affected by the banks' repayment abilities. Therefore, including deposits in the model does affect our results as long as banks also borrow from noninsured creditors. The timing of our model is depicted in Fig. 3.

$t = 0$	$t = 1$
Investors provide equity $\underline{e}$ Banks raise debt capital c Banks exchange K Banks invest in loan portfolio	Cash flows are realized Possibly government bailouts

Figure 3: Timing of the model

Each bank has access to two scalable investment possibilities in two different industries (denoted 1 and 2) at $t = 0$, as in Acharya and Yorulmazer (2007). One can think of these investment opportunities as portfolios of loans to firms in one of the two industries. More precisely, bank B_A (B_B) can lend to firms in industry A_1 or A_2 (B_1 and B_2). If in equilibrium banks decide to lend to firms in the same industry, that is, they either lend to A_1 and B_1 or to A_2 and B_2, then the returns of their loan portfolios are assumed to be perfectly correlated ($\rho = 1$). However, if they decide to invest in different industries, we assume that the returns are uncorrelated ($\rho = 0$). Similar to Rochet and Tirole (1996), we assume that both investments are stochastic decreasing-returns-to-scale technologies, which return $I \cdot R$ with probability λ (where $\lambda R > 1$) and yield a return of zero with probability $(1 - \lambda)$ at $t = 1$. The costs for an initial investment of size I are $\psi(I)$ at $t = 0$, where $\psi(0) = 0$, $\psi'(0) = 1$, and $\psi'' > 0$. Consequently, the decision in which industry to invest only affects the correlation of returns, but not their magnitude. This structure allows us to determine how interbank connections influence the banks' incentive to invest, that is, the size and the correlation of their loan portfolios.

In line with Allen and Gale (2000), the banks can establish an interbank market (network) by exchanging an arbitrary amount of interbank deposits K at $t = 0$, which have to be repaid at $t = 1$. When increasing interbank deposits, the banks incur transaction costs $\tau(K)$, where $\tau(0) = \tau'(0) = 0$ and $\tau'' > 0$. These costs include a variety of expenses associated with trading funds, such as brokerage and CHIPS or Fedwire transaction fees or the costs of searching for banks with matching liquidity needs. The convex form of $\tau(K)$ represents the increasing marginal costs of searching for trade partners and those resulting from the need to split large interbank transactions into many small ones to work around credit lines (e.g., Neyer and Wiemers (2004)).

Lastly, we assume that, due to regulatory requirements, banks need at least an equity contribution of $\underline{e}$. To model equity investors we follow Allen and Gale (2005) and Brusco and Castiglionesi (2007) in that we assume that the equity investor E_A (E_B) in region A (B) is endowed with $e \geq \underline{e}$ units of capital at $t = 0$ and has no endowment at date $t = 1$. The investors can use their endowment for either consumption or to buy bank shares. In the latter case the investors are entitled to receive dividends at $t = 1$ (denoted by d_1). Their utility is then given by

$$u(d_0, d_1) = d_0 \lambda R + d_1 \tag{1}$$

Since an investor can obtain a utility of $e\lambda R$ by immediately consuming his initial endowment (consumption at $t = 0$ is denoted by d_0), he has to earn an expected return of at least λR on the invested capital to give up consumption at $t = 0$. By investing an amount $\underline{e}$ at $t = 0$, the equity investor obtains a lifetime utility of $(e - \underline{e})\lambda R + d_1$. Hence, the investors will only buy bank shares if the expected utility from doing so is higher than the utility they would get from immediately consuming their endowment, that is, if

$$(e - \underline{e})\lambda R + E[d_1] \geq e\lambda R \tag{2}$$

holds. This setup leads to the following participation constraint for investors:

$$E[d_1] \geq \underline{e}\lambda R \tag{3}$$

Under the assumption of perfect competition in the banking market (i.e., creditors have all the bargaining power), this constraint will be binding.[3] Therefore, if a bank wants to invest I and have an interbank exposure of K, it has to raise $c = \psi(I) + \tau(K) - \underline{e} > 0$ from the uninsured creditor. Increasing the equity level above the required minimum can not be

[3]Shifting the bargaining power to the equity investors only changes the distribution of the benefits from implicit government guarantees, but does not affect bank behavior.

optimal, since equity raises the marginal investment costs and thus decreases the expected creditor payment. In the following, we assume that $\psi'(\underline{e}) < \lambda R$, which ensures that it is always optimal to raise debt from the uninsured creditor.

If both investments are successful, the banks are able to settle their interbank claims, repay the uninsured creditors, and pay the investors a positive dividend. If, however, the investment of one or both banks fails, either one or both banks may not be able to meet their liabilities and will consequently default. In case of a default, we assume that there is a positive probability α that the government of the respective country will step in and bail out the bank, that is, take over the bank and repay all its liabilities.[4] It would be reasonable to assume that α is increasing in the interconnectedness of the bank (too interconnected to fail), its balance sheet size (too big to fail), and the number of failing banks (too many to fail). However, to isolate the direct effect that cycle flows have on the expected repayment of uninsured creditors, we assume that the bailout probability is not increasing in either the balance sheet size of the bank or its interconnectedness or the number of failing banks. Setting up the model such that the bailout probability increases with one of these factors (i.e., the governments decide whether to bail out a defaulted bank or not) would reinforce our results, since then higher interconnectedness increases the wealth transfer from the government to the private sector even further.

Consequently, the payments to the uninsured creditors and investors depend on the performance of the loan portfolio and on whether a bank is bailed out if a default occurs. Due to perfect competition in the banking sector, banks thus seek to maximize the repayment of uninsured creditors by choosing the parameters R_D, ρ, I, and K. Having described the setup, we now return to our main questions in this Section: What level of interbank exposure do banks choose, which investment size (and, in turn, which amount of creditor funds) is optimal, and do banks prefer to invest in correlated or uncorrelated assets to optimally exploit implicit bailout guarantees? Furthermore, we analyze how these decisions are interrelated.

All aspects are important to consider, since they all increase systemic risk. On the one hand, interconnectedness and high leverage lead to systemic risk resulting from spillover

[4]For our mechanism to work, it is sufficient that the bailout probabilities are not perfectly positively correlated, which is certainly true if the banks are established in different countries. Furthermore, anecdotal evidence shows that bailout decisions seem to be negatively correlated within the same country (e.g., the bailout of Bear Stearns after the default of Lehman Brothers), which reinforces the mechanism.

effects that are transmitted through the interbank market (even without correlation on the asset side of the banks' balance sheet). On the other hand, even without being interconnected, investment correlation increases systemic risk due to possible joined bank failures. The following analysis investigates the interaction between these sources of systemic risk and determines how interconnectedness influences the banks' investment decision, that is, whether they invest in correlated or uncorrelated loan portfolios. To analyze this issue, we derive the highest expected repayment banks can achieve with an investment correlation of zero and one, respectively. Then we compare the resulting repayments to determine which of the two yields a higher return for uninsured creditors.

4.1. Positively correlated investments

Consider first the situation in which bank investments are perfectly positively correlated, that is, $\rho = 1$. In this case there are five different outcomes (depending on the success of the investments and whether the banks are bailed out or not), depicted in Table 1.

$\rho = 1$	Prob.	L_A	L_B	B_A	B_B	C_A	C_B	E_A	E_B
S_1	λ	S	S	N	N	R_D	R_D	$IR - R_D$	$IR - R_D$
S_2	$(1-\lambda)\alpha^2$	F	F	B	B	R_D	R_D	0	0
S_3	$(1-\lambda)(1-\alpha)\alpha$	F	F	B	N	R_D	$K\frac{R_D}{R_D+K}$	0	0
S_4	$(1-\lambda)(1-\alpha)\alpha$	F	F	N	B	$K\frac{R_D}{R_D+K}$	R_D	0	0
S_5	$(1-\lambda)(1-\alpha)^2$	F	F	N	N	0	0	0	0

Table 1: Capital flows for investment correlation of $\rho = 1$

Column 1 presents the five different states, while Column 2 presents the probability of each given state occurring. Columns L_A and L_B show whether the investments of banks B_A and B_B are successful (S) or not (F). Columns B_A and B_B indicate whether banks B_A and B_B are bailed out by the government (B) or not (N). The Columns C_A and C_B show the repayment of uninsured creditors, while Columns E_A and E_B show the dividends the equity holders receive. To understand the cash flows presented in Table 1, first note that if either both investments are successful (S_1) or both banks are bailed out (S_2), the uninsured creditors of both banks will receive their full repayment. These states only differ with respect to the dividend paid to the investor, since in the case of a bailout the government takes over the bank and thus has the residual claim. Assuming that equity is only partially wiped out after a default would only reinforce our results, since this would relax the participation constraint of the equity investor. If only one bank is bailed out (S_3 and S_4), then the creditor of this bank will receive the full repayment whereas the creditor of the other bank will still receive a fraction $K/(R_D + K)$ of his claim R_D, despite the fact that his own bank is not bailed out (network insurance). Since the model is symmetric, it is sufficient to focus on the optimization problem of one of the banks (we relax this assumption in Section 5). Hence, we only analyze the behavior of bank B_A. Due to perfect competition, bank B_A wants to maximize the expected utility of its uninsured creditor C_A. Thus, the optimization problem at $t = 0$ becomes:

$$\max_{R_D,I,K} U_1 = \lambda R_D + (1-\lambda)\left[\alpha R_D + (1-\alpha)\alpha K \frac{R_D}{R_D + K}\right] - c \tag{4}$$

subject to

$$E[d_1] \geq \underline{e}\lambda R \tag{5}$$

The objective function consists of the following parts: With probability λ the investment of the bank is successful and creditors receive their contractually specified repayment R_D. With probability $(1-\lambda)$ the investment fails. In this case the return of the creditors depends on whether the banks are bailed out or not. Specifically, if bank B_A is bailed out (which happens with probability α), the government repays all liabilities and hence its creditors again receive the full repayment. If, however, the government does not bail out bank B_A, the repayment depends on whether bank B_B is bailed out. If bank B_B is not bailed out either, the repayment is clearly zero. However, if bank B_B is bailed out, the government injects $R_D + K$. This bailout then allows bank B_B to settle all its claims. Therefore, B_A receives K and has to split these proceeds between its uninsured creditor C_A and bank B_B. As described before, in bankruptcy proceedings this splitting is usually done on a pro rata basis, that is, the uninsured creditor of bank B_A receives a share $R_D/(R_D + K)$ of the funds bank B_A received from B_B.

Furthermore, the binding participation constraint of the equity holder implies

$$E[d_1] = \underline{e}\lambda R \Rightarrow \lambda\left[IR - R_D\right] = \underline{e}\lambda R \Rightarrow R_D = (I - \underline{e})R \tag{6}$$

Inserting $R_D = (I - \underline{e})R$ and c into Eq. (4) yields the following maximization problem:

$$\max_{I,K} U_1 = \lambda(I-\underline{e})R + (1-\lambda)\left[\alpha(I-\underline{e})R + (1-\alpha)\alpha K \frac{(I-\underline{e})R}{(I-\underline{e})R + K}\right] - \psi(I) - \tau(K) + \underline{e} \tag{7}$$

The first-order conditions lead to:

Proposition 4.1.

a) If the banks choose correlated investments, there exist unique, interior optimal levels of the investment size, I_1^*, the creditor liabilities, $(I_1^* - \underline{e})R$, and the interbank exposure, K_1^*.

b) Larger interbank exposure K incentivizes banks to have more creditor liabilities and to invest more and vice versa.

c) Higher equity requirements $\underline{e}$ incentivize banks to lower their interbank exposure and to invest more.

Proof See the Appendix. QED

For high interbank exposures, the governments have to inject more funds in the banking sector in case of a bailout. Hence, if the banks default and only B_B is bailed out, the amount B_A receives from B_B increases with the interbank exposure. If this amount is large, bank B_A is also incentivized to increase its creditor liabilities and invest more funds since then a larger share $(I - \underline{e})R/((I - \underline{e})R + K)$ of the funds bank B_A receives from B_B is paid to the creditor of B_A. Hence, creditor liabilities and interbank exposure are mutually reinforcing each other, implying that banks with a high interbank exposure have an incentive to increase creditor liabilities and vice versa. This mechanism thus results in high bank leverage and interconnectedness. Furthermore, higher equity requirements decrease the face value of creditor liabilities and, in turn, the share of the other bank's bailout funds that is paid to the creditor of B_A. This incentivizes banks to lower their interbank exposure and, due to its higher marginal effect on the creditor's bailout share, they are incentivized to increase the investment size. We derive comparative statics for changes in the bailout probability α in Section 5.1., where the implications of a change in the bailout probability can be analyzed in more detail due to asymmetric bail probabilities.

Therefore, the highest expected utility for the creditor that can be achieved when choosing correlated investments is:

$$\overline{U_1} = \lambda(I_1^* - \underline{e})R + (1-\lambda)\left[\alpha(I_1^* - \underline{e})R + \alpha(1-\alpha)K_1^* \frac{(I_1^* - \underline{e})R}{(I_1^* - \underline{e})R + K_1^*}\right] - \psi(I_1^*) - \tau(K_1^*) - \underline{e} \quad (8)$$

4.2. Uncorrelated investments

We next turn to the case in which banks decide to invest in different industries, that is, $\rho = 0$. Here, two scenarios must be considered. On the one hand, the interbank exposure can be chosen such that even if one bank's investment is successful but the other bank's investment fails, the first bank will be unable to repay its obligations and hence financial contagion will occur. On the other hand, if the exposure is low enough, a successful bank will stay solvent no matter what happens to the other bank. Let K^t denote the "switching point", that is, the level of interbank exposure where a successful bank will just stay solvent, even if the other bank fails (see the Appendix for the derivation of K^t). The different possibilities for the cash flows are presented in Tables 2 and 3, where the notation is as described before. It is crucial to note that the interest rate R_D differs between the two possibilities, since the participation constraints of the equity investors differ. Table 2 presents the cash flows for $K < K^t$.

$\rho = 0$	Prob.	L_A	L_B	B_A	B_B	C_A	C_B	E_A	E_B
S_1	λ^2	S	S	N	N	R_D^{nc}	R_D^{nc}	$IR - R_D^{nc}$	$IR - R_D^{nc}$
S_2	$(1-\lambda)^2\alpha^2$	F	F	B	B	R_D^{nc}	R_D^{nc}	0	0
S_3	$(1-\lambda)^2(1-\alpha)\alpha$	F	F	B	N	R_D^{nc}	$K\frac{R_D^{nc}}{R_D^{nc}+K}$	0	0
S_4	$(1-\lambda)^2(1-\alpha)\alpha$	F	F	N	B	$K\frac{R_D^{nc}}{R_D^{nc}+K}$	R_D^{nc}	0	0
S_5	$(1-\lambda)^2(1-\alpha)^2$	F	F	N	N	0	0	0	0
S_6	$\lambda(1-\lambda)\alpha$	S	F	N	B	R_D^{nc}	R_D^{nc}	$IR - R_D^{nc}$	0
S_7	$\lambda(1-\lambda)\alpha$	F	S	B	N	R_D^{nc}	R_D^{nc}	0	$IR - R_D^{nc}$
S_8	$\lambda(1-\lambda)(1-\alpha)$	S	F	N	N	R_D^{nc}	$K\frac{R_D^{nc}}{R_D^{nc}+K}$	X_0	0
S_9	$\lambda(1-\lambda)(1-\alpha)$	F	S	N	N	$K\frac{R_D^{nc}}{R_D^{nc}+K}$	R_D^{nc}	0	X_0

Table 2: Outcomes for $K < K^t$, where $X_0 = IR - R_D^{nc} - K\frac{R_D^{nc}}{R_D^{nc}+K}$ - No contagion

States $S_1 - S_5$ parallel the respective outcomes in Table 1. Things differ from the results of Table 1 if only one investment fails, depending on whether the successful bank stays solvent (no contagion; see Table 2) or also becomes insolvent (see Table 3). If the interbank exposure is low enough ($K < K^t$) such that there is no contagion, then the successful bank can always fully repay its uninsured creditor, whereas the creditor of the unsuccessful bank will only receive the full amount if this bank is bailed out (S_6 and S_7 in Table 2). If the unsuccessful bank is not bailed out, its creditor will get just a fraction of his repayment (S_8 and S_9 in Table 2). If, on the other hand, the interbank exposure is higher than the threshold K^t, the successful bank will not be able to settle its interbank liabilities and, on top of that, will be unable to fully repay its creditor. Depending on which bank (if any) is bailed out, the creditors of both the successful and the failed bank receive either their full repayment or just a fraction ($S_6 - S_{11}$ in Table 3). In a next step, we compare the expected repayments of the uninsured creditor in these two scenarios, that is, $K < K^t$ and $K \geq K^t$.

$\rho = 0$	Prob.	L_A	L_B	B_A	B_B	C_A	C_B	E_A	E_B
S_1	λ^2	S	S	N	N	R_D^c	R_D^c	$IR - cR_D^c$	$IR - cR_D^c$
S_2	$(1-\lambda)^2\alpha^2$	F	F	B	B	R_D^c	R_D^c	0	0
S_3	$(1-\lambda)^2(1-\alpha)\alpha$	F	F	B	N	R_D^c	$K\frac{R_D^c}{R_D^c+K}$	0	0
S_4	$(1-\lambda)^2(1-\alpha)\alpha$	F	F	N	B	$K\frac{R_D^c}{R_D^c+K}$	R_D^c	0	0
S_5	$(1-\lambda)^2(1-\alpha)^2$	F	F	N	N	0	0	0	0
S_6	$\lambda(1-\lambda)\alpha$	S	F	N	B	R_D^c	R_D^c	$IR - cR_D^c$	0
S_7	$\lambda(1-\lambda)(1-\alpha)\alpha$	S	F	B	N	R_D^c	$K\frac{R_D^c}{R_D^c+K}$	0	0
S_8	$\lambda(1-\lambda)(1-\alpha)^2$	S	F	N	N	$IR\frac{R_D^c+K}{R_D^c+2K}$	$IR\frac{K}{R_D^c+2K}$	0	0
S_9	$\lambda(1-\lambda)\alpha$	F	S	B	N	R_D^c	R_D^c	0	$IR - cR_D^c$
S_{10}	$\lambda(1-\lambda)(1-\alpha)\alpha$	F	S	N	B	$K\frac{R_D^c}{R_D^c+K}$	R_D^c	0	0
S_{11}	$\lambda(1-\lambda)(1-\alpha)^2$	F	S	N	N	$IR\frac{K}{R_D^c+2K}$	$IR\frac{R_D^c+K}{R_D^c+2K}$	0	0

Table 3: Outcomes for $K \geq K^t$ - Contagion

The interest rate R_D^{nc} (no contagion) follows from the binding participation constraint of the equity holder. If $K < K^t$, Constraint (5) implies that

$$\lambda^2 \left(IR - R_D^{nc}\right) + \lambda(1-\lambda)\left[\alpha\left(IR - R_D^{nc}\right) + (1-\alpha)\left(IR - R_D^{nc} - K\frac{R_D^{nc}}{R_D^{nc}+K}\right)\right] = \underline{e}\lambda R \quad (9)$$

Therefore, if the investment correlation is zero and $K < K^t$, the overall utility of the uninsured creditors is

$$\begin{aligned} U_0(K < K^t) &= [\lambda + (1-\lambda)\alpha]R_D^{nc} + (1-\lambda)(1-\alpha)[\lambda + (1-\lambda)\alpha]K\frac{R_D^{nc}}{R_D^{nc}+K} \\ &- \psi(I) - \tau(K) + \underline{e} \end{aligned} \quad (10)$$

Rearranging Eq. (9) to

$$(1-\lambda)(1-\alpha)[\lambda + (1-\lambda)\alpha]K\frac{R_D^{nc}}{R_D^{nc}+K} = [\lambda + (1-\lambda)\alpha]\left[(I-\underline{e})R - R_D^{nc}\right] \quad (11)$$

and plugging this expression into Eq. (10) yields

$$U_0(K < K^t) = [\lambda + (1-\lambda)\alpha](I-\underline{e})R - \psi(I) - \tau(K) + \underline{e} \quad (12)$$

Eq. (12) implies that, for $K < K^t$, the expected repayment of the creditor C_A does not depend on the interbank exposure. The reason is that, due to the participation constraint of the equity investor E_A, his loss in dividends caused by the payment to the failed bank B_B (S_8 in Table 2) has to be offset by a reduction in the creditor's interest rate, R_D^{nc}. The resulting decrease in the repayment to C_A in the success states S_1, S_6, and S_8 is exactly offset by the additional payment in S_9 that arises due to the interbank exposure:

$$\left(\lambda^2 + \lambda(1-\lambda)\alpha + \lambda(1-\lambda)(1-\alpha)\right)(R_D - R_D^{nc}) = \lambda(1-\lambda)(1-\alpha)K\frac{R_D^{nc}}{R_D^{nc}+K} \tag{13}$$

Furthermore, the reduction in the interest rate has the disadvantage that it lowers the value of the implicit government guarantees, since the face value of the creditor's liabilities is decreased. This reduces the expected repayment to C_A in states S_2, S_3, and S_7, when his own bank is bailed out. However, this loss is exactly offset by the additional payment in state S_4 that arises from the interbank exposure and the bailout of B_B. Due to the symmetry of our model, the same holds for investor E_B and creditor C_B. Taken together, the expected repayment to the creditors becomes independent from the interbank exposure. Hence, due to transaction costs, it is always optimal to choose $K_0^{nc} = 0$ when $K < K^t$.

Furthermore, the Appendix shows that there exists a unique and interior maximum I_0^{nc} for the investment size. Hence, for $K < K^t$, the highest expected utility for the noninsured creditor is

$$\overline{U_0}(K < K^t) = [\lambda + (1-\lambda)\alpha](I_0^{nc} - \underline{e})R - \psi(I_0^{nc}) + \underline{e} \tag{14}$$

Next, we derive the interest rate R_D^c for the contagion case. When $K \geq K^t$, the equity investors do not receive a dividend payment as soon as one of the banks fails and is not bailed out and we thus obtain for the interest rate

$$\begin{aligned} \lambda^2 (IR - R_D^c) + \lambda(1-\lambda)\alpha\,(IR - R_D^c) \quad &\geq \quad \underline{e}\lambda R \\ \Rightarrow R_D^c = \left[I - \frac{\underline{e}}{\lambda + (1-\lambda)\alpha}\right] R \quad &< \quad R \end{aligned} \tag{15}$$

Therefore, as soon as $K \geq K^t$, a change in K does not alter the dividend payment to E_A and hence no longer changes the interest rate R_D^c. Compared to the no contagion case, where investor E_A receives at least a partial repayment if B_A is successful and B_B defaults and is not bailed out (S_8 in Table 2), E_A receives nothing in this situation in the contagion case. Hence, the interest rate R_D^c is even lower than R_D^{nc}. In the contagion case, the overall utility of the uninsured creditors will be:

$$\begin{aligned} U_0(K \geq K^t) &= \left[(1+\lambda)\alpha + \lambda^2(1-2\alpha) - (1-\lambda)\lambda\alpha^2\right] R_D^c + \lambda(1-\lambda)(1-\alpha)^2 IR \\ &+ (1-\lambda)(1-\alpha)\alpha K \frac{R_D^c}{R_D^c + K} - \psi(I) - \tau(K) + \underline{e} \qquad (16) \\ &= [\lambda + (1-\lambda)\alpha]\,(I - \underline{e})R + (1-\lambda)(1-\alpha)\alpha K \frac{R_D^c}{R_D^c + K} \\ &- \alpha\underline{e}R\frac{(1-\lambda)(1-\alpha)}{\lambda + (1-\lambda)\alpha} - \psi(I) - \tau(K) + \underline{e} \qquad (17) \end{aligned}$$

Since $R_D^c < R_D^{nc}$, the face value of the creditors' liabilities is even further reduced and thus the value of bailouts of their own banks is smaller than in the no contagion case (third term in Eq. (17)). However, bailouts of the other bank become more valuable since higher interbank exposure implies that a higher fraction of the other banks bailout funds is transferred to the creditor (second term in Eq. (17)).

Again, the Appendix shows that also for $K \geq K^t$ there exists a unique and interior maximum I_0^c for the investment size and a unique optimal level of interbank exposure K_0^c. Hence, for $K \geq K^t$, the highest expected utility for the noninsured creditor that can be achieved is

$$\begin{aligned} \overline{U_0}(K \geq K^t) &= [\lambda + (1-\lambda)\alpha]\,(I_0^c - \underline{e})R + (1-\lambda)(1-\alpha)\alpha K_0^c \frac{\left(I_0^c - \frac{\underline{e}}{\lambda+(1-\lambda)\alpha}\right)R}{\left(I_0^c - \frac{\underline{e}}{\lambda+(1-\lambda)\alpha}\right)R + K_0^c} \\ &- \alpha\underline{e}R\frac{(1-\lambda)(1-\alpha)}{\lambda + (1-\lambda)\alpha} - \psi(I_0^c) - \tau(K_0^c) + \underline{e} \qquad (18) \end{aligned}$$

To determine whether banks choose a level of interbank exposure that leads to contagion, we now compare the utility of creditors for the different levels of interbank deposits from Eq. (14) and Eq. (18). In the Appendix, we show that choosing (I_0^c, K_0^c) dominates the alternative of having no interbank exposure and choosing I_0^{nc} if the expected additional gain from the interbank exposure, due to the higher value of a bailout of the other bank, outweighs the loss in value of the own bank's bailout due to the contagion risk and the resulting lower interest rate $R_D^c < R_D^{nc} < (I - \underline{e})R$. Otherwise, the banks do not enter into interbank connections and choose an investment size of I_0^{nc}. These findings can be summarized in the following proposition.

Proposition 4.2. *If banks invest in uncorrelated portfolios (given a positive bailout probability), they choose an interbank exposure of K_0^c and an investment size of I_0^c if $\overline{U_0}(K \geq K^t) > \overline{U_0}(K < K^t)$ and $K_0^c \geq K^t$. Otherwise, the banks choose to have no interbank exposure and an investment size of I_0^{nc}.*

Proof See the Appendix. QED

4.3. Comparison of correlated and uncorrelated investments

What remains is to show under which correlation structure uninsured creditors receive a higher expected repayment. In the Appendix, we formally prove that $\overline{U_1} > \overline{U_0}$ always holds, implying that banks will always choose perfectly correlated investments. This main finding can be summarized in the following proposition.

Proposition 4.3. *If banks are connected via an interbank market and there is a nonzero bailout probability, it is optimal for them to invest in correlated assets. Moreover, they have an incentive to increase their interbank exposure to $K_1^* > 0$ and choose the investment size I_1^* and the creditor liabilities $(I_1^* - \underline{e})R$.*

Proof See the Appendix. QED

To understand why this result holds, recall that the investment correlation only alters the expected bailout funds and not the investment returns. Hence, the banks choose the investment correlation that maximizes the value of implicit government guarantees and, in turn, the total expected inflows into the private sector. Given a positive interbank exposure, which is beneficial for creditors since it increases the value of the other bank's bailout, uncorrelated investments decrease the expected dividend payments to the equity investors compared to the case when the banks invest in correlated assets. Hence, the creditors' interest rates have to be lowered, which, in turn, decreases the face value of debt and thus the value of the implicit government guarantees. This relation incentivizes banks to invest in correlated portfolios.

In this Section, we demonstrate that banks always have an incentive to take on a positive interbank exposure to increase the value of government guarantees. The benefit of being connected to other banks can be further enhanced by choosing correlated assets, which gives banks an incentive to herd. We can thus provide an additional explanation for the herding behavior of banks besides the effect discussed by Acharya and Yorulmazer (2007). In their paper correlated investments increase the bailout probability of each bank. Even if we abstract from the fact that correlated investments increase the bailout probability, we find an additional incentive for herding behavior. Furthermore, interbank exposure incentivizes banks to invest more and, in turn, to take on more creditor liabilities and vice versa. This positive link between interbank exposure and creditor liabilities might help explain the increase in the density of the interbank network in the last decades. According to our model, this might be a result of the increase in bank leverage and size in this period.

Hence, the mechanism described in this paper leads to an overall increase in systemic risk that results from interconnectedness, higher leverage, and herding behavior. However, the incentive of being highly interconnected can be mitigated by raising the minimum equity requirements. Since banks always choose correlated investment, given interbank connections and a positive bailout probability, we restrict our analysis to this case in the next section.

5. Extensions

This Section provides two extensions to our main model. In the first part, we introduce asymmetric bailout probabilities and in the second part, we analyze the effect of interbank connections on risk shifting incentives.

5.1. Asymmetric bailout probabilities

In this Section, we analyze the implications of banks having different bailout probabilities on their incentive to be interconnected. Without loss of generality, we now assume that bank B_A (B_B) has the probability α_A (α_B) of being bailed out in case of a default with $\alpha_A = \alpha + \delta$ and $\alpha_B = \alpha - \delta$. The parameter $\delta > 0$ thus captures the difference in bailout probabilities. Furthermore, for simplicity, we assume from now on that the investment is not scalable and instead needs an initial amount of one unit of capital (with $\psi(1) = 1$), such that $c = 1 + \tau(K) - \underline{e}$.

Thus, the optimization problems at $t = 0$ now become:

$$\max_{R_D,I,K} U_A = \lambda R_D + (1-\lambda)\left[\alpha_A R_D + (1-\alpha_A)\alpha_B K \frac{R_D}{R_D + K}\right] - c \tag{19}$$

$$\max_{R_D,I,K} U_B = \lambda R_D + (1-\lambda)\left[\alpha_B R_D + (1-\alpha_B)\alpha_A K \frac{R_D}{R_D + K}\right] - c \tag{20}$$

subject to the participation constraints of the equity investors

$$E[d_1] \geq \underline{e}\lambda R \Rightarrow \lambda(R - R_D) = \underline{e}\lambda R \tag{21}$$

and the interest rate thus becomes $R_D = (1-\underline{e})R$. Therefore, the desired interbank exposure of bank B_B, K_B^α, implied by the first-order condition:

$$\frac{\partial U_B}{\partial K} = (1-\lambda)(1-(\alpha-\delta))(\alpha+\delta)R^2 \frac{(1-\underline{e})^2}{((1-\underline{e})R + K_B^\alpha)^2} - \tau'(K_B^\alpha) = 0 \tag{22}$$

is higher than the desired exposure of B_A, K_A^α:

$$\frac{\partial U_A}{\partial K} = (1-\lambda)(1-(\alpha+\delta))(\alpha-\delta)R^2 \frac{(1-\underline{e})^2}{((1-\underline{e})R + K_A^\alpha)^2} - \tau'(K_A^\alpha) = 0 \tag{23}$$

The reason is that interbank exposure increases the value of the other bank's bailout, in case the creditor's own bank is not bailed out. Given that the creditor's own bank's bailout probability is very low and the other bank's bailout probability is very high, the likelihood of this case occurring is very high and, in turn, so is the additional value of having interbank exposure. Therefore, the desired interbank exposure increases with the other bank's bailout probability and decreases with the own bank's bailout probability:

$$\frac{\partial K_A^\alpha}{\partial \delta} = -\frac{(1-\lambda)(1-2\delta)R^2 \frac{(1-\underline{e})^2}{((1-\underline{e})R+K_A^\alpha)^2}}{(1-\lambda)(1-(\alpha+\delta))(\alpha-\delta)R^2 \frac{2(1-\underline{e})^2}{((1-\underline{e})R+K_A^\alpha)^3} + \tau''(K_A^\alpha)} < 0 \tag{24}$$

$$\frac{\partial K_B^\alpha}{\partial \delta} = \frac{(1-\lambda)(1+2\delta)R^2 \frac{(1-\underline{e})^2}{((1-\underline{e})R+K_B^\alpha)^2}}{(1-\lambda)(1-(\alpha+\delta))(\alpha-\delta)R^2 \frac{2(1-\underline{e})^2}{((1-\underline{e})R+K_B^\alpha)^3} + \tau''(K_B^\alpha)} > 0 \tag{25}$$

To incentivize B_A to enter into higher cyclical liabilities, B_B can compensate B_A for the additional costs by paying the amount η:

$$\begin{aligned}\eta &= \tau(K)-\tau(K_A^\alpha)\\ &- (1-\lambda)(1-(\alpha+\delta))\alpha\left[K\frac{(1-\underline{e})R}{(1-\underline{e})R+K}-K_A^\alpha\frac{(1-\underline{e})R}{(1-\underline{e})R+K_A^\alpha}\right]\end{aligned}\tag{26}$$

The right hand side of Eq. (26) represents the additional costs for B_A of having an interbank exposure of $K > K_A^\alpha$ instead of K_A^α, which are given by the additional transaction costs minus the additional benefit of a higher interbank exposure.

Hence, incorporating the additional payment, the optimization problem of bank B_B becomes:

$$\begin{aligned}\max_K U_B &= \lambda(1-\underline{e})R+(1-\lambda)(\alpha-\delta)(1-\underline{e})R\\ &+ (1-\lambda)(1-(\alpha-\delta))(\alpha+\delta)K\frac{(1-\underline{e})R}{(1-\underline{e})R+K}-\eta-\tau(K)-(1-\underline{e})\end{aligned}\tag{27}$$

This optimization problem yields the following proposition.

Proposition 5.1. *If bank B_A has a higher probability of being bailed out than B_B then*

a) B_B desires more interbank exposure than B_A, where the banks' desired level increases with the other bank's bailout probability and decreases with the bank's own bailout probability.

b) bank B_B incentivizes B_A to increase the interbank exposure to $K^\alpha = K_A^\alpha + \Delta^\alpha$ by paying the amount η^α, where Δ^α is the interbank exposure that B_B wants to have in addition to K_A^α.

c) the desired additional interbank exposure Δ^α and the respective compensation payment η^α both increase with the difference between the bailout probabilities δ.

Proof See the Appendix. QED

Hence, by paying an additional fee and thereby incentivizing other banks to enter into higher interbank exposure, banks with lower bailout probabilities can utilize higher bailout probabilities of other banks. The simplest way to implement such a compensation fee is an interest payment on interbank deposits/loans, such that banks with lower bailout probabilities (i.e., smaller, non-systemic banks in poorer countries) pay higher interbank interest rates than banks with higher bailout probabilities (i.e., larger, systemic banks in richer countries). This interest rate gap then increases with the difference of the banks' bailout probabilities, implying that implicit bailout guarantees are priced into a bank's interbank liabilities.

5.2. The interbank network and risk shifting

In the following, we show that the incentive to engage in risk-shifting increases with K. To model the riskiness of the investment decision, we consider two assets: a risk-free storage technology that transfers one unit of wealth today into one unit of wealth tomorrow, and a risky negative NPV investment that generates a return $R_R > 1$ with probability $\lambda_R < 1$ where $\lambda_R R_R < 1$.

For ease of illustration, we neglect transaction costs and thus $c = 1 - \underline{e}$. Given that there is no bailout possibility, the bank can offer creditors either a repayment of c (if it invests in the safe asset) or R_D^R with probability λ_R if it invests in the risky negative NPV asset. The promised repayment R_D^R results from the binding participation constraint of the equity holder. We assume that the outside option of the equity holder is now given by the risk-free storage technology. Therefore, the participation constraint for an investment in the risky asset becomes

$$E[d_1] = \underline{e} \Rightarrow \lambda_R \left[R_R - R_D^R\right] = \underline{e} \Rightarrow R_D^R = R_R - \frac{\underline{e}}{\lambda_R} \tag{28}$$

In the following, we assume that R_R is at least high enough such that the uninsured creditor receives a return larger than one in the success state (i.e., $R_D^R > c$). We first consider a scenario without a bailout possibility and no interbank network. Here, it can be easily seen that the expected repayment of the creditors is higher if the bank invests in the safe asset since

$$c > \lambda_R R_D^R = \lambda_R R_R - \underline{e} \tag{29}$$

Hence, without the possibility of a bailout, banks will always choose the safe investment.

Next, we consider the case in which the bank has a positive probability of being bailed out by the government but still no connections to other banks. Now it can become profitable to switch to the negative NPV investment if the bailout probability is high enough. More precisely, a bank will switch to the negative NPV investment if the expected repayment of creditors for this investment is higher than for the safe repayment c, that is,

$$\lambda_R R_D^R + (1-\lambda_R)\alpha R_D^R > c \tag{30}$$

Besides the state of nature in which the investment is successful, creditors now also receive the higher return R_D^R when the bank is bailed out by the government. The critical α, that is, the bailout probability where the bank is indifferent between the two investments is given by

$$\alpha^* = \frac{c - \lambda_R R_D^R}{(1-\lambda_R) R_D^R} < 1 \tag{31}$$

which is true since $R_D^R > c$. Hence, for $\alpha > \alpha^*$ it is always profitable to switch to the negative NPV investment.

Now, we again allow the bank to exchange funds with the bank in the other region. Whether banks will switch to the negative NPV investment again depends on α. Whenever the expected repayment of the uninsured creditor from investing in the negative NPV investment opportunity is higher, banks will shift away from the risk-free investment. Formally, the following condition must be satisfied:

$$\lambda_R R_D^R + (1-\lambda_R)\left[\alpha R_D^R + \alpha(1-\alpha)K\frac{R_D^R}{R_D^R + K}\right] > c \tag{32}$$

Rearraning this equation yields

$$\alpha^{**} = \frac{c - \lambda_R R_D^R}{(1-\lambda_R) R_D^R \left(1 + (1-\alpha^{**})\frac{K}{R_D^R + K}\right)} < \alpha^* \tag{33}$$

Hence, the critical α^{**} is strictly smaller if a bank is connected (i.e., $K > 0$) to another bank on the interbank market, that is, $\alpha^* > \alpha^{**}$. Hence, the critical threshold α is lower once a bank enters into connections with other banks. Put differently, a lower bailout probability is sufficient to make the bank switch to the negative NPV investment. The positive bailout probability can turn a negative NPV investment into a positive NPV investment from the perspective of the uninsured creditors since they will receive the high repayment with a higher probability. This effect is reinforced once the bank is connected to another bank if this other bank has a positive bailout probability as well. Our results are summarized in the following proposition.

Proposition 5.2. *The more interconnected a bank becomes, the lower the critical bailout probability that makes it profitable for the bank to engage in risk shifting, that is, to switch to risky negative NPV investments.*

Risk shifting thus becomes more attractive for banks since the downside risk is limited by two factors. First, the downside risk is limited by the positive bailout probability because creditors receive their full repayment after the bank is bailed out. Second, the interbank connection further reduces the downside risk, since it adds an additional state in which the creditor receives a positive repayment. These two effects turn a negative NPV investment into a positive NPV investment (from the perspective of the uninsured creditors).

6. Discussion and policy implications

This paper shows that banks have an incentive to create a high degree of interconnectedness by engaging in circular lending activities. This holds true even if we allow the interbank market to exist for other reasons than simply exploiting implicit government guarantees (e.g., liquidity co-insurance, see the Appendix for further details). Several policy implications can be derived from our results. Generally, each of these policy implications aims at reducing the banks' incentive to create excessive interbank exposures by entering into cyclical liabilities and therefore aims at reducing systemic risk.

First of all, as shown in Section 4.1., raising the the minimum equity requirement reduces the banks' incentive to by highly interconnected. Similarly, one can think about increasing the risk weights for interbank loans under the Basel accord and thereby increase the amount of equity necessary to satisfy minimum capital requirements. Currently banks do not have to hold high amounts of capital for most of their interbank exposure. If interbank loans get a higher risk weight, banks are incentivized to reduce their circular lending activities and hence reduce systemic risk in the interbank market. However, banks could potentially counter this regulatory measure by creating equity cycle flows in addition to cyclical debt liabilities. By investing equity in a cyclical way, banks can reach any desired equity ratio without being dependent on outside investors.

Second, as long as the cyclical interbank liabilities only exist bilaterally between two banks, regulators could potentially net out these exposures before deciding upon bank bailouts. In reality, however, the interbank cycle flows of course involve more than two banks, implying that regulators would need to know the entire network topology to be able to cancel out cycle flows. Since interbank exposures are highly complex, intransparent, and often involve banks in different countries, canceling out these flows before bailing out a bank is impossible. However, the creation of a centralized clearing house for interbank activities can potentially eliminate the perverse incentives described in the paper. If all interbank activities are channeled through a clearing house, the regulator knows the complete interbank network topology and is thus able to cancel matching interbank deposits of the various banks. However, this approach would require a global clearing house and thus a collaboration of all involved bank regulators.

Furthermore, one of the key topics in the current discussion in the European Union is the introduction of a financial transaction tax to limit speculative trading activities. Since interconnectedness can not only be created via interbank loans, but also by using derivatives like for example CDS, such a tax could be a potential mechanism to reduce the high interconnectedness by adding additional transaction costs and therefore mitigate the systemic risk problems that result from investing in highly correlated low-quality assets.

A fourth possibility to mitigate the incentives to create large cycle flows would be the introduction of the widely discussed bank levy. Charging banks with large balance sheets (that can very well result from high amounts of cyclical liabilities) higher taxes for their systemic risk can potentially mitigate the incentive to create these large cycle flows in the first place.

Finally, if a government lowers bailout expectations, this actually leads to higher interconnectedness if other governments do not act in a similar way. The reason is that the incentive to be interconnected increases if a bank's own bailout probability is lowered. Hence, if the bailout probability of banks is reduced in only one country, these banks then want to have more interbank exposure to banks in other countries to benefit from their bailouts. Hence, reducing the interconnectedness on the interbank market by lowering bailout exceptions can only be realized when governments use a coordinated approach and expectations are lowered in all countries simultaneously.

7. Conclusion

This paper sheds light on the puzzle why banks have an incentive to be highly interconnected on the interbank market and why it can be rational to engage in circular lending activities, although this behavior considerably increases systemic risk and leverage without altering the aggregate relation with the real economy. We show that banks create these cyclical liabilities because they increase the value of implicit government bailout guarantees. Such guarantees shift the probability distribution of the returns of risky investments and thereby increase the expected repayment of uninsured creditors. Furthermore, the mechanism we derive in this paper is able to explain why banks prefer correlated risky investments. Hence, the presented mechanism leads to an overall increase in systemic risk that results from interconnectedness, risky assets, as well as herding behavior. Therefore, our model helps explain why banks invested in risky correlated investments (e.g., US subprime loans) in the run-up to the financial crisis.

Appendix

A.1. Proof of Proposition 4.1

The first-order condition with respect to I for an optimum is:

$$\frac{\partial U_1}{\partial I} = \lambda R + (1-\lambda)\left[\alpha R + (1-\alpha)\alpha R \frac{K^2}{((I-\underline{e})R+K)^2}\right] - \psi'(I) = 0 \tag{34}$$

The corresponding second-order condition is:

$$SOC_I \equiv \frac{\partial^2 U_1}{\partial^2 I} = -(1-\lambda)(1-\alpha)\alpha R^2 \frac{2K^2}{((I_1^*-\underline{e})R+K)^3} - \psi''(I) < 0 \tag{35}$$

which is satisfied since $\psi'' > 0$. Treating K as exogenous and using the implicit function theorem yields for the partial derivative of I with respect to interbank exposure K:

$$\frac{\partial I}{\partial K} \quad = \quad \frac{(1-\lambda)(1-\alpha)\alpha R^2 \frac{2(I-\underline{e})K}{((I-\underline{e})R+K)^3}}{(1-\lambda)(1-\alpha)\alpha R^2 \frac{2K^2}{((I-\underline{e})R+K)^3} + \psi''(I)} > 0 \tag{36}$$

Hence, the size of the investment depends positively on the interbank exposure.

The first-order condition with respect to K for an optimum is:

$$\frac{\partial U_1}{\partial K} = (1-\lambda)(1-\alpha)\alpha R^2 \frac{(I-\underline{e})^2}{((I-\underline{e})R+K)^2} - \tau'(K) = 0 \tag{37}$$

The corresponding second-order condition is:

$$SOC_K \equiv \frac{\partial^2 U_1}{\partial^2 K} = -(1-\lambda)(1-\alpha)\alpha R^2 \frac{2(I-\underline{e})^2}{((I_1^*-\underline{e})R+K)^3} - \tau''(K) < 0 \tag{38}$$

which is satisfied since $\tau'' > 0$. Treating I as exogenous and using the implicit function theorem yields for the derivative of K with respect to the size of the investment I:

$$\frac{\partial K}{\partial I} = \frac{(1-\lambda)(1-\alpha)\alpha R^2 \frac{2(I-\underline{e})K}{((I-\underline{e})R+K)^3}}{(1-\lambda)(1-\alpha)\alpha R^2 \frac{2(I-\underline{e})^2}{((I_1^*-\underline{e})R+K)^3} + \tau''(K)} > 0 \tag{39}$$

Hence, the interbank exposure depends positively on the size of the investment.

Hence, the optimal choice (I_1^*, K_1^*) must satisfy the equations:

$$\lambda R + (1-\lambda)\left[\alpha R + (1-\alpha)\alpha R \frac{(K_1^*)^2}{((I_1^* - \underline{e})R + K_1^*)^2}\right] - \psi'(I_1^*) = 0 \tag{40}$$

$$(1-\lambda)(1-\alpha)\alpha R^2 \frac{(I_1^* - \underline{e})^2}{((I_1^* - \underline{e})R + K_1^*)^2} - \tau'(K_1^*) = 0 \tag{41}$$

which is a unique and interior maximum since

$$\Delta \equiv \frac{\partial^2 U_1}{\partial^2 I}\frac{\partial^2 U_1}{\partial^2 K} - \left(\frac{\partial^2 U_1}{\partial I \partial K}\right)^2 > 0 \tag{42}$$

and $\partial^2 U_1 / \partial^2 I < 0$ with

$$\frac{\partial^2 U_1}{\partial I \partial K} = (1-\lambda)(1-\alpha)\alpha R^2 \frac{2(I_1^* - \underline{e})K_1^*}{((I_1^* - \underline{e})R + K_1^*)^3} \tag{43}$$

Lastly, we determine the total derivative of I_1^* and K_1^* with respect to $\underline{e}$. The total derivative of Eq. (34) with respect to $\underline{e}$ is given by

$$\begin{aligned}
\frac{d\,\frac{\partial U_1}{\partial I}}{d\,\underline{e}} &= \frac{\partial \frac{\partial U_1}{\partial I}}{\partial \underline{e}} + \frac{\partial \frac{\partial U_1}{\partial I}}{\partial K}\frac{\partial K}{\partial \underline{e}} + \frac{\partial \frac{\partial U_1}{\partial I}}{\partial I}\frac{\partial I}{\partial \underline{e}} = 0 \\
&\Rightarrow \frac{\partial K}{\partial \underline{e}}\left(\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K(I-\underline{e})}{((I-\underline{e})R+K)^3}\right) \\
&- \frac{\partial I}{\partial \underline{e}}\left(\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K^2}{((I-\underline{e})R+K)^3} + \psi''(I)\right) = -\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K^2}{((I-\underline{e})R+K)^3}
\end{aligned} \tag{44}$$

Furthermore, the total derivative of Eq. (37) with respect to $\underline{e}$ is given by

$$\begin{aligned}
\frac{d\,\frac{\partial U_1}{\partial K}}{d\,\underline{e}} &= \frac{\partial \frac{\partial U_1}{\partial K}}{\partial \underline{e}} + \frac{\partial \frac{\partial U_1}{\partial K}}{\partial K}\frac{\partial K}{\partial \underline{e}} + \frac{\partial \frac{\partial U_1}{\partial K}}{\partial I}\frac{\partial I}{\partial \underline{e}} = 0 \\
&\Rightarrow -\frac{\partial K}{\partial \underline{e}}\left(\frac{2(1-\lambda)(1-\alpha)\alpha R(I-\underline{e})^2}{((I-\underline{e})R+K)^3} + \tau''(K)\right) \\
&+ \frac{\partial I}{\partial \underline{e}}\left(\frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)^3}\right) = \frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)^3}
\end{aligned} \tag{45}$$

Hence, the derivative of I_1^* with respect to $\underline{e}$ becomes

$$
\begin{aligned}
\frac{\partial I_1^*}{\partial \underline{e}} &= \frac{\begin{vmatrix} -\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K^2}{((I-\underline{e})R+K)^3} & \left(\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K(I-\underline{e})}{((I-\underline{e})R+K)^3}\right) \\ \frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)^3} & -\left(\frac{2(1-\lambda)(1-\alpha)\alpha R(I-\underline{e})^2}{((I-\underline{e})R+K)^3} + \tau''(K)\right) \end{vmatrix}}{\begin{vmatrix} -\left(\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K^2}{((I-\underline{e})R+K)^3} + \psi''(I)\right) & \left(\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K(I-\underline{e})}{((I-\underline{e})R+K)^3}\right) \\ \left(\frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)^3}\right) & -\left(\frac{2(1-\lambda)(1-\alpha)\alpha R(I-\underline{e})^2}{((I-\underline{e})R+K)^3} + \tau''(K)\right) \end{vmatrix}} && (46) \\
&= \frac{\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K^2}{((I-\underline{e})R+K)^3}\tau''(K)}{\frac{\partial^2 U_1}{\partial^2 I}\frac{\partial^2 U_1}{\partial^2 K} - \left(\frac{\partial^2 U_1}{\partial I \partial K}\right)^2} > 0 && (47)
\end{aligned}
$$

and the derivative of K_1^* with respect to $\underline{e}$ becomes

$$
\begin{aligned}
\frac{\partial K_1^*}{\partial \underline{e}} &= \frac{\begin{vmatrix} -\left(\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K^2}{((I-\underline{e})R+K)^3} + \psi''(I)\right) & -\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K^2}{((I-\underline{e})R+K)^3} \\ \left(\frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)^3}\right) & \frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)^3} \end{vmatrix}}{\begin{vmatrix} -\left(\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K^2}{((I-\underline{e})R+K)^3} + \psi''(I)\right) & \left(\frac{2(1-\lambda)(1-\alpha)\alpha R^2 K(I-\underline{e})}{((I-\underline{e})R+K)^3}\right) \\ \left(\frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)^3}\right) & -\left(\frac{2(1-\lambda)(1-\alpha)\alpha R(I-\underline{e})^2}{((I-\underline{e})R+K)^3} + \tau''(K)\right) \end{vmatrix}} && (48) \\
&= -\frac{\frac{2(1-\lambda)(1-\alpha)\alpha RK(I-\underline{e})}{((I-\underline{e})R+K)}\psi''(I)}{\frac{\partial^2 U_1}{\partial^2 I}\frac{\partial^2 U_1}{\partial^2 K} - \left(\frac{\partial^2 U_1}{\partial I \partial K}\right)^2} < 0 && (49)
\end{aligned}
$$

A.2. Switching point K^t in Section 4.2

Here, we will formally derive the critical threshold of interbank deposits K^t that just allows a successful bank to stay solvent if the bank it is connected to defaults and is not bailed out. The critical cases to derive this threshold are those in which only one investment fails and neither of the banks is bailed out, i.e., S_8 and S_{11}. Here, the bank with the successful investment will pay the following amount to the bank with the failed investment:

$$\min\left\{K, IR\frac{K(R_D+K)}{R_D(R_D+2K)}\right\} \tag{50}$$

The first term represents the amount the successful bank owes to the failed bank and the second term results from:

$$\sum_{i=0}^{\infty} IR\left(\frac{K}{R_D+K}\right)^{(1+2i)} = IR\frac{K}{R_D+K}\frac{1}{1-\frac{K^2}{(R_D+K)^2}} = IR\frac{K(R_D+K)}{R_D(R_D+2K)} \tag{51}$$

Hence, the failing bank receives either its full repayment (if there are enough funds available to settle all claims), i.e., $K \leq IR\ K(R_D + K)/(R_D(R_D + 2K))$ or receives a payment of $IR\ K(R_D + K)/(R_D(R_D + 2K))$. The critical threshold up to which the bank receives its full repayment can be written as:

$$K_1^t = IR\frac{K_1^t(R_D + K_1^t)}{R_D(R_D + 2K_1^t)} \Rightarrow K_1^t = \frac{R_D\,[IR - R_D]}{2R_D - IR} \tag{52}$$

From Eq. (52) we can see that the successful bank can always pay back its liabilities to the unsuccessful bank as long as $IR > 2R_D$. Thus, it will never default in this case. In what follows we will focus on the more interesting case in which a default is possible depending on the level of K. Hence, from now on we will assume that $IR < 2R_D$. We next consider the repayment the uninsured creditor gets from the successful bank, which is given by:

$$\min\left\{R_D, IR\frac{R_D + K}{R_D + 2K}\right\} \tag{53}$$

The first term is the total amount owed to the uninsured creditor and the second term comes from:

$$\sum_{i=0}^{\infty} IR\frac{R_D}{R_D + K}\left(\frac{K}{R_D + K}\right)^{2i} = IR\frac{R_D}{R_D + K}\frac{1}{1 - \frac{K^2}{(R_D+K)^2}} = IR\frac{(R_D + K)}{R_D + 2K} \tag{54}$$

Hence, as long as K is small enough such that $R_D \leq IR\ (R_D + K)/(R_D + 2K)$ the successful bank can fully repay its uninsured creditor. However if K exceeds a critical threshold, the bank is unable to settle all its claims and can only repay $IR\ (R_D + K)/(R_D + 2K)$ to its creditor. The critical switching point is given by:

$$R_D = IR\frac{(R_D + K_2^t)}{R_D + 2K_2^t} \Rightarrow K_2^t = \frac{R_D\,[IR - R_D]}{2R_D - IR} \tag{55}$$

As can be seen from Eq. (52) and Eq. (55), the thresholds K_1^t and K_2^t are equal. We now turn to the repayment of the uninsured creditor of the failed bank, which is given by:

$$\min\left\{R_D, K\frac{R_D}{R_D + K}, IR\frac{K}{R_D + 2K}\right\} = \min\left\{K\frac{R_D}{R_D + K}, IR\frac{K}{R_D + 2K}\right\} \tag{56}$$

where the first term is again the total amount owed to the uninsured creditor, the second term is the maximal payment from the bank with the successful investment to the bank with the failed investment times the fraction the insured creditor gets from this payment, and the last term comes from:

$$\sum_{i=0}^{\infty} IR \frac{R_D}{R_D+K}\left(\frac{K}{R_D+K}\right)^{(1+2i)} = IR\frac{R_D K}{(R_D+K)^2}\frac{1}{1-\frac{K^2}{(R_D+K)^2}} = IR\frac{K}{R_D+2K} \tag{57}$$

One can immediately see that the unsuccessful bank can never fully repay its uninsured creditors. Furthermore, as long as K is small enough such that

$$K\frac{R_D}{R_D+K} \leq IR\frac{K}{R_D+2K} \ , \tag{58}$$

the payment of the unsuccessful bank to its uninsured creditors is $KR_D \,/(R_D+K)$. If K is too high, the payment is $IR\;K/(R_D+2K)$. The critical switching threshold is given by

$$R_D\frac{K_3^t}{R_D+K_3^t} = IR\frac{K_3^t}{R_D+2K_3^t} \Rightarrow K_3^t = \frac{R_D\left[IR-R_D\right]}{2R_D-IR} \tag{59}$$

Hence, all three thresholds are the same, which is why we will denote them in the following K^t. Plugging the value of R_D^{nc} (since we approach K_t from below) into the formula for the contagion threshold K_t in Eq. (59) yields for this threshold

$$K^t = \frac{R_D^{nc}\left[IR-R_D^{nc}\right]}{2R_D^{nc}-IR} = \frac{\underline{e}R}{\lambda+(1-\lambda)\alpha}\,\frac{I\left[\lambda+(1-\lambda)\alpha\right]-\underline{e}}{I\left[\lambda+(1-\lambda)\alpha\right]-2\underline{e}} \tag{60}$$

Hence, there exists a positive interbank exposure K^t for which the successful bank stays solvent (in case one bank is successful and the other is not) if

$$I\left[\lambda+(1-\lambda)\alpha\right]-2\underline{e} > 0 \tag{61}$$

Conversely, if Condition (61) does not hold, we can restrict our analysis to the contagion case $K \geq K^t$. This completes the derivation of K^t.

A.3. Proof of Proposition 4.2

For $K < K^t$, the first-order condition implies for the optimal investment size I_0^{nc}:

$$\frac{\partial U_0}{\partial I}(K < K^t) = [\lambda + (1-\lambda)\alpha]R - \psi'(I_0^{nc}) = 0 \tag{62}$$

where the second order derivative is negative and the determinant positive. Thus, I_0^{nc} is a unique and interior maximum.

For $K \geq K^t$, the first-order condition with respect to the level of creditor funds I yields:

$$\begin{aligned} \frac{\partial U_0}{\partial I}(K \geq K^t) &= [\lambda + (1-\lambda)\alpha]R + (1-\lambda)(1-\alpha)\alpha R \frac{K^2}{\left(\left(I - \frac{e}{\lambda(1-\lambda)\alpha}\right) + K\right)^2} \\ &- \psi'(I) = 0 \end{aligned} \tag{63}$$

The respective second-order condition is:

$$\frac{\partial^2 U_0}{\partial^2 I}(K \geq K^t) = -(1-\lambda)(1-\alpha)\alpha R^2 \frac{2K^2}{\left(\left(I - \frac{e}{\lambda(1-\lambda)\alpha}\right) + K\right)^3} - \psi''(I) < 0 \tag{64}$$

which is satisfied since $\psi'' > 0$.

Furthermore, the first-order condition with respect to the interbank exposure K implies:

$$\frac{\partial U_0}{\partial K}(K \geq K^t) = (1-\lambda)(1-\alpha)\alpha R^2 \frac{\left(I - \frac{e}{\lambda(1-\lambda)\alpha}\right)^2}{\left(\left(I - \frac{e}{\lambda(1-\lambda)\alpha}\right) R + K\right)^2} - \tau'(K) = 0 \tag{65}$$

The second-order condition is:

$$\frac{\partial^2 U_0}{\partial^2 K}(K \geq K^t) = -(1-\lambda)(1-\alpha)\alpha R^2 \frac{2\left(I - \frac{e}{\lambda(1-\lambda)\alpha}\right)^2}{\left(\left(I - \frac{e}{\lambda(1-\lambda)\alpha}\right) R + K\right)^3} - \tau''(K) < 0 \tag{66}$$

which is satisfied since $\tau'' > 0$.

Hence, the optimal choice (I_0^c, K_0^c) must satisfy the equations:

$$[\lambda + (1-\lambda)\alpha]R + (1-\lambda)(1-\alpha)\alpha R \frac{(K_0^c)^2}{\left(\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right) + K_0^c\right)^2} \quad - \quad \psi'(I_0^c) = 0 \tag{67}$$

$$(1-\lambda)(1-\alpha)\alpha R^2 \frac{\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)^2}{\left(\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)R + K_0^c\right)^2} - \tau'(K_0^c) = 0 \tag{68}$$

which is a unique and interior maximum if $K_0^c \geq K^t$ since

$$\Delta \equiv \frac{\partial^2 U_0}{\partial^2 I}\frac{\partial^2 U_0}{\partial^2 K} - \left(\frac{\partial^2 U_0}{\partial I \partial K}\right)^2 > 0 \tag{69}$$

and $\partial^2 U_0 / \partial^2 I < 0$ with

$$\frac{\partial^2 U_0}{\partial I \partial K} = (1-\lambda)(1-\alpha)\alpha R^2 \frac{2\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)K_0^c}{\left(\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)R + K_0^c\right)^3} \tag{70}$$

If $K_0^c \geq K^t$ and

$$\begin{aligned}
\overline{U_0}(K \geq K^t) \quad &> \quad \overline{U_0}(K < K^t) && (71)\\
\left[\begin{array}{l} [\lambda + (1-\lambda)\alpha]\left(I_0^c - \underline{e}\right)R \\ +(1-\lambda)(1-\alpha)\alpha K_0^c \frac{\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)R}{\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)R + K_0^c} \\ -\alpha \underline{e} R \frac{(1-\lambda)(1-\alpha)}{\lambda + (1-\lambda)\alpha} - \psi(I_0^c) - \tau(K_0^c) + \underline{e} \end{array}\right] \quad &> \quad [\lambda + (1-\lambda)\alpha](I_0^{nc} - \underline{e})R - \psi(I_0^{nc}) + \underline{e} \\
(1-\lambda)(1-\alpha)\alpha K_0^c \frac{\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)R}{\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)R + K_0^c} - \tau(K_0^c) \quad &> \quad \left[\begin{array}{l} \int_{I_0^{nc}}^{I_0^c} \psi(x)dx \\ -[\lambda + (1-\lambda)\alpha](I_0^c - I_0^{nc})R \\ +\alpha \underline{e} R \frac{(1-\lambda)(1-\alpha)}{\lambda + (1-\lambda)\alpha} \end{array}\right] && (72)
\end{aligned}$$

hold, choosing the amount K_0^c of interbank deposits dominates the alternative of having no interbank exposure. As shown by Condition (72), choosing K_0^c dominates if the expected additional gain from the interbank exposure, due to the higher value of a bailout of the other bank, outweighs the loss in value of the own bank's bailout due to the contagion risk and the resulting lower interest rate $R_D^c < R_D^{nc} < (I - \underline{e})R$.

Hence, if $K_0^c \geq K^t$ and Condition (71) holds, the banks will choose to have the interbank exposure K_0^c. If, on the other hand, on of these conditions does not hold, they will chose to have no interbank exposure.

A.4. Proof of Proposition 4.3

First, we prove that $\overline{U_1} > \overline{U_0}(K < K^t)$ and then that $\overline{U_1} > \overline{U_0}(K \geq K^t)$. We prove that $\overline{U_1} > \overline{U_0}(K < K^t)$ in two steps. First, we show that correlated investments yield the same expected utility as uncorrelated investments when the banks choose $(I_0^{nc}, 0)$ instead of (I_1^*, K_1^*) when investing in correlated portfolios (and $K < K^t$). Second, due to the fact that (I_1^*, K_1^*) is a unique optimum, it has to hold that the expected utility when choosing (I_1^*, K_1^*) has to be higher than for the case that the bank chooses $(I_0^{nc}, 0)$ when investing in correlated investments. Comparing $U_1(I_0^{nc}, 0)$ and $\overline{U_0}(K < K^t)$ from Eq. (14) it follows that

$$\begin{aligned} U_1(I_0^{nc}, 0) &= \overline{U_0}(K < K^t) \\ \lambda(I_0^{nc} - \underline{e})R + (1-\lambda)\alpha(I_0^{nc} - \underline{e})R - \psi(I_0^{nc}) + \underline{e} &= [\lambda + (1-\lambda)\alpha](I_0^{nc} - \underline{e})R - \psi(I_0^{nc}) + \underline{e} \end{aligned} \tag{73}$$

Since $\overline{U_1} > U_1(I_0^{nc}, 0)$, it follows that $\overline{U_1} > \overline{U_0}(K < K^t)$.

Similarly, we show that $\overline{U_1} > \overline{U_0}(K \geq K^t)$ in two steps. First, we show that correlated investments yield a higher expected utility than uncorrelated investments even when the banks choose (I_0^c, K_0^c) instead of (I_1^*, K_1^*) when investing in correlated investments (and $K \geq K^t$). Second, due to the fact that (I_1^*, K_1^*) is a unique optimum, it has to hold that the expected utility when choosing (I_1^*, K_1^*) has to be higher than for the case that the bank chooses (I_0^c, K_0^c) when investing in correlated investments.

Comparing $U_1(I_0^c, K_0^c)$ and $\overline{U_0}(K \geq K^t)$ from Eq. (18) it follows that

$$\begin{aligned} U_1(I_0^c, K_0^c) &> \overline{U_0}(K \geq K^t) \\ \begin{bmatrix} \lambda(I_0^c - \underline{e})R + (1-\lambda)\alpha(I_0^c - \underline{e})R \\ +(1-\lambda)(1-\alpha)\alpha K_0^c \frac{(I_0^c - \underline{e})R}{(I_0^c - \underline{e})R + K_0^c} \\ -\psi(I_0^c) - \tau(K_0^c) + \underline{e} \end{bmatrix} &> \begin{bmatrix} [\lambda + (1-\lambda)\alpha]\,(I_0^c - \underline{e})R \\ +(1-\lambda)(1-\alpha)\alpha c_0^c K_0^c \frac{\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)R}{\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)R + K_0^c} \\ -\alpha \underline{e} R \frac{(1-\lambda)(1-\alpha)}{\lambda + (1-\lambda)\alpha} - \psi(I_0^c) - \tau(K_0^c) + \underline{e} \end{bmatrix} \\ \begin{bmatrix} (1-\lambda)(1-\alpha)\alpha K_0^c \frac{(I_0^c - \underline{e})R}{(I_0^c - \underline{e})R + K_0^c} \\ +\alpha \underline{e} R \frac{(1-\lambda)(1-\alpha)}{\lambda + (1-\lambda)\alpha} \end{bmatrix} &> (1-\lambda)(1-\alpha)\alpha K_0^c \frac{\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)R}{\left(I_0^c - \frac{\underline{e}}{\lambda(1-\lambda)\alpha}\right)R + K_0^c} \end{aligned} \tag{74}$$

which is true since $\lambda(1-\lambda)\alpha < 1$. Since $\overline{U_1} > U_1(I_0^c, K_0^c)$, it follows that $\overline{U_1} > \overline{U_0}(K \geq K^t)$.

A.5. Proof of Proposition 5.1

Inserting the expression from Eq. (26) and $K = K_A^\alpha + \Delta$ into Eq. (27) yields

$$\begin{aligned}
\max_{\Delta} U_B &= \lambda(1-\underline{e})R + (1-\lambda)(\alpha-\delta)(1-\underline{e})R \\
&+ (1-\lambda)(1-(\alpha-\delta))(\alpha+\delta)\frac{(K_A^\alpha+\Delta)(1-\underline{e})R}{(1-\underline{e})R+(K_A^\alpha+\Delta)} \\
&+ (1-\lambda)(1-(\alpha+\delta))(\alpha-\delta)\left[\frac{(K_A^\alpha+\Delta)(1-\underline{e})R}{(1-\underline{e})R+(K_A^\alpha+\Delta)} - \frac{K_A^\alpha(1-\underline{e})R}{(1-\underline{e})R+K_A^\alpha}\right] \\
&- 2\tau(K_A^\alpha+\Delta) + \tau(K_A^\alpha) \qquad (75)
\end{aligned}$$

where Δ is the interbank exposure that B_B wants to have in addition to K_A^α. The first order condition of Eq. (75) with respect to Δ yields the optimal additional interbank exposure Δ^α:

$$\begin{aligned}
\frac{\partial U_B}{\partial \Delta} &= 2(1-\lambda)\frac{(1-\underline{e})^2R^2}{((1-\underline{e})R+(K_A^\alpha+\Delta^\alpha))^2}\left[\delta^2+(1-\alpha)\alpha\right] \\
&- 2\tau'(K_A^*+\Delta^\alpha) = 0 \qquad (76)
\end{aligned}$$

Solving Eq. (76) for Δ^α and plugging it into Eq. (26) gives the optimal compensation fee η^α. From Eq. (76) follows for the derivative of Δ^α with respect to δ:

$$\frac{\partial \Delta^\alpha}{\partial \delta} = \frac{-\frac{2(1-\lambda)(1-\underline{e})^2R^2}{((1-\underline{e})R+(K_A^\alpha+\Delta^\alpha))^3}\frac{\partial K_A^\alpha}{\partial \delta}\left[\delta^2+(1-\alpha)\alpha\right] + \frac{4(1-\lambda)(1-\underline{e})^2R^2}{((1-\underline{e})R+(K_A^\alpha+\Delta^\alpha))^2}\delta}{\frac{2(1-\lambda)(1-\underline{e})^2R^2}{((1-\underline{e})R+(K_A^\alpha+\Delta^\alpha))^3}\left[\delta^2+(1-\alpha)\alpha\right]+\tau''(K_A^\alpha+\Delta^\alpha)} > 0 \qquad (77)$$

where the derivative is positive since $\partial K_A^*/\partial\delta < 0$. Therefore, the interbank exposure that B_B wants to have in addition to K_A^α increases with the difference between the bailout probabilities of the two banks, δ. Furthermore, from Eq. (26) it follows directly that η^α increases with δ.

A.7. Risk averse creditors

In the following, we allow uninsured creditors to be risk averse (in line with the literature on interbank networks and financial contagion, e.g., Allen and Gale, 2000; Brusco and Castiglionesi, 2007) to demonstrate the robustness of our results. Here, the interbank market not only is present for the reasons discussed previously, but also allows banks to coinsure against regional liquidity shocks as in Allen and Gale (2000). We show that even if the interbank market has a different reason to exist, our main mechanism is still present. Specifically, we show that banks have an incentive to increase their interbank exposure beyond the level that would be sufficient to perfectly coinsure against liquidity shocks. Our economy in this Section now consists of three dates $t = 0, 1, 2$ and, again, two regions A and B, each with a continuum of identical banks that all adopt the same behavior and can thus be described by a representative bank (protected by limited liability). Furthermore, there are now n ex ante identical uninsured creditors and again one risk-neutral investor in each region. Creditors have Diamond-Dybvig (1983) preferences, that is,

$$U(c_1, c_2) = \begin{cases} u(c_1) \text{ with probability } \omega^i \text{ (early creditors)} \\ u(c_2) \text{ with probability } 1 - \omega^i \text{ (late creditors)} \end{cases} \tag{78}$$

where $i \in \{A, B\}$ and the utility function $u(\cdot)$ is defined for nonnegative numbers, strictly increasing, strictly concave, and twice continuously differentiable and satisfies Inada conditions. Consumption at $t = 1$ ($t = 2$) is denoted by c_1 (c_2). Each creditor is endowed with one unit of capital at $t = 0$. Of the n creditors in each region there are n_e^i early creditors and n_l^i late creditors. Thus, $\omega^i \equiv n_e^i/n$ represents the fraction of early creditors, where ω^i can be either high or low ($\omega_H > \omega_L$). There are two equally likely states S_1 and S_2. At $t = 1$ state-dependent liquidity preferences are revealed (see Table 4).

	ω^A	ω^B
S_1	ω_H	ω_L
S_2	ω_L	ω_H

Table 4: Liquidity shocks

Each region has the same ex ante probability of facing a high liquidity shock. A creditor's type is private information and the proportion of early creditors in the whole economy is given by $\gamma = (\omega_H + \omega_L)/2$. Thus, there is no aggregate uncertainty. At $t = 1$ all liquidity-related uncertainty is resolved and creditors learn their type.

There are two types of investment opportunities: a risk-free, liquid type and a risky, illiquid one (generating only a return of $r < 1$ if liquidated at $t = 1$). The risk-free asset is a storage technology that transfers one unit of capital at a certain period into one unit of capital in the following period. The illiquid asset is only available at $t = 0$ and generates a return of either $R > 1$ with probability λ or zero with probability $(1 - \lambda)$ at $t = 2$ for each unit of capital invested. We assume that the illiquid asset has a positive NPV, that is, $\lambda R > 1$, and that investment outcomes are perfectly positively correlated across regions.

Since our model now has three dates, the equity investors are entitled to receive dividends at $t = 1$ and $t = 2$. Hence, the investor's utility is now

$$u(d_0, d_1, d_2) = \lambda R d_0 + d_1 + d_2 \tag{79}$$

As before, since investors can obtain a utility of $\lambda e R$ by immediately consuming the initial endowment, they must earn an expected return of at least λR on their invested money to give up consumption at $t = 0$. Hence, the participation constraint for investors becomes

$$E[d_1 + d_2] \geq \lambda \underline{e} R \tag{80}$$

As shown by Brusco and Castiglionesi (2007), we can restrict attention to policies paying no dividends at $t = 1$. Therefore, the bank has to invest the whole equity contribution into the illiquid asset and the full proceeds from this investment have to be paid to the equity investors to satisfy their participation constraint. Hence, we only have to analyze the bank's decision regarding the allocation of the debt contribution.

Central planner economy

In this economy the Pareto-efficient allocation can be characterized as the solution to the problem of a planner maximizing the creditors' expected utility. By pooling resources the planner can overcome the problem of the regions' asymmetric liquidity needs. Let y and x denote the per capita amounts invested in the risk-free and risky assets, respectively. Furthermore, let c and cr_d denote the amounts creditors can withdraw to satisfy their liquidity needs at $t = 1$ and $t = 2$, respectively. In this context, r_d can be understood as the interest rate creditors earn by not withdrawing their funds for an additional period. The planner's problem can then be written as

$$\max_{x,y,c,r_d} U = \gamma u(c) + (1 - \gamma)\, \lambda u(cr_d) \tag{81}$$

subject to

$$2x + 2y \leq 2n,\ \ \gamma\, 2nc \leq 2y,\ \ (1 - \gamma)\, 2ncr_d \leq 2xR, \tag{82}$$

$$x \geq 0,\ y \geq 0,\ c \geq 0,\ r_d \geq 0. \tag{83}$$

The first set of constraints represents budget constraints for periods 0, 1 and 2. Since optimality requires that the constraints be binding, the optimization problem can be rewritten as

$$\max_{y} \gamma u\left(\frac{y}{\gamma n}\right) + (1-\gamma)\, \lambda u\left(\frac{R(n-y)}{(1-\gamma)n}\right) \tag{84}$$

Given the utility function's properties this optimization problem has a unique interior solution. The optimal value $y^* \in (0,1)$ can be obtained from the first-order condition

$$u'\left(\frac{y^*}{\gamma n}\right) = \lambda R u'\left(\frac{R(n-y^*)}{(1-\gamma)n}\right) \tag{85}$$

Once y^* has been determined, we can use the remaining constraints to determine the optimal values of the other variables. Hence, we obtain

$$c^* = \frac{y^*}{\gamma n},\ r_d^* = \frac{R(n-y^*)}{(1-\gamma)nc^*},\ \text{and } x^* = n - y^* \tag{86}$$

Since $\lambda R > 1$, we can conclude that $u'(c) > u'(cr_d)$ and hence $r_d > 1$, implying that consumption is higher at $t = 2$ than at $t = 1$. Consequently, late creditors have no incentive to mimic early creditors. We denote the first-best allocation as $\delta^* = (y^*, x^*, c^*, r_d^*)$.

Decentralized economy with an interbank market and no bailout possibility

Allen and Gale (2000) show that this first-best allocation can be achieved by allowing banks in a decentralized economy to coinsure against liquidity shocks. Coinsurance is possible since the liquidity needs of the two regions are negatively correlated. In contrast to Allen and Gale (2000), we again allow banks to exchange an arbitrary amount of deposits K at $t = 0$, and not only the amount necessary to achieve first-best. However, we show that exchanging funds above the level of the first best solution does not increase the utility of uninsured creditors if there is no bailout possibility. Let k denote the amount of interbank deposits that is withdrawn by the bank that faces a high liquidity shock at $t = 1$.

The capital flows are depicted in Fig. 4. At $t = 0$ the two banks exchange deposits K. At $t = 1$ the bank with the high liquidity shock (B_A in Fig. 4) withdraws an amount k from

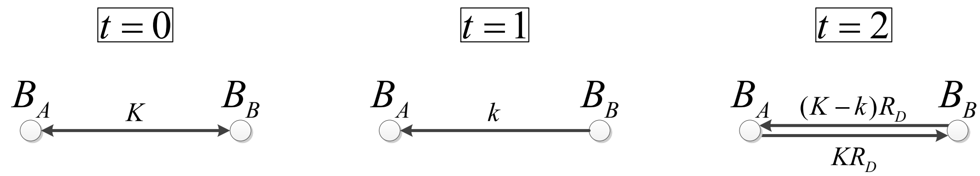

Figure 4: Capital flows in the two region economy

the other bank to satisfy the liquidity needs of its creditors. In the final period bank B_A receives its remaining deposits $(K - k)$ from bank B_B and pays back the deposits that bank B_B deposited in bank B_A. From now on, we follow Allen and Gale (2000) in that we assume that these remaining deposits also yield the interest r_d and that interbank deposits incur no transaction costs. Hence, each bank can offer a contract $\delta = (y, x, c, r_d, K)$ to its creditors and the bank in the other region. With perfect competition in the banking sector, the banks will offer their creditors a contract that replicates the first-best outcome. The optimization problem of a bank can then be written as

$$\max_{x,y,c,r_d,K,k} U = \frac{1}{2}[\omega_H u(c) + (1 - \omega_H)\lambda u(cr_d)] + \frac{1}{2}[\omega_L u(c) + (1 - \omega_L)\lambda u(cr_d)] \tag{87}$$

subject to

$$\omega_H nc \leq y + k \tag{88}$$

$$\omega_L nc + k \leq y \tag{89}$$

$$(1 - \omega_H)ncr_d + Kr_d \leq Rx + (K - k)r_d \tag{90}$$

$$(1 - \omega_L)ncr_d + (K - k)r_d \leq Rx + Kr_d \tag{91}$$

$$x \geq 0,\ y \geq 0,\ c \geq 0,\ r_d \geq 0,\ x + y \leq n,\ k \leq K \tag{92}$$

Constraints (88) and (89) represent budget constraints at $t = 1$ and Constraints (90) and (91) represent budget constraints at $t = 2$. As shown by Allen and Gale (2000), optimality requires that $k^* = (\omega_H - \gamma)nc^*$. As long as there is no positive bailout probability, the actual amount of funds exchanged, K, does not alter the utility of the creditors as long as $K \geq k^*$. These findings lead to the following proposition.

Proposition 7.1. *If there is no possibility for banks to be bailed out and the two representative banks exchange an amount K of deposits, then the first-best allocation δ^* can be implemented by a decentralized banking system offering standard deposit contracts. Moreover, banks have no incentive to exchange more funds than required to achieve first-best, that is, they will only exchange $K = k^* = (\omega_H - \gamma)nc^*$.*

Proof For the proof of the first part of the proposition, we refer to the proof of Proposition 3 of Brusco and Castiglionesi (2007). To see why the second part is true, that is, why banks do not exchange more than necessary to achieve first-best, note that optimality again requires the constraints to be binding. Then the amount of funds actually exchanged, K, drops out of the optimization problem. Hence, the amount that is actually exchanged does not influence the utility of the creditors. Therefore, banks have no incentive to exchange more funds than necessary to achieve first-best, which implies that $K = k^* = (\omega_H - \gamma)nc^*$. QED

Decentralized economy with an interbank market and positive bailout probability

So far we have assumed that after a bank failure occurs, creditors receive no repayment at $t = 2$. Now we investigate how the results change if there is the possibility that a bank will be bailed out by the government after a default. As before, we assume that a bailout happens with probability α. Therefore, the optimization problem becomes

$$\begin{aligned} \max_{x,y,c,r_d,K,k} U &= \frac{1}{2}\left[\omega_H u(c) + (1-\omega_H)\left[\begin{array}{l} \lambda u(cr_d) + (1-\lambda)[(1-\alpha)^2 u(0) \\ +\alpha(1-\alpha)u(cr_d) \\ +(1-\alpha)\alpha u\left(\theta_1 cr_d\right) + \alpha^2 u(cr_d)] \end{array}\right]\right] \\ &+ \frac{1}{2}\left[\omega_L u(c) + (1-\omega_L)\left[\begin{array}{l} \lambda u(cr_d) + (1-\lambda)[(1-\alpha)^2 u(0) \\ +\alpha(1-\alpha)u(cr_d) \\ +(1-\alpha)\alpha u\left(\theta_2 cr_d\right) + \alpha^2 u(cr_d)] \end{array}\right]\right] \end{aligned} \tag{93}$$

with

$$\theta_1 = \frac{K-k}{(1-\omega_H)ncr_d + K} \quad \text{and} \quad \theta_2 = \frac{K}{(1-\omega_L)ncr_d + (K-k)}$$

subject to

$$\omega_H nc \leq y + k \tag{94}$$

$$\omega_L nc + k \leq y \tag{95}$$

$$(1-\omega_H)ncr_d + Kr_d \leq Rx + (K-k)r_d \tag{96}$$

$$(1-\omega_L)ncr_d + (K-k)r_d \leq Rx + Kr_d \tag{97}$$

$$x \geq 0,\ y \geq 0,\ c \geq 0,\ r_d \geq 0,\ x+y \leq n,\ k \leq K \tag{98}$$

Eq. (93) is the objective function of the optimization problem of the representative bank in region i. The bank in region i is equally likely to face a high or a low liquidity shock. If a high liquidity shock occurs in, for example, region A, a fraction ω_H of the creditors will withdraw their funds at $t=1$ and the remaining creditors will demand repayment in $t=2$. At $t=2$ several cases must be considered. The risky asset yields a positive return R with probability λ and creditors receive their promised repayment cr_d. If the risky asset yields a zero payoff, the return of the creditor depends on whether the banks are bailed out or not. If neither of the two banks is bailed out, creditors receive no payment. If the bank in region A is bailed out, the government steps in and creditors receive their full repayment cr_d. If only the bank in region B is bailed out, bank B_A receives the funds still owed to it by B_B (see Fig. 4). Since B_A has already withdrawn an amount k at $t=1$, it receives the remaining funds $(K-k)r_d$. Since B_A has two creditors, namely, its uninsured creditor and bank B_B, funds are again split on a pro rata basis. Hence, creditors receive a fraction θ_1 of their promised repayment. Finally, if both banks are bailed out, then creditors again receive the full amount. The second case (where B_A faces a low liquidity shock) can be described analogously.

All constraints are as in the previous case without a bailout possibility. By examining the optimization problem, it becomes obvious that the amount of funds exchanged, K, now has an influence on the utility of the creditors. Although K again drops out of the constraints (optimality again requires the constraints to be binding), it now also enters the objective function directly because it determines the amount that creditors receive in the case of a default if only one bank is bailed out. Before the repayment in this state of nature was zero.

Again, optimality requires that banks choose $k^* = (\omega_H - \gamma)nc^*$. Hence, the optimization problem (93) can be simplified to

$$\begin{aligned}\max_{x,y,c,r_d,K} U &= \gamma u(c) + (1-\gamma)\left[\lambda + (1-\lambda)\alpha\right] u(cr_d) \\ &+ \frac{1}{2}(1-\lambda)(1-\alpha)\alpha\left[(1-\omega_H)u(\theta_1 cr_d) + (1-\omega_L)u(\theta_2 cr_d)\right] \qquad (99)\end{aligned}$$

subject to

$$x + y \leq n,\ \gamma nc \leq y,\ (1-\gamma)ncr_d \leq xR, \qquad (100)$$

$$x \geq 0,\ y \geq 0,\ c \geq 0,\ r_d \geq 0. \qquad (101)$$

Since the constraints in the respective periods again have to be binding, we can solve them for c and r_d, respectively and can plug these values into the objective function, which yields:

$$\begin{aligned}\max_{y,K} U &= \gamma u\left(\frac{y}{\gamma n}\right) + (1-\gamma)\left[\lambda + (1-\lambda)\alpha\right] u\left(\frac{R(n-y)}{(1-\gamma)n}\right) \\ &+ \frac{1}{2}(1-\omega_H)(1-\lambda)(1-\alpha)\alpha u\left(\theta_1\frac{R(n-y)}{(1-\gamma)n}\right) \\ &+ \frac{1}{2}(1-\omega_L)(1-\lambda)(1-\alpha)\alpha u\left(\theta_2\frac{R(n-y)}{(1-\gamma)n}\right) \qquad (102)\end{aligned}$$

The first order condition with respect to y then yields:

$$\begin{aligned}u'\left(\frac{y}{\gamma n}\right) &= [\lambda + (1-\lambda)\alpha]u'\left(\frac{R(n-y)}{(1-\gamma)n}\right)R \\ &+ \frac{1}{2}\frac{(1-\omega_H)}{(1-\gamma)}(1-\lambda)(1-\alpha)\alpha u'\left(\theta_1\frac{R(n-y)}{(1-\gamma)n}\right)\theta_1 R \\ &+ \frac{1}{2}\frac{(1-\omega_L)}{(1-\gamma)}(1-\lambda)(1-\alpha)\alpha u'\left(\theta_2\frac{R(n-y)}{(1-\gamma)n}\right)\theta_2 R \qquad (103)\end{aligned}$$

where the second order conditions are satisfied. Looking at this first order condition one can see that the marginal utility of consumption at $t = 1$ is higher now, implying that consumption is lower. Hence, if it is more likely to get the higher repayment at $t = 2$ creditors want to shift more consumption to this later period. Hence, the optimal amount of funds withdrawn at $t = 1$ is now smaller than in the situation without bailout. Furthermore, we obtain the following first-order condition for K:

$$\begin{aligned}\frac{\partial U}{\partial K} &= \frac{1}{2}\frac{(1-\omega_H)}{(1-\gamma)}(1-\lambda)(1-\alpha)\alpha u'\left(\theta_1\frac{R(n-y)}{(1-\gamma)n}\right)\frac{\partial\theta_1}{\partial K}\frac{R(n-y)}{(1-\gamma)n} \\ &+ \frac{1}{2}\frac{(1-\omega_L)}{(1-\gamma)}(1-\lambda)(1-\alpha)\alpha u'\left(\theta_2\frac{R(n-y)}{(1-\gamma)n}\right)\frac{\partial\theta_2}{\partial K}\frac{R(n-y)}{(1-\gamma)n} > 0 \end{aligned} \tag{104}$$

which is true since $\partial\theta_1/\partial K > 0$ and $\partial\theta_2/\partial K > 0$. As we can see from the first-order condition, the utility of the creditor is now increasing in K (i.e., the funds exchanged at $t = 0$), since K increases the amount that the creditor receives in case of default of the risky asset (although the amount needed to satisfy the consumption needs of creditors is now actually smaller, banks have an incentive to increase their interbank exposure). Therefore, banks have an incentive to increase the amount of interbank deposits and hence their connectivity to a level that exceeds the first-best solution derived before. These findings yield the following proposition.

Proposition 7.2. *Given a positive bailout probability, banks have an incentive to increase their interbank exposure beyond the first-best level.*

Hence, even if the interbank market does not exist only as an insurance for noninsured creditors but also to coinsure against regional liquidity shocks, as in Allen and Gale (2000), the main mechanism is still present.

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