Entrepreneurs Heterogeneity and New Venture Financing

PRELIMINARY AND INCOMPLETE

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Abstract

In this article, we study the market for entrepreneurial finance and explain the coexistence of different financiers like Venture Capital (VC) and Angel investors. Our premise is that entrepreneur heterogeneity plays a major role in determining the structure of the market for entrepreneurial finance. We capture entrepreneur heterogeneity by assuming that entrepreneurs utility include *motivational factors* beyond that provided by *value considerations*.

In our model, the difference between VC and Angel investors is that VC investors acquire better information. It seems natural that VC investors, being better informed, will dominate the market and wipe out the less informed Angel investors. When we consider entrepreneur heterogeneity, however, the outcome is different, as Angel investors offer entrepreneurs an avenue to better capture their motivational factors.

Our model yields several empirical implications: 1. If entrepreneurs switch financiers, they will switch from Angel financing to VC financing. 2. On average, Angel-backed ventures are smaller, have higher ex-ante expected values, and are less likely to be liquidated compared to VC-backed ventures. 3. Industries with more attractive characteristics tend to have more Angel financing. 4. Boom periods are characterized by more Angel financing. 5. Locations with better entrepreneurial ecosystems exhibit more Angel financing.

1. **Introduction**

In this article, we study the market for entrepreneurial finance and explain the coexistence of different financiers like Venture Capital (VC) and Angel investors. This issue has become increasingly important because new ventures are the major source of growth in GDP and job creation in the economy,[1] and because some estimates suggest that Angel investors are as important as VC investors in financing new ventures.[2]

Our premise is that entrepreneur heterogeneity plays a major role in determining the structure of the market for entrepreneurial finance. We capture entrepreneur heterogeneity by assuming that entrepreneurs utility include *motivational factors* beyond that provided by *value considerations*.

The entrepreneurship literature identifies personal attributes of entrepreneurs, including attitude toward risk (Shane and Venkataraman (2000)), need for achievement (McClelland (1961)), tolerance for ambiguity (Schere (1982)), and locus of control (Rotter (1966)). See Shane, Locke, and Collins (2003) for a detailed description of these factors. Clearly, these attributes affect entrepreneur utility beyond value considerations. While there is no generally accepted theory that connects these attributes to a specific utility function, we postulate that they enter the utility function in the form of motivational factors[3]. For example, entrepreneurs with need for achievement derive utility from developing, say, a successful treatment for a previously incurable disease.

[1] See Haltiwanger, Jarmin, and Miranda (2009).
[2] See Goldfarb et al. (2007), Ibrahim (2008), Shane (2012), Sudek et al. (2008), Kerr et al. (2014).
[3] Almlund et al. (2011) discuss the connection between personal traits and utility functions. Benabou and Tirole (2003) introduce "intrinsic motivation" into the utility function of an agent, and show that its presence may cause monetary rewards to adversely affect the agent motivation.

We consider a dynamic financing model with learning, where venture takes two stages to complete. First is the *design stage*, which requires an initial investment at the outset. Second is the *development stage*, which requires additional investment. Entrepreneurs may secure the entire investment at the outset or use stage financing, raising the initial investment at the outset and the additional investment after the design stage. Following completion of the development stage, the venture's product is brought to the market and the venture *exit value* is realized. The exit value can be either high or low. We assume that entrepreneurs enjoy motivational factors whenever the venture continues to the development stage.

To finance the venture, entrepreneurs must raise the required investment in the market for entrepreneurial finance. There are three types of investors in this market: VC, Expert Angel, and General Angel. All investors share the same information at the outset and are similar in all aspects except their information acquisition capabilities during the design stage. VC investors are the most informed as they learn during the design stage whether the exit value will be high or low. General Angel investors are the least informed, learning no new information during that stage. Expert Angel investors are partially informed, learning some information about the likelihood of a high exit value.

Value maximization calls for VC financing because VC investors optimally refuse to finance the development stage when they learn that the exit value will be low. Nevertheless, entrepreneurs with high enough motivational factors would not want VC financing, thereby deviating from value maximization. They prefer General Angel financing, because it enables them to enjoy their motivational factors because such investors, who do not acquire new information during the design stage, may finance also the development stage, even when the exit value will be low. Expert Angel investors acquire partial information and, thus, sometimes refuse

to finance the development stage. Entrepreneurs resort to Expert Angel financing when their motivational factors are high but General Angel investors refuse to provide financing.

Perhaps surprisingly, the better informed VC investors do not force the less informed Angel investors from the market. This would be the case if entrepreneurs only cared about value. However, Angel investors offer entrepreneurs an avenue to enjoy their motivational factors. This implies that the structure of the market for entrepreneurial finance caters to the needs of heterogeneous entrepreneurs, as it provides entrepreneurs with their investor of choice. A possible implication of the above is that Angel investors design their business to avoid learning new information so that they can appeal to entrepreneurs with high motivational factors.

Our model yields several empirical implications:

1. If entrepreneurs switch financiers, they will switch from Angel financing to VC financing.
2. On average, Angel investors will invest smaller amounts than VC investors.
3. On average, Angel-backed ventures have higher expected values at the time of the initial investment compared to VC-backed ventures.
4. VC-backed ventures are more likely to be liquidated in early stages than Angel-backed ventures.
5. An industry with more attractive venture characteristics, e.g., lower investments and higher expected exit values at the time of the initial investment, is likely to have more Angel-backed ventures.
6. Better periods in the business cycle ("boom periods") are characterized by more Angel-backed ventures.

7. Geographic locations with better entrepreneurial ecosystems tend to have more Angel-backed ventures.

"Conventional wisdom" holds that VC investors invest in later-stage larger ventures because earlier stage investments are too small for them. On the other hand, Angel investors invest in earlier-stage smaller ventures because they are less sophisticated and have fewer funds (Ibrahim, (2008)). In contrast, in the equilibrium of our model Angel investors support smaller early-stage ventures even when they are fully rational and have unlimited funds and no venture is "too small" for VC investors.

Our theory provides a new rationale for government intervention to promote entrepreneurship. Current government justification for intervention in the market for small businesses is the generation of important knowledge spillovers, such as new ideas, new products, new technologies, and new entrepreneurial know-how. Our model reveals an additional rationale for government intervention: "discouraged entrepreneurs" with negative motivational factors may stay out of the market even though they have high net present value (NPV) ventures.[4] This causes a significant welfare loss that is not due to knowledge spillovers. Thus, government interventions that enhance entrepreneurs' motivational factors or subsidize positive NPV ventures may greatly improve social welfare.

Our theory also provides guidance for more efficient implementation of policy measures when the government wants to enhance entrepreneurship by subsidizing ventures.[5] The government should not subsidize Angel-backed ventures because this leads to social waste by

[4] An example for a "discouraged entrepreneurs" with negative motivational factors, is an entrepreneur with a significant "fear of failure".

[5] The government may want to subsidize positive NPV ventures when entrepreneurs have negative motivational factors or negative NPV ventures when there are positive externalities.

only increasing value to entrepreneurs that would get financing anyway. In contrast, a subsidy that is given to VC-backed ventures may increase new venture activity.

The theoretical entrepreneurial finance literature generally abstracts away from the differences between Angel and VC investors. An exception is the Conti et al. (2011) signaling model, in which entrepreneurs use excessive patenting to signal their quality to VC investors and the level of family and friend financing to signal to Angel investors their commitment to the venture. Our theory is based on entrepreneur heterogeneity without signaling considerations.

Bergemann and Hege (2005) consider the financing of an R&D (research and development) project where Angel investors are less informed than VC investors. In their model, entrepreneurs receive money from the investor and decide whether to invest it or to consume it. The informed VC investors can observe the allocation decision of the entrepreneur while the uninformed Angel investors cannot. Consequently, they obtain that Angel investors will stop financing sooner than VC investors. In our model, the choice of financing is driven by entrepreneur heterogeneity. We show that Angel investors are more likely to continue financing than VC investors because, being less informed, they can better cater to the motivational factor of the entrepreneur.

Admati and Pfleiderer (1994), Berglöf (1994), Bergmann and Hege (1998), Hellmann (1998), and Cornelli and Yosha (2003) focus only on VC investors and examine various contractual terms between VC investors and entrepreneurs. We focus on the structure of the market for entrepreneurial finance and explain the coexistence of Angel and VC investors.

The empirical literature investigates the contribution of early-stage financiers to venture value, focusing mostly on VC financing. Hellemann and Puri (2000), Puri and Zarutski (2012), and Chemmanur et al. (2011) find that VC-backed ventures have better long-term performance

than non-VC-backed ventures. Other studies identify particular traits of VC-backed financing that lead to superior venture performance (Hochberg et al. (2007), Sorensen (2007)). Other studies try to better identify the ability of VC to enhance value by looking at exogenous shocks to the supply of VC financing (Kortum and Lerner (2000), Samila and Sorenson (2011), Mollica and Zingales (2007)).

Unlike the VC financing literature, Angel financing has received much less attention, even though Angel investors have a significant role in financing new ventures. An exception is Kerr, Lerner, and Schoar (2014) who study investment decisions by Angel groups. Our theory provides a useful guide for future empirical work on the choice between Angel financing and VC financing.

The rest of the article is organized as follows. In section 2, we introduce the model. In Section 3, we analyze the game in which VC financing is the only option. In Sections 4 and 5, we do the same for Expert Angel and General Angel financing, respectively. In Section 6, we analyze the simultaneous choice between VC, Expert Angel, and General Angel financing. Sections 7 and 8 discuss the empirical and policy implications of the model. In Section 9, we discuss the robustness of our model and possible extensions, and Section 10 concludes. All proofs are in the Appendix.

2. Model

We envision a market for entrepreneurial finance consisting of many entrepreneurs and potential financiers. Entrepreneurs develop new ventures and need to raise funds. For simplicity, we assume that entrepreneurs have no personal funds and need to raise the entire sum from outside

investors. We assume equity financing.[6] New venture development consists of two stages over three dates, $t = 0, 1, 2$, until the product is brought to the market. The initial stage is the *design stage*. It starts at $t = 0$ and requires an *initial investment* I_0. At $t = 1$, the design stage ends and the product *development stage* begins. This stage requires a *follow-up investment* I_1 at $t = 1$. The entrepreneur may raise the entire investment $I = I_0 + I_1$ at $t = 0$, or raise I_0 at $t = 0$ and I_1 at $t = 1$. At $t = 2$, the product is brought to the market and the venture *exit value* is realized.

We assume that the exit value at $t = 2$ can be either high or low. While we formally refer to the venture value at $t = 2$ as an exit value, our model equally applies to situations where the venture continues to operate beyond $t = 2$ as an independent company, in which case the exit value represents the present value of all future cash flows at $t = 2$. We normalize the low exit value to zero, and denote the high exit value by V. The exit value V represents the estimate at $t = 0$ of what will be the high exit value at $t = 2$.

The design stage determines the probability distribution over the exit value. The exit value is V with probability P, and 0 with probability $1 - P$. A higher probability P corresponds to a more productive design stage. At $t = 0$, the probability P is unknown and is characterized by a distribution function $F(P)$ with an expected value $\bar{P}$.

Our premise is that entrepreneurs utility include *motivational factors* (MF) beyond that provided by *value considerations*. Value considerations imply that entrepreneur utility increases with the value of the venture. Motivational factors imply that entrepreneurs draw utility from other aspects of the venture that interact with their personal attributes. Some examples are:

[6] Other forms of financing like debt or preferred stocks would not change the main results of our model.

1. An entrepreneur who derives positive or negative utility from being in control. The economic and finance literature focuses on positive "private benefits" of control.[7] Negative utility from being in control could arise when entrepreneurs dislike controlling or managing others
2. An entrepreneur who cares about the environment draws positive utility from environmentally friendly ventures and negative utility from environmentally abusive ventures.
3. An entrepreneur with a "need for achievement" derives utility from developing a successful treatment for a previously incurable disease, regardless of the value of this venture.

The entrepreneurship literature identifies various motivations for entrepreneurs, including entrepreneurs' attitude towards risk (Shane and Venkataraman (2000)), need for achievement (McClelland (1961)), tolerance for ambiguity (Schere (1982)), locus of control (Rotter (1966)) and others. See Shane, Locke, and Collins (2003) for a detail description of these factors.

In general, an entrepreneur's utility from developing and bringing a venture to the market depends on the venture exit value and the motivational factors that he derives from the venture. An entrepreneur derives a vector of motivational factors $(MF_1, MF_2, MF_3 \ldots\ldots MF_n)$ from the venture.[8] For simplicity, we let $MF = U(MF_1, MF_2, MF_3 \ldots\ldots MF_n)$ represent the utility equivalence of an entrepreneur's motivational factors, where $MF \in (-\infty, \infty)$. The economy consists of a large pool of entrepreneurs with different MFs. We denote the cumulative distribution function of MF across entrepreneurs by $Q(MF)$.

[7] See Grossman and Hart (1988), Harris and Raviv (1988), Harris and Raviv (1990), and Berkovitch and Israel (1999).

[8] To the extent that the entrepreneur has more than one potential venture, he selects the venture that maximizes his utility.

Consider an entrepreneur with a given MF. We assume that the entrepreneur's utility function is additively separable and linear in the venture value and his motivational factors:

$$U = \upsilon + MF, \tag{1}$$

where υ represents the value to the entrepreneur from his stake in the venture. When the venture exit value is low, then $\upsilon = 0$. When the venture exit value is high, then υ equals the entrepreneur's share of the exit value V.

The game evolves as follows. At $t = 0$, the structure of the game, $V, I_0, I_1, F(P)$, and MF are common knowledge to the entrepreneur and all investors that consider the venture.[9] The entrepreneur can raise funds at $t = 0$ and $t = 1$ from three types of investors. These investors differ in their information acquisition capabilities at $t = 1$; the most informed, the least informed, and the partially informed investors. The most informed investors learn at $t = 1$ if the exit value at $t = 2$ will be high or low.[10] The least informed investors do not learn new information at $t = 1$ and maintain their prior beliefs $F(P)$. The partially informed investors learn at $t = 1$ the probability P but do not know whether the exit value at $t = 2$ will be high or low. We assume that the entrepreneur learns the probability P at $t = 1$.[11] In addition, once the initial investment is made at $t = 0$, the identity of the investor becomes public information.[12]

We refer to the most informed investors as "Venture Capital" (VC) investors, the least informed investors as "General Angel" (GA) investors, and the partially informed investors as

[9] The analysis is identical if the entrepreneur motivational factors MF are private information held only by the entrepreneur.

[10] This assumption is for the ease of exposition. A more realistic assumption is that the most informed investors are not "perfectly informed" but are better informed than all other investors. The results do not depend on whether the investors are perfectly informed or better informed. The results go through because the better informed investors make better investment decisions.

[11] This assumption implies that the entrepreneur has the same information as the partially informed investors. In Section 9.2, we discuss the implications of changing this assumption.

[12] In practice, the identity of all existing investors is summarized in a "cap table" that is available for current and future investors.

"Expert Angel" (EA) investors. VC investors employ technological, product, and market specialists to gather and analyze information about ventures' products and their markets; thus, these investors are likely to be the most informed. Expert Angel investors are financiers with a limited ability to gather information about ventures' products and markets because they do not employ professional analysts. They do have, however, some knowledge of ventures' products, likely because they are or have been involved in companies from the same or closely related industries. General Angel investors are wealthy individuals with little or no knowledge of ventures' products and their markets.[13] We assume a competitive financing market with many GA, EA, and VC investors.

We next solve the financing game by first analyzing the case where only VC financing is available in the economy (Section 3), where only EA financing is available (Section 4), and where only *GA* financing is available (Section 5). In Section 6 we analyze the case where all type of financiers are available.

3. Venture Capital Financing

In this section, we consider the case where only VC financing is available for the two investments, I_0 and I_1. We denote by $(1-\alpha)$ the equity ownership of the VC investor of $t=0$

[13] To focus on the implications of entrepreneur heterogeneity on the financing of new ventures, we abstract away from the advising and networking roles of start-up investors by implicitly assuming that all investors have the same contribution, so the exit value is unaffected by the identity of the financier. This assumption is without loss of generality and is made for the ease of exposition. See discussion in Section 9.3.

and by $(1-\gamma)$ the equity ownership of the VC investor of $t=1$.[14] It follows that the entrepreneur gets the remaining equity stake, $\alpha\gamma$.[15]

We first solve the financing problem at $t=1$, taking the solution of the financing problem at $t=0$, α, as given. Recall that at $t=1$ a VC investor knows whether the exit value will be zero or V at $t=2$. Since ventures with $I_1 > V$ are not fundable, the analysis that follows is constrained to ventures with $V \geq I_1$. When the VC learns that the venture value will be V, she can recover her investment I_1 and thus is willing to invest. When the VC learns that the venture value will be zero, she cannot recover her investment and thus does not invest. Consequently, the entrepreneur's expected utility at $t=1$ after learning P but before approaching a VC investor is

$$U_{vc}^{1} = P \times (\alpha\gamma V + MF). \tag{2}$$

In Equation (2), the term $P\alpha\gamma V$ is the entrepreneur's expected share of the venture value. It consists of the product of the entrepreneur's share of the exit value at $t=1$, $\alpha\gamma V$, and the probability P. Similarly, $P \times MF$ is the expected value of the entrepreneur's motivational factors. The assumption that financial markets are competitive implies that the VC share of the venture equals her investment I_1. Consequently, γ is the solution to

$$(1-\gamma)V = I_1. \tag{3}$$

Rearranging Equation (3) yields

$$\gamma = \frac{V-I_1}{V}. \tag{4}$$

Substituting Equation (4) into Equation (2) yields the entrepreneur's expected utility at $t=1$:

$$U_{vc}^{1} = P(\alpha(V-I_1) + MF). \tag{5}$$

[14] While the investor of $t=0$ gets a stake of $(1-\alpha)$, it will be diluted at $t=1$ by $(1-\gamma)$, resulting in a final stake of $(1-\alpha)\gamma$.

[15] The resulting claims on the venture are $\alpha\gamma$, $(1-\alpha)\gamma$, and $(1-\gamma)$ for the entrepreneur, the investor of $t=0$, and investor of $t=1$, respectively.

Equation (5) implies that the entrepreneur expects to obtain his share of the venture value net of the investment of $t = 1$, $\alpha(V - I_1)$, plus the utility from his motivational factors, MF, all multiplied by the probability P.

The expected payoff of the initial VC investor, conditional on the probability P, is

$$\pi_{vc}^{1} = P(1 - \alpha)(V - I_1). \tag{6}$$

Equation (6) implies that the expected payoff of the initial VC investor equals its share of the venture, $(1 - \alpha)$, times the venture exit value net of the investment at $t = 1$, $V - I_1$, times the probability P.

At the time the initial VC investor decides whether to invest, she does not know the realized probability P. Consequently, she compares her expected payoff from the venture to the initial investment:

$$\int_0^1 \pi_{vc}^{1}\, dF(P) = (1 - \alpha)(V - I_1) \int_0^1 P\, dF(P) = (1 - \alpha)\bar{P} \times (V - I_1) \; = I_0. \tag{7}$$

Equation (7) is the expected payoff to the VC investor, where the expectation is taken over the probability P at $t = 0$. Rearranging Equation (7) yields

$$\alpha = \frac{\bar{P} \times (V - I_1) - I_0}{\bar{P} \times (V - I_1)}. \tag{8}$$

Equation (8) reveals that the venture is fundable at $t = 0$ by a VC investor whenever

$$\bar{P} \times (V - I_1) \geq I_0. \tag{9}$$

Substituting α from Equation (8) into Equation (5) and taking the expectation with respect to P reveals that the entrepreneur's expected utility at $t = 0$, provided that the venture is fundable, is

$$U_{vc}^{0} = \bar{P} \times (V - I_1) - I_0 + \bar{P} \times MF. \tag{10}$$

Equation (10) implies that the entrepreneur obtains the expected NPV of the venture plus the expected utility from his motivational factors.

4. Expert Angel Financing

We now consider the case where only EA financing is available for the two investments, I_0 and I_1. As before, we first solve the financing problem at $t = 1$, taking the solution of the financing problem at $t = 0$ as given. Recall that at $t = 1$ an EA knows the realized value of P but does not know what the exit value will be. Consequently, an EA invests at $t = 1$ according to the venture's expected value $P \times V$. The EA invests if and only if $P \times V \geq I_1$, that is, when she recovers her investment I_1. We define by P^c the critical value of P above which an EA will agree to fund the venture at $t = 1$. An EA breaks even if $P \times V = I_1$, implying that $P^c = \frac{I_1}{V}$. For any $P < P^c$, an EA does not finance the venture at $t = 1$, and the venture is liquidated. When $P \geq P^c$, an EA finances the venture. Next, we analyze this case.

The entrepreneur's expected utility at $t = 1$ after learning P, provided that $P \geq P^c$, is

$$U_{EA}^1 = P\alpha\gamma V + MF. \tag{11}$$

The term $P\alpha\gamma V$ is the entrepreneur's expected share of the venture value. Note that the entrepreneur's share of the venture, $\alpha\gamma$, is defined as in Section 3, but it assumes different values based on the pricing under EA financing. The assumption that financial markets are competitive implies that the EA share of the venture value at $t = 1$ equals her investment I_1. Consequently, γ is the solution to

$$(1 - \gamma)P \times V = I_1. \tag{12}$$

Solving for γ yields

$$\gamma = \frac{P \times V - I_1}{P \times V}. \tag{13}$$

Substituting Equation (13) into Equation (11) tells us that the entrepreneur's expected utility at $t = 1$ is

$$U_{EA}^1 = \alpha(P \times V - I_1) + MF. \tag{14}$$

Equation (14) states that at $t = 1$ the entrepreneur obtains his share of the expected venture value net of the required investment I_1 plus the utility of his motivational factors.

The EA investor of $t = 0$ obtains her share of the expected venture value net of the required investment of $t = 1$,[16]

$$\pi_{EA}^{1} = (1 - \alpha)(P \times V - I_1). \tag{15}$$

The *EA* breaks even at $t = 0$ if her expected payoff equals her investment,

$$\int_{Pc}^{1} \pi_{EA}^{1}\, dF(P) = (1 - \alpha) \int_{Pc}^{1} (P \times V - I_1)\, dF(P) = I_0. \tag{16}$$

Equation (16) equates the initial investment to the expected payoff to the EA investor at $t = 0$ where the expectation is taken over the probability P, provided that $P \geq P^c$. For any $P < P^c$, the venture is liquidated and the payoff is zero. Solving for α yields

$$\alpha = \frac{\int_{Pc}^{1} (P \times V - I_1) dF(P) - I_0}{\int_{Pc}^{1} (P \times V - I_1) dF(P)}. \tag{17}$$

It follows from Equation (17) that the venture is fundable at $t = 0$ if

$$\int_{Pc}^{1} (P \times V - I_1)\, dF(P) - I_0 \geq 0. \tag{18}$$

Substituting α from Equation (17) into Equation (14) and taking expectation with respect to P implies that the entrepreneur's expected utility at $t = 0$, provided that the venture is fundable at $t = 0$, is

$$U_{EA}^{0} = \int_{Pc}^{1} (P \times V - I_1)\, dF(P) - I_0 + MF(1 - F(P^c)). \tag{19}$$

Equation (19) implies that the entrepreneur's expected utility consists of the venture's expected NPV at $t = 0$ plus the entrepreneur's expected motivational factors.

[16] Note that since the EA does not know the exit value, there are two possible inefficiencies at $t = 1$. First, when $P < P^c$, the venture is not financed, resulting in a value loss when the exit value is V. Second, when $P \geq P^c$, the venture is financed, leading to a loss of I_1 when the exit value is zero.

5. General Angel Financing

We now consider the case where only GA financing is available for the two investments, I_0 and I_1. Recall that a GA does not learn any new information beyond what she knows at $t = 0$. Therefore, at $t = 0$ a GA has to decide whether to fund the entire investment $I = I_0 + I_1$ or refuse to fund it. A GA funds the venture at $t = 0$ if the expected venture value at $t = 0$, $\int_0^1 P \times V \, dF(P) = \bar{P} \times V$ is higher than the total investment:

$$\overline{\mathrm{P}} \times \mathrm{V} \geq I_0 + I_1 \tag{20}$$

The resulting expected utility at $t = 0$ for the entrepreneur is

$$U_{GA}^0 = \overline{\mathrm{P}} \times \mathrm{V} - I_0 - I_1 + MF. \tag{21}$$

Equation (21) implies that the entrepreneur obtains the expected NPV of the venture plus his motivational factors. In this scenario, the entrepreneur enjoys his motivational factors, once he secures financing.

6. The Entrepreneur's Choice of Investors: Venture Capital, Expert Angel, General Angel

Thus far, we have considered VC financing, EA financing, or GA financing in isolation. We now allow the entrepreneur to choose the type of investor each period. The entrepreneur will choose the investor that provides him the highest expected utility. We first consider the case where the entrepreneur chooses VC financing at $t = 0$. Since the VC at $t = 0$ becomes an insider of the firm, she learns at $t = 1$ the future exit value. This fact is common knowledge, implying that if the entrepreneur approaches an Angel investor at $t = 1$, the Angel investor will infer that the *VC* knows that the exit value will be zero, and she will not provide financing. Therefore, the only remaining alternative for the entrepreneur is to seek VC financing at $t = 1$, implying that the analysis of this case is identical to that presented in Section 3 above. The entrepreneur obtains

VC financing at $t = 1$ only when the VC learns that the exit value will be V, resulting in expected utility U_{vc}^{0} to the entrepreneur at $t = 0$.

We now consider the case where the entrepreneur chooses EA financing at $t = 0$. The analysis of this case is different from that of Section 4 because now the entrepreneur can also seek GA or VC financing at $t = 1$. GA financing is irrelevant at $t = 1$, as a GA will not finance a venture that a better informed EA refuses to finance. Thus, we only need to consider the entrepreneur's choice between VC and EA financing at $t = 1$. If the entrepreneur chooses VC financing at $t = 1$, his expected utility is given by U_{vc}^{1} of Equation (5).[17] In contrast, the entrepreneur's expected utility from EA financing at $t = 1$ is given by U_{EA}^{1} of Equation (14), provided that $P \geq P^{c}$. Otherwise, if $P < P^{c}$, the EA does not provide financing and the entrepreneur's only option is to seek VC financing. In this case, his expected utility is given by Equation (5).

Finally, the solution of the case where the entrepreneur chooses GA financing at $t = 0$ is equivalent to the analysis of Section 5, because the GA provides all funds necessary for periods $t = 0$ and $t = 1$ and thus the entrepreneur does not need to seek financing at $t = 1$.[18]

Lemma 1 characterizes the entrepreneur's choice at $t = 1$:

Lemma 1

1. Suppose the entrepreneur obtains VC financing at $t = 0$. Then, at $t = 1$ the entrepreneur seeks VC financing and obtains it iff the VC learns that the exit value will be V.

[17] Note that the equity ownership α is determined according to EA financing.

[18] It is also possible that a GA will provide financing at $t = 0$ for only the design stage, and the entrepreneur will seek VC or EA financing at $t = 1$. This possibility is similar to only VC and thus, we do not explicate it.

2. Suppose the entrepreneur obtains EA financing at $t = 0$. Then, at $t = 1$, if $P \geq P^c$ and $MF - \alpha I_1 \geq 0$, the entrepreneur obtains EA financing. Otherwise, if $P < P^c$ or $MF - \alpha I_1 < 0$, the entrepreneur seeks VC financing and obtains it iff the VC learns that the exit value will be V.

Proof – See Appendix.

Lemma 1.1 states that an entrepreneur that starts with VC financing will continue with VC financing at $t = 1$ whenever the exit value will be V. Lemma 1.2 states that an entrepreneur that starts with EA financing will continue with EA financing under two conditions: First, the EA is willing to finance the venture ($P \geq P^c$). Second, the entrepreneur prefers EA financing to VC financing. The benefit for the entrepreneur from VC financing is from exercising the option to abandon at $t = 1$ when the VC learns that the exit value will be zero, saving I_1, out of which the entrepreneur's part is αI_1. In contrast, EA financing enables the entrepreneur to realize his motivational factors MF. When the motivational factors are larger than the value to abandon, $MF - \alpha I_1 \geq 0$, the entrepreneur prefers Angel financing.

We now consider the financing game at $t = 0$. There are two cases to consider, depending on the entrepreneur's preferences at $t = 1$:

Case 1: $MF - \alpha I_1 \geq 0$. In this case, the entrepreneur prefers EA financing. When the EA is willing to finance the venture, $P \geq P^c$, the entrepreneur obtains financing from the EA. But if $P < P^c$, the entrepreneur is forced to seek VC financing. Consequently, the entrepreneur's expected utility at $t = 0$ from EA financing is

$$U_{EA}^0 = \int_0^{P^c} P(V - I_1 + MF)\, dF(P) + \int_{P^c}^1 (P \times V - I_1 + MF)\, dF(P) - I_0. \quad (22)$$

The first integral in Equation (22) is the expected utility to the entrepreneur from VC financing at $t = 1$. The second integral represents his expected utility from EA financing at $t = 1$.

The venture is fundable by an EA at $t = 0$ if

$$\int_0^{P^c} P(V - I_1)\, dF(P) + \int_{P^c}^1 (P \times V - I_1)\, dF(P) - I_0 \geq 0. \quad (23)$$

Equation (23) represents the NPV of the venture under EA financing. The first integral is the expected value net of investment under VC financing at $t = 1$, while the second integral represents the expected value net of investment under EA financing.

Case 2: $MF - \alpha I_1 < 0$. In this case, the entrepreneur prefers VC financing at $t = 1$. The analysis of this case is equivalent to that of Section 3, where only VC financing is available. The entrepreneur's expected utility is given by Equation (10), and the venture is fundable if Inequality (9) holds.

Proposition 1 characterizes the equilibrium of the game:

Proposition 1

1. Suppose $\bar{P} \times MF + \bar{P}(V - I_1) - I_0 > 0$ and Inequality (20) holds.
    a. If $MF > I_1$, the entrepreneur obtains $I_0 + I_1$ from a GA at $t = 0$.
    b. If $I_1 > MF$, the entrepreneur obtains VC financing at $t = 0$. At $t = 1$ the entrepreneur seeks financing from a VC and obtains it if the VC learns that the exit value will be V.
2. Suppose $\bar{P} \times MF + \bar{P}(V - I_1) - I_0 > 0$, Inequality (20) does not hold and Inequality (23) holds.

a. If $MF > I_1$, the entrepreneur obtains EA financing at $t = 0$. At $t = 1$ the entrepreneur obtains EA financing if $P \geq P^c$, and VC financing if $P < P^c$ and the VC learns that the exit value will be V.

b. If $I_1 > MF$, the entrepreneur obtains VC financing at $t = 0$. At $t = 1$ the entrepreneur seeks financing from a VC and obtains it if the VC learns that the exit value will be V.

3. Suppose $\bar{P} \times MF + \bar{P}(V - I_1) - I_0 > 0$ and Inequality (23) does not hold.

   a. If Inequality (9) holds, the entrepreneur obtains VC financing at $t = 0$. At $t = 1$ the entrepreneur seeks financing from the VC and obtains it if the VC learns that the exit value will be V.

   b. If Inequality (9) does not hold, the venture cannot be funded.

4. Suppose $0 > \bar{P} \times MF + \bar{P}(V - I_1) - I_0$. Then, the entrepreneur does not initiate the venture.

Proof – See Appendix.

The financing of new ventures described in Proposition 1 is the outcome of the interaction between the financing preferences of the entrepreneur and the willingness of the various financiers to fund the venture. Figure 1 shows the results of Proposition 1. The 45-degree line from the origin through A represents the zero expected utility for the entrepreneur under VC financing. The entrepreneur does not bring to the market any venture that falls below this line, in accordance with part 4 of the proposition. The line from the origin through B (the X axis) represents all ventures with zero NPV under VC financing. All ventures below this line are not

fundable, as indicated by part 3b of the proposition. It follows that all ventures that are above these two lines are brought to the market by the entrepreneur and are fundable by investors.

The vertical dotted line represents all ventures where $MF = I_1$. The entrepreneur prefers VC financing for any venture that falls to the left of this line, in keeping with parts 1b, 2b, and 3b of the proposition. For these ventures, the value of the option to abandon (saving I_1) is higher than MF. For any venture that falls to the right of this line, the entrepreneur prefers GA financing, but who ultimately funds the venture will depend on the willingness of the various financiers.

Any venture that falls above the top horizontal line (where $NPV = (1 - \bar{P}) \times I_1$) will be financed by GA investors because the NPV under GA financing is positive, that is, Inequality (20) holds, in line with part 1a of Proposition 1. The term $(1 - \bar{P}) \times I_1$ is the "price protection" that GA investors demand for the fact that they do not stop the venture when the exit value will be zero. Since GA investors do not finance ventures below this line, the entrepreneur can get financing from either EA or VC investors. In accordance with part 2a, EA investors finance all ventures above the horizontal line where $NPV = \int_{PC}^{1}(1 - P)I_1 dF(P)$, which is the "price protection" that EA investors demand for not stopping the venture optimally. Finally, below this horizontal line the entrepreneur obtains VC financing when $NPV > 0$, as shown in part 3a of the proposition.

7. Empirical Implications

The equilibrium results of Proposition 1 apply to a given entrepreneur with a given venture. To derive empirical implications, consider an economy with many entrepreneurs and ventures with different characteristics. We assume that the distribution of each parameter is independent of the distributions of all other parameters. That is, the random variables MF, P, $I_0, I_1,$ and V are all

independent. As before, at t = 0, each entrepreneur knows his MF and the parameters of his venture, I_0, I_1, V, and $F(P)$. We assume that $F(P)$ is common for all entrepreneurs. An analysis of this economy yields several empirical implications that we summarize below.

7.1. **The Dynamics of New Venture Financing**

The first empirical implication relates to the financing dynamics of our model. A shift from one type of financier at $t = 0$ to another at $t = 1$ is possible but is limited to a switch from less informed investors to more informed investors. The financing dynamics are given by Proposition 2.

Proposition 2

For any distribution of the model parameters, there are three possible sequences of venture financing:

1. Ventures that start with EA financing at $t = 0$ will either stay with EA financing or switch to VC financing at $t = 1$.
2. Ventures that start with VC financing at $t = 0$ will continue with VC financing at $t = 1$.
3. Ventures that start with GA financing will get the entire investment I at $t = 0$.

Proof – See Appendix.

Proposition 2.1 is consistent with the conventional wisdom that Angel investors tend to invest before VC investors. The rationales behind this assumption are that either Angel investors do not have enough resources to support both the initial and follow-up investments or that the initial investment is "too small" for VC funds. In contrast, this same result emerges as part of the equilibrium outcome of our model in which Angel investors have unlimited resources and no funding is "too small" for VC investors.

We now discuss the implication of our theory for venture terminations at $t = 1$. A venture is terminated whenever it fails to secure financing at $t = 1$. Proposition 3 describes the likelihood of venture terminations at $t = 1$, depending upon the identity of the financier at $t = 0$.

Proposition 3

For any distribution of the model parameters, GA-backed ventures are less likely to be shut down than EA-backed ventures, which are, in turn, less likely to be shut down than VC-backed ventures.

Proof – See Appendix.

VC-backed ventures are shut down whenever the VC learns that the exit value will be low. GA-backed ventures are never shut down because GA investors provide the entire investment at $t = 0$. For given values of $I_0, I_1,$ and V, EA-backed ventures are shut down less frequently than VC-backed ventures because when $P \geq P^c$, they secure financing at $t = 1$, even when the exit value will be low. Clearly, GA-backed ventures are less likely to be shut down than *VC*-backed ventures. Taken together, it follows that the termination probability at $t = 1$ is higher for VC-backed ventures than for Angel-backed ventures.

7.2. The Implications of Motivational Factors Heterogeneity for New Venture Financing

Entrepreneur motivational factors heterogeneity is in the heart of our theory. We now discuss the impact of motivational factors on new venture financing. We start by considering a benchmark economy where MF equals zero for all entrepreneurs, and compare it to economies where MF is positive or negative for all entrepreneurs.

Starting with the benchmark economy where $MF = 0$ for all entrepreneurs, Proposition 1 implies that the most efficient investors, VC investors, will finance all positive NPV ventures and that entrepreneurs' participation decisions are fully aligned with the VC financing decisions.

In contrast, in economies where motivational factors are always positive, entrepreneurs prefer Angel financing over VC financing whenever $MF > I_1$, resulting in the coexistence of both Angel and VC investors.

In economies where motivational factors are always negative, entrepreneurs with positive NPV ventures may choose not to participate in the entrepreneurship game even though their ventures are fundable.

Proposition 4 presents the implications of entrepreneur motivational factors heterogeneity on the financing of new ventures for economies where MF are positive or negative for all entrepreneurs.

Proposition 4

1. Consider two economies where all entrepreneurs have positive MF and the probability distribution of MF in the first economy, Q_1, stochastically dominates that of the second economy, Q_2. All other venture parameters are the same in the two economics. In such a case, the proportion of Angel-backed ventures is higher in the first economy. The total number of ventures being financed is the same.
2. Consider two economies where all entrepreneurs have negative MF and the probability distribution of MF in the first economy, Q_1, stochastically dominates that of the second economy, Q_2. All other venture characteristics are the same in the two economies. In such a case, more ventures are being financed in the first economy. In both economies, all ventures are financed by VCs.

Proof – See Appendix.

Proposition 4 demonstrates that the effect of a change in the distribution of MF has very different implications for new venture financing, depending on the specific entrepreneurial environment. In an entrepreneurship-oriented economy that is characterized by entrepreneurs with positive MF, the existence of entrepreneurs with higher MF will result in more Angel financing without altering the total number of new ventures.

In an economy populated with potential entrepreneurs with negative MF - strong "fear of failure," lack of entrepreneurial aspirations, or entrepreneurial aversion - the existence of entrepreneurs with higher MF will bring into the market potential entrepreneurs that otherwise would stay out of entrepreneurship.

7.3. The Implications of Variation in Investment and Exit Value

We now consider the relations among new venture financing, the required level of investments, and exit values. This analysis enables us to understand new venture financing in different industries, locations, and business cycles. We first consider a cross-section variation of the initial investment I_0, while keeping all other parameters unchanged. Proposition 5 provides the empirical implications for different values of initial investment.

Proposition 5

Consider an economy with given distributions $Q(MF)$ and $F(P)$ and any given values of I_1 and V. Suppose also that the only source of variation among ventures is the initial investment, I_0. GA investors finance ventures with a lower average initial investment I_0 and a lower average total investment I compared to EA and VC investors.

1. Angel investors, defined as the union of GA and EA investors, fund ventures with a lower average initial investment I_0 and a lower average total investment I compared to VC investors.

Proof – See Appendix.

The intuition behind the first result of Proposition 5 is that a lower I_0 is associated with a higher NPV. Thus, ventures with lower I_0 are more likely to meet the higher NPV required by GA investors. Consequently, GA investors finance ventures with the lowest average I_0. Part 2 of Proposition 5 stems from VC investors having the lowest required NPV. Thus, some ventures with high initial investment I_0 that are rejected by Angel investors are financed by VC investors. Consequently, Angel investors finance ventures with a lower average initial investment I_0 compared to VC investors. Figures 2 and 3 depict this result in detail.

We now consider a cross-section variation of the follow-up investment I_1, while keeping all other parameters unchanged. Proposition 6 provides the empirical implications of different follow-up investment values.

Proposition 6

Consider an economy with given distributions $Q(MF)$ and $F(P)$ and any given values of I_0 and V. Suppose the only source of variation among ventures is the follow-up investment I_1.

1. GA investors finance ventures with a lower average follow-up investment I_1 and a lower average total investment I compared to EA and VC investors.
2. Angel investors, defined as the union of GA and EA investors, fund ventures with a lower average follow-up investment I_1 and a lower average total investment I compared to VC investors.

Proof – See Appendix.

The intuition behind the results of Proposition 6 is similar to that of Proposition 1: a lower investment I_1 is associated with a higher NPV. It has, however, another dimension because the level of the follow-up investment I_1 also affects the entrepreneur's choice of financiers. Specifically, a lower level of follow-up investment I_1 increases the likelihood that the entrepreneur prefers GA financing over VC financing. This additional dimension enhances the previous result. Figure 4 depicts the results of Proposition 6.

We now consider a cross-section variation in V, while keeping all other parameters unchanged. Proposition 7 provides the empirical implications for different values of V.

Proposition 7

Consider an economy with given distributions $Q(MF)$ and $F(P)$ and any given values of I_0 and I_1. Suppose the only source of variation among ventures is V.

1. GA investors finance ventures with a higher average V compared to EA and VC investors.
2. Angel investors, defined as the union of GA and EA investors, fund ventures with a higher average V compared to VC investors.

Proof – See Appendix.

The intuition for this result is analogous to that of Proposition 5, with the only difference being that now a higher V is associated with a higher venture NPV. This result is depicted in Figures 5 and 6.

Propositions 5-7 are the result of a simple feature of our model, namely, that the least informed investors demand a higher stake in the firm. In other words, they price-protect against their inferior information. Consequently, Angel investors tend to finance ventures of higher

quality than the average VC-backed venture.[19] This result has important implications for the matching of ventures with financiers in different industries, at different points in the business cycles, and for different geographical locations.

1. **Different Industries** - An industry with more attractive venture characteristics, e.g., lower investment and higher expected exit value at the time of the initial investment, will exhibit a larger fraction of Angel-backed ventures and a lower fraction of VC-backed ventures.
2. **Business Cycles** - Periods in the business cycle that exhibit higher expected exit values will have a larger fraction of Angel-backed ventures and a lower fraction of VC-backed ventures.
3. **Geographic Locations** - Geographic locations with better entrepreneurial ecosystems will experience a larger fraction of Angel-backed ventures and a lower fraction of VC-backed ventures.

The result that, in equilibrium, Angel investors finance ventures with higher quality on average than VC-backed ventures may suggest that the average dilution of entrepreneurs' equity required by the initial Angel investors is lower than that required by VC investors. This is not the case, however, because VC financing is more efficient due to the more efficient exercise of the option to abandon. Consequently, for a given venture, VC financing would result in a lower dilution. Which of these two opposing effects dominates is an empirical question.

8. Policy Implications

It is widely acknowledged that a successful start-up industry is key for economic development, growth, and job creation. Numerous studies have shown that small businesses generate important

[19] Note that the average quality relates to ex-ante expected value at the time of the investment. Nevertheless, this result is reversed when comparing ventures that secured follow-up investment and brought their product to the market.

externalities such as knowledge spillover (in new ideas, new products, new technologies, etc.) among firms within the same industry (Baptista and Swann (1998), Breschi and Lissoni, (2001) and Niosi and Zhegu (2005)) or across different industries (Glaeser, Kallal, Scheinkman and Shleifer (1992), Feldman and Audretsch (1999), and Szirmai and Goedhuys 2011)). Therefore, the idea of government intervention to promote small business is very popular in academia and among politicians. This intervention can take the form of monetary incentives (differential tax treatment, regulatory relief, anti-trust exemptions, etc.) or encouragement of entrepreneurs through nonmonetary means. For example, Lerner (2010) describes five ways by which Singapore enhanced entrepreneurship:

1. Providing public funds for venture investors seeking to locate in the city-state,
2. Offering subsidies to firms in targeted technologies,
3. Encouraging potential entrepreneurs and mentoring fledgling ventures,
4. Providing subsidies to leading biotechnology researchers to move their laboratories to Singapore, and
5. Offering awards to failed entrepreneurs (with the hope of encouraging risk-taking).

This list includes both value motivations (provision of funds and subsidies) and nonvalue motivations (encouragement of potential entrepreneurs and awards for failed ones).

Our theory provides an additional rationale for government intervention beyond spillover externalities. Specifically, "discouraged entrepreneurs" - entrepreneurs with negative motivational factors - may stay out of the entrepreneurship game even when they have high NPV ventures, resulting in a significant welfare loss. This is the case when the social welfare function

includes ventures' NPV but not entrepreneurs' motivational factors.[20] Figure 7 demonstrates this social welfare loss. Any venture below the heavy line from the origin through point A has positive NPV but is not brought to the market due to the entrepreneur's negative motivational factors. For example, Venture J is not brought to the market. The welfare loss is NPV_J, which is significant in this case. A subsidy that increases the venture NPV from NPV_J to NPV_J^1 or enhancement of entrepreneurs' motivational factors from MF_J to MF_J^1 will increase social welfare by NPV_J net of the cost of implementing the policy measure.

Our theory also provides guidance for more efficiently implementing policy measures. When the government wants to increase new venture activity through monetary incentives, it should avoid subsidizing Angel-backed ventures. Such subsidies would only increase value to entrepreneurs that would get financing anyway, leading to social waste. In contrast, a subsidy that is given to VC-backed ventures increases new venture activity. Ventures that are just below the lines that go from the origin through points A and B of Figure 7 will enter the market and will be financed by VC investors. This policy is especially effective when the government cannot distinguish between marginal ventures and positive NPV ventures.

9. Model Robustness and Extensions

In this section, we examine the robustness of the model to changes in some of the underlying assumptions, including how the availability of financial contracts interacts with the structure of the market for new venture financing and how changes in the model information structure affects our results.

[20] If the social welfare function also included entrepreneurs' motivational factors, spillover effects could still justify government intervention.

9.1 Financial Contracts

In our model, different investor types cater to the needs of different entrepreneurs. Financial contracting cannot replace investor types in catering to different entrepreneurs due to differences in the information that the various investors have at $t = 1$. Clearly, the less informed Angel investors cannot mimic the contract of the more informed VC investors, as they do not have the required information. The remaining question then is whether the more informed VC investors can offer contracts that mimic that of the less informed Angel investors. The answer is no in regard to EA investors because VC investors do not know the probability P that EA investors learn. Therefore, even though VC investors are better informed about the outcome of the venture, they cannot replicate EA contracts.

VC investors, however, can offer the same contract as GA investors, but they may have trouble committing to them because they would be compelled to not use information they possess. In this respect, the market structure we identify may be the best way to commit to the most efficient financing contracts.

9.2 Different Information Structures

We have assumed the following information structure:

1. All players share the same information at the outset ($t = 0$).
2. The entrepreneur has the same information as the EA at $t = 1$.

The most interesting case of different information at $t = 0$ is when entrepreneurs have superior information about the venture. Asymmetric information about MF is not important, as investors do not consider it. However, if the entrepreneur has superior information about the probability distribution $F(P)$ or the high exit value V, the financing game may evolve in a different way. Entrepreneurs may want to signal their type, or financiers may set up a screening

mechanism at $t = 0$ to separate the different type of entrepreneurs. With such separation, each type will follow the financing game in our model.[21]

The most interesting case of different information at $t = 1$ is when entrepreneurs obtain different information than EA investors. If entrepreneurs obtain better information than EA investors, the investors will try to infer the information from the entrepreneurs' behavior. For example, consider the case where entrepreneurs learn the exit value with certainty, exactly like VC investors. In this case, an entrepreneur that obtains EA financing at $t = 0$ and seeks EA financing at $t = 1$ will be rejected by EA investors at $t = 1$ due to adverse selection. When an informed entrepreneur seeks EA financing at $t = 1$, EA investors must infer that the venture exit value will definitely be low. This implies that there is no equilibrium where EA investors finance both the initial and follow-up investments. Nevertheless, GA financing is still feasible because GA investors provide the entire investment I at $t = 0$ when information is still symmetric.

When entrepreneurs obtain at $t = 1$ inferior information to that of EA investors, all of our model results hold because competition between EA investors will result in the equilibrium outcome of our model.[22]

9.3 Model Robustness

We believe that our results are general and robust to changes and modifications of the basic model and, therefore, are applicable to many different economic environments. For example, the high exit value V is modeled here as independent of the identity of the financier. This reflects the assumption that different financiers make the same contribution to a venture's success and value.

[21] Alternatively, in some cases the choice between the VC investor and Angel investor is part of a signaling equilibrium (see Conti et al. (2011)).

[22] This happens because competition between EA investors guarantees that the pricing at $t = 1$, given by Equation (12), will be based on the realized probability P.

The model can be extended to accommodate different contributions by VC and Angel investors. If VC investors can enhance a venture's value by more than Angel investors, they become more attractive to entrepreneurs. This would only increase the level of motivational factors above which entrepreneurs prefer Angel financing.[23]

Our model can also be extended to include investors' motivational factors. For example, investors may like environmentally friendly ventures or dislike investments in online gambling. Thus, they may be willing to invest in negative NPV ventures that are environmentally friendly or demand a higher NPV hurdle from investments in the online gambling industry. This change in investors' fundability conditions may affect the final choice between Angel and VC investors. For example, if Angel investors like environmentally friendly ventures, they may be willing to finance otherwise unfundable ventures.

We have assumed a linearly separable utility function for entrepreneurs. This is the reason why entrepreneurs have a "corner solution": they prefer GA financing when $MF > I_1$ and VC financing when $MF < I_1$. With a concave utility function for entrepreneurs, an "interior solution" whereby entrepreneurs prefer EA financing for some parameter values would be possible.

10. Conclusions

In this article, we show how entrepreneur heterogeneity explains important features of the market for entrepreneurial finance. Using our theory, we derive several empirical predictions regarding financier selections under different market conditions and venture characteristics as well as new policy implications. We argue that the market for new venture financing structures itself to cater

[23] The same reasoning is valid for the case where VC investors can improve the probability distribution $F(P)$. Moreover, the same logic holds if Angel investors are better at enhancing ventures' valuation.

to different entrepreneurs. In this respect, heterogeneous entrepreneurs affect the structure of financial markets.

The question remains whether some of these results can be carried over to financial markets of larger, established corporations. In most cases, the dominant role that entrepreneurs play in their ventures is more pronounced than that of a CEO, dominant chairperson, or controlling shareholder of a large corporation. Nevertheless, these latter business people are important and powerful, and it is conceivable that they have an influence over important decisions in a way that affects the structure of financial markets. For example, the choice between a bank loan and issuing public debt may depend on the willingness of the manager to be monitored. This is not merely an agency issue, as some managers will be more efficient if left unmonitored.

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Appendix – Proofs

Proof of Lemma 1

1. Suppose the entrepreneur obtains VC financing at $t = 0$. Then, from the discussion preceding Lemma 1, an Angel investor will not finance the venture at $t = 1$. Consequently, the entrepreneur's only option is to seek VC financing, which he obtains if the VC learns that the venture will succeed.
2. Suppose the entrepreneur obtains EA financing at $t = 0$ and $P \geq P^c$. Then, by the definition of P^c, there is $\gamma \in [0,1]$ such that Equation (12) holds and the EA is willing to finance the required investment I_1 at $t = 1$. Therefore, at this juncture, the entrepreneur has to decide between EA and VC financing. Comparing U_{vc}^1 of Equation (5) to U_{EA}^1 of Equation (14) reveals that the entrepreneur prefers EA financing if $MF - \alpha I_1 \geq 0$. When $MF - \alpha I_1 < 0$, the entrepreneur prefers VC financing. Since the VC learns if the exit value will be high before she finances the venture, she is only willing to provide funding when she learns that the value will be high.

   Suppose now that $P < P^c$. Then, by the definition of P^c, the EA does not finance the venture. The entrepreneur's only option is to seek VC financing, which he obtains if the VC learns that the exit value will be high.

Q.E.D.

Proof of Proposition 1

1. Suppose $\bar{P} \times MF + \bar{P}(V - I_1) - I_0 > 0$ and Inequality (20) holds. It follows that Inequalities (23) and (9) will also hold. Thus, all investors are willing to finance the venture. The entrepreneur selects the investor that maximizes his utility at $t = 0$.

Consequently, we must compare U_{vc}^0 of Equation (10) to U_{GA}^0 of Equation (21) and U_{EA}^0 of Equation (22).

A comparison of U_{GA}^0 to U_{EA}^0 reveals that $U_{GA}^0 > U_{EA}^0$ if

$$\overline{\mathrm{P}}\mathrm{V} - I_0 - I_1 + MF$$

$$> \int_0^{P^c} P(V - I_1 + MF)\, dF(P) + \int_{P^c}^1 (P \times V - I_1 + MF)\, dF(P) - I_0.$$

Rearranging terms yields that the entrepreneur prefers GA financing at $t = 0$, if $MF > I_1$ and EA financing if $I_1 > MF$.

A comparison of U_{EA}^0 to U_{vc}^0 reveals that $U_{GA}^0 > U_{vc}^0$ if

$$\int_0^{P^c} P(V - I_1 + MF)\, dF(P) + \int_{P^c}^1 (P \times V - I_1 + MF)\, dF(P) - I_0$$

$$> \bar{P} \times (V - I_1) - I_0 + \bar{P} \times MF.$$

Rearranging terms yields that the entrepreneur prefers EA financing at $t = 0$, if $MF > I_1$ and VC financing if $I_1 > MF$.

The two comparisons above yield that

$$U_{GA}^0 > U_{EA}^0 > U_{vc}^0 \qquad \text{if } MF > I_1\,. \qquad \text{(A-1)}$$

This proves part (1.a) of the proposition.

Likewise, the two comparisons yield that

$$U_{vc}^0 > U_{EA}^0 > U_{GA}^0 \qquad \text{if } I_1 > MF. \qquad \text{(A-2)}$$

The Inequality $\bar{P} \times MF + \bar{P}(V - I_1) - I_0 > 0$ implies that the entrepreneur is interested in initiating the venture under VC financing. Inequality (A-1) implies that his utility under GA financing is the highest. This proves part 1a.

Inequality (A-2) proves that the entrepreneur obtains VC financing at $t = 0$. The equilibrium at $t = 1$ is proven in Lemma 1.1. This proves part 1b.

2. Suppose $\bar{P} \times MF + \bar{P}(V - I_1) - I_0 > 0$, Inequality (20) does not hold but Inequality (23) holds. It follows that Inequality (9) holds. Thus, only EA and VC investors are willing to finance the venture.

   Inequality (A-1) shows that $U_{EA}^0 > U_{vc}^0$ when $MF > I_1$. This proves that the entrepreneur obtains EA financing at $t = 0$. Since $I_1 > \alpha \times I_1$, the equilibrium at $t = 1$ is proven in Lemma 1.2. This proves part 2a.

   Inequality (A-2) states that $U_{vc}^0 > U_{EA}^0$ when $I_1 > MF$. This proves that the entrepreneur obtains VC financing at $t = 0$. The equilibrium at $t = 1$ is proven in Lemma 1.1. This proves part 2b.

3. Suppose $\bar{P} \times MF + \bar{P}(V - I_1) - I_0 > 0$ and Inequality (22) does not hold. If Inequality (9) holds, then at $t = 0$ the venture is only fundable by a VC. The equilibrium at $t = 1$ is proven in Lemma 1.1. This proves part 3a. If Inequality (9) does not hold, VC investors (and obviously Angel investors) are unwilling to finance the venture. This proves part 3b.

4. Suppose $0 > \bar{P} \times MF + \bar{P}(V - I_1) - I_0$. This inequality implies that $I_1 > MF$. From Inequality (A-2), the entrepreneur prefers VC financing when $I_1 > MF$. The condition $0 > \bar{P} \times MF + \bar{P}(V - I_1) - I_0$ states that the entrepreneur's utility under VC financing is negative. This proves part 4.

Q.E.D.

Proof of Proposition 2

It follows from Proposition 1 that a venture that starts with VC financing will continue with VC financing, regardless of its parameter values. A venture that starts with GA financing raises the entire financing at the outset and does not switch to another financier. A venture that starts with

EA financing switches to VC financing whenever $P < P^c$. This proves parts 1 through 3 of the proposition.

Q.E.D.

Proof of Proposition 3

A GA provides the entire financing at the outset, and thus she never shuts down a venture. A VC investor shuts down a venture whenever she learns that the exit value will be low. This happens with probability $1 - \overline{\mathrm{P}}$. An EA-backed venture is shut down whenever a VC-backed venture is shut down, except when $P \geq P^c$, in which case the EA finances the venture even when the exit value will be low. Thus, the probability that an EA investor shuts a venture is $(1 - \overline{\mathrm{P}})F(P^c) < (1 - \overline{\mathrm{P}})$.

Q.E.D.

Proof of Proposition 4

1. With $MF > 0$, the entrepreneur participates whenever the venture is fundable. The fundability condition is independent of the entrepreneur's motivational factors. Hence, the number of ventures being financed is the same in the two economies. Angel financing happens when $MF > I_1$. Stochastic dominance implies that $1 - Q_1(I_1) > 1 - Q_2(I_1)$ for any given I_1. Since the distribution of I_1 is the same in the two economies, the result follows.
2. Proposition 1 implies that when $MF < 0$, all fundable ventures are financed by VC investors. Entrepreneurs bring their ventures to the market whenever $\bar{P} \times MF + \bar{P}(V - I_1) - I_0 > 0$, or equivalently whenever $MF > I_1 - V + \frac{I_0}{\bar{P}}$. Stochastic dominance implies that for any given I_1, $1 - Q_1\left(I_1 - V + \frac{I_0}{\bar{P}}\right) > 1 - Q_2\left(I_1 - V + \frac{I_0}{\bar{P}}\right)$. Since the distribution of I_1 is the same in the two economies, the result follows.

Q.E.D.

Proof of Proposition 5

Let Z_{I_0} be the cumulative distribution function of I_0 with a positive support over [0, ∞]. From Inequality (20), GA financing at $t = 0$ is feasible for any $I_0 \leq I_0^*$, where

$$I_0^* \equiv \overline{\mathrm{P}} \times \mathrm{V} - I_1. \tag{A-3}$$

Rearranging Inequality (23) yields that EA financing is feasible for any $I_0 \leq I_0^{**}$, where

$$I_0^{**} \equiv \overline{\mathrm{P}}(\mathrm{V} - I_1) - \int_{Pc}^{1}\big((1-P)I_1\big)\, dF(P). \tag{A-4}$$

From Inequality (9), VC financing is feasible for any $I_0 \leq I_0^{***}$, where

$$I_0^{***} \equiv \overline{\mathrm{P}}(\mathrm{V} - I_1). \tag{A-5}$$

Equations (A-3) through (A-5) imply that $I_0^{***} > I_0^{**} > I_0^*$.

Consider all I_0 satisfying $I_0 \leq I_0^*$. In this case, the venture is fundable by GA, EA, and VC investors. Inequality (A-1) implies that $U_{GA}^0 > U_{EA}^0$ if $MF - I_1 > 0$, and inequality (A-2) implies that $U_{vc}^0 > U_{EA}^0$ if $MF - I_1 < 0$. Therefore, the entrepreneur will seek financing from either a VC or a GA. Since the distribution function Q over MF is independent of the distribution function Z_{I_0} over I_0, it follows that the probability of VC investment in this region is $Q(I_1)$ and of GA investment is $1 - Q(I_1)$.

Consider all I_0 satisfying $I_0^{**} \geq I_0 > I_0^*$. In this case the venture is only fundable by EA and VC investors. Proposition 1.2 implies that the entrepreneur obtains investment from an EA if $MF - I_1 > 0$ and from a VC if $MF - I_1 < 0$. Again, the independence of the distribution function Q over MF and the distribution function Z_{I_0} over I_0 implies that the probability of VC investment in this region is $Q(I_1)$ and of EA investment is $1 - Q(I_1)$.

Finally, for all I_0 satisfying $I_0^{***} \geq I_0 > I_0^{**}$, the venture is only fundable by a VC, so the entrepreneur definitely obtains investment from a VC.

Since GA financing is possible for all I_0 satisfying $I_0^* \geq I_0$, the average investment for a GA is given by

$$\bar{I}_{GA} = \frac{\int_0^{I_0^*} I_0 dZ_{I_0}(I_0)}{Z_{I_0}(I_0^*)}. \tag{A-6}$$

Since EA financing is possible for all I_0 satisfying $I_0^{**} \geq I_0 > I_0^*$, the average investment for a EA is given by

$$\bar{I}_{EA} = \frac{\int_{I_0^*}^{I_0^{**}} I_0 dZ_{I_0}(I_0)}{Z_{I_0}(I_0^{**}) - Z_{I_0}(I_0^*)}. \tag{A-7}$$

Since VC financing is possible for all I_0 satisfying $I_0^{***} \geq I_0$, the average investment for a VC is given by

$$\bar{I}_{VC} = \frac{\int_0^{I_0^{**}} I_0 dZ_{I_0}(I_0) \times Q(I_1) + \int_{I_0^{**}}^{I_0^{***}} I_0 dZ_{I_0}(I_0)}{Z_{I_0}(I_0^{**}) \times Q(I_1) + (Z_{I_0}(I_0^{***}) - Z_{I_0}(I_0^{**}))}. \tag{A-8}$$

1. We first prove that $\bar{I}_{EA} > \bar{I}_{GA}$. Given that for a GA, $I_0 \in [0, I_0^*]$, it follows that $\bar{I}_{GA} < I_0^*$. Similarly, given that for an EA, $I_0 \in (I_0^*, I_0^{**}]$, it follows that $\bar{I}_{EA} > I_0^*$. Consequently, $\bar{I}_{EA} > I_0^* > \bar{I}_{GA}$.

   Next we show that $\bar{I}_{VC} > \bar{I}_{GA}$. Equation (A-8) can be written as a weighted average, as follows:

$$\bar{I}_{VC} = \omega_1 \times \bar{I}_{GA} + \omega_2 \times \bar{I}_{EA} + (1 - \omega_1 - \omega_2) \frac{\int_{I_0^{**}}^{I_0^{***}} I_0 dZ_{I_0}(I_0)}{(Z_{I_0}(I_0^{***}) - Z_{I_0}(I_0^{**}))}, \tag{A-9}$$

   where $\omega_1 = \frac{Z_{I_0}(I_0^*) \times Q(I_1)}{Z_{I_0}(I_0^{**}) \times Q(I_1) + (Z_{I_0}(I_0^{***}) - Z_{I_0}(I_0^{**}))} \in (0,1)$

and $\omega_2 = \frac{(Z_{I_0}(I_0^{**}) - Z_{I_0}(I_0^{*})) \times Q(I_1)}{Z_{I_0}(I_0^{**}) \times Q(I_1) + (Z_{I_0}(I_0^{***}) - Z_{I_0}(I_0^{**}))} \in (0,1)$.

Since $\bar{I}_{GA} < I_0^*$ and $\bar{I}_{EA} > I_0^*$ and since $I_0 > I_0^*$ for all $I_0 \in [I_0^{**}, I_0^{***}]$, it follows that the weighted average on the right-hand side of Equation (A-9) exceeds its lowest argument, $\bar{I}_{GA}$. This proves the results regarding the initial investment, I_0. The result about total investment I follows since I_1 is the same for all ventures.

2. Since Angel financing is possible for all $I_0 \in [0, I_0^{**}]$, the average investment for Angel investors is

$$\bar{I}_A = \frac{\int_0^{I_0^{**}} I_0 dZ_{I_0}(I_0)}{Z_{I_0}(I_0^{**})}. \tag{A-10}$$

As before, Equation (A-8) can be written as a weighted average, as follows:

$$\bar{I}_{VC} = \omega_3 \times \bar{I}_A + (1 - \omega_3) \frac{\int_{I_0^{**}}^{I_0^{***}} I_0 dZ_{I_0}(I_0)}{(Z_{I_0}(I_0^{***}) - Z_{I_0}(I_0^{**}))}, \tag{A-11}$$

where $\omega_3 = \frac{Z_{I_0}(I_0^{**}) \times Q(I_1)}{Z_{I_0}(I_0^{**}) \times Q(I_1) + (Z_{I_0}(I_0^{***}) - Z_{I_0}(I_0^{**}))} \in (0,1)$.

Again, the weighted average on the right-hand side of Equation (A-11) exceeds its lowest argument, $\bar{I}_A$. This proves the results regarding the initial investment, I_0. The result about total investment I follows since I_1 is the same for all ventures.

Q.E.D.

Proof of Proposition 6

Let Z_{I_1} be the cumulative distribution function of I_1 with a positive support over [0, ∞].

From Inequality (20), GA financing at $t = 0$ is feasible for any $I_1 \leq I_1^*$, where

$$I_1^* \equiv \overline{\mathrm{P}} \times \mathrm{V} - I_0. \tag{A-12}$$

Rearranging Inequality (23) yields that EA financing is feasible for any $I_1 \leq I_1^{**}$, where

$$I_1^{**} \equiv \frac{\bar{P} \times V - I_0}{\bar{P} + \int_{Pc}^{1} (1-P) dF(P)}. \quad \text{(A-13)}$$

From Inequality (9), VC financing is feasible for any $I_1 \leq I_1^{***}$, where

$$I_1^{***} \equiv \frac{\bar{P} \times V - I_0}{\bar{P}}. \quad \text{(A-14)}$$

Equations (A-12) through (A-14) imply that $I_1^{***} > I_1^{**} > I_1^{*}$.

Consider all I_1 satisfying $I_1 \leq I_1^{*}$. In this case, the venture is fundable by GA, EA, and VC investors. Proposition 1.1 implies that the entrepreneur obtains GA financing if $MF - I_1 \geq 0$ and VC financing if $MF - I_1 < 0$.

Consider all I_1 satisfying $I_1^{**} \geq I_1 > I_1^{*}$. In this case, the venture is only fundable by EA and VC investors. Proposition 1.2 implies that the entrepreneur obtains EA financing if $MF - I_1 \geq 0$ and VC financing if $MF - I_1 < 0$.

Finally, for all I_1 satisfying $I_1^{***} \geq I_1 > I_1^{**}$, the venture is only fundable by VC investors, so the entrepreneur obtains investment from a VC.

We denote by $\bar{I}_{1GA}$, $\bar{I}_{1EA}$, $\bar{I}_{1A}$, and $\bar{I}_{1VC}$ the average expected follow-up investment level of GA, EA, all Angels, and VC investors at $t = 0$, respectively.

1. We first prove that $\bar{I}_{1EA} > \bar{I}_{1GA}$. Given that for GA investors $I_1 \in [0, I_1^{*}]$, it follows that $I_1^{*} > \bar{I}_{1GA}$. Similarly, given that for EA investors $I_1 \in (I_1^{*}, I_1^{**}]$, it follows that $\bar{I}_{1EA} > I_1^{*}$. Consequently, $\bar{I}_{1EA} > I_1^{*} > \bar{I}_{1GA}$.

   Next we show that $\bar{I}_{1VC} > \bar{I}_{1GA}$. For all $I_1 \in [0, I_1^{*}]$, the expected investment of VC investors exceeds that of GA investors. This is so because as I_1 increases within the range, the likelihood of VC financing increases and the likelihood of GA financing decreases. Since VC investors also invest in the range $I_1 \in (I_1^{*}, I_1^{***}]$, it follows that the

unconditional average investment of VC investors, $\bar{I}_{1VC}$, exceeds that of GA investors $\bar{I}_{1GA}$.

2. Next, we show that $\bar{I}_{1VC} > \bar{I}_{1A}$. As in the proof of part 1 of the proposition, the expected investment of VC investors exceeds that of Angel investors for all $I_1 \in [0,\ I_1^{**}]$. Since VC investors also invest in the range $I_1 \in (I_1^{**},\ I_1^{***}]$, it follows that the unconditional average investment of VC investors, $\bar{I}_{1VC}$, exceeds that of Angel investors, $\bar{I}_{1A}$.

Q.E.D.

Proof of Proposition 7

Let Z_V be the cumulative distribution function of V with a positive support over [0, ∞].

Inequality (9) implies that VC financing at $t = 0$ is feasible for any $V \geq V^*$, where

$$V^* \equiv \frac{I_0 + \bar{P} I_1}{\bar{P}}. \tag{A-15}$$

From Inequality (23), EA financing is feasible for any $V \geq V^{**}$, where

$$V^{**} \equiv \frac{I_0 + \bar{P} I_1 + \int_{P^C(V^{**})}^{1} (1-P) I_1 \, dF(P)}{\bar{P}}. \tag{A-16}$$

From Inequality (20), GA financing is feasible for any $V \geq V^{***}$, where

$$V^{***} \equiv \frac{I_0 + I_1}{\bar{P}}. \tag{A-17}$$

Equations (A-15) through (A-17) imply that $V^{***} > V^{**} > V^*$.

Consider all V satisfying $V \geq V^{***}$. In this case, the venture is fundable by GA, EA, and VC investors. Inequality (A-1) implies that $U_{GA}^0 > U_{EA}^0$ if $MF - I_1 > 0$, and inequality (A-2) implies that $U_{vc}^0 > U_{EA}^0$ if $MF - I_1 < 0$. Therefore, the entrepreneur seeks financing from either a VC or a GA. Since the distribution function Q over MF is independent of the

distribution function Z_V over V, it follows that the probability of VC investment in this region is $Q(I_1)$ and of GA investment is $1 - Q(I_1)$.

Consider all V satisfying $V^{***} > V \geq V^{**}$. In this case, the venture is fundable only by EA and VC investors. Proposition 1.2 implies that the entrepreneur obtains investment from EA investors if $MF - I_1 > 0$ and from VC investors if $MF - I_1 < 0$. Again, the independence of the distribution function Q over MF and the distribution function Z_V over V implies that the probability of VC investment in this region is $Q(I_1)$ and of EA investment is $1 - Q(I_1)$.

Finally, for all V satisfying $V^{**} > V \geq V^{*}$, the venture is fundable only by VC investors, so the entrepreneur definitely obtains investment from VC.

Since GA financing is possible for all V satisfying $V \geq V^{***}$, the average exit value for GA-backed ventures is

$$\overline{V_{GA}} = \frac{\int_{V^{***}}^{\infty} V dZ_V(V)}{1 - Z_V(V^{***})}. \tag{A-18}$$

Since EA financing is possible for all V satisfying $V^{***} > V \geq V^{**}$, the average exit value for EA-backed ventures is

$$\overline{V_{EA}} = \frac{\int_{V^{**}}^{V^{***}} V dZ_V(V)}{Z_V(V^{***}) - Z_V(V^{**})}. \tag{A-19}$$

Since VC financing is possible for all V satisfying $V \geq V^{*}$, the average exit value for VC-backed ventures is

$$\overline{V_{VC}} = \frac{\int_{V^{**}}^{\infty} V dZ_V(V) \times Q(I_1) + \int_{V^{*}}^{V^{**}} V dZ_V(V)}{(1 - Z_V(V^{**})) \times Q(I_1) + (Z_V(V^{**}) - Z_V(V^{*}))}. \tag{A-20}$$

1. We first prove that $\overline{V_{GA}} > \overline{V_{EA}}$. Given that for GA, $V \geq V^{***}$, it follows that $\overline{V_{GA}} > V^{***}$. Similarly, given that for EA-backed ventures, $V \in [V^{**}, V^{***})$, it follows that $V^{***} > \overline{V_{EA}}$. Consequently, $\overline{V_{GA}} > V^{***} > \overline{V_{EA}}$.

   We now show that $\overline{V_{GA}} > \overline{V_{VC}}$. Equation (A-20) can be written as a weighted average, as follows:

$$\overline{V_{VC}} = \omega_1 \times \overline{V_{GA}} + \omega_2 \times \overline{V_{EA}} + (1 - \omega_1 - \omega_2) \frac{\int_{V^*}^{V^{**}} V dZ_V(V)}{(Z_V(V^{**}) - Z_V(V^*))}, \quad \text{(A-21)}$$

   where $\omega_1 = \frac{(1-Z_V(V^{***}))\times Q(I_1)}{(1-Z_V(V^{**}))\times Q(I_1)+(Z_V(V^{**})-Z_V(V^*))} \in (0,1)$

   and $\omega_2 = \frac{(Z_V(V^{***})-Z_V(V^{**}))\times Q(I_1)}{(1-Z_V(V^{**}))\times Q(I_1)+(Z_V(V^{**})-Z_V(V^*))} \in (0,1)$.

   Since $\overline{V_{GA}} > \overline{V_{EA}}$, and $\overline{V_{EA}} > V^{**}$ and since $V^{**} \geq V$ for all $V \in [V^*, V^{**}]$, it follows that the weighted average on the right-hand side of Equation (A-21) is below its highest argument, $\overline{V_{GA}}$. This proves that $\overline{V_{GA}} > \overline{V_{VC}}$.

2. Since Angel financing is possible for all $V \geq V^{**}$, the average exit value for Angel-backed ventures is

$$\overline{V_A} = \frac{\int_{V^{**}}^{\infty} V dZ_V(V)}{1-Z_V(V^{**})}. \quad \text{(A-22)}$$

   As before, Equation (A-20) can be written as a weighted average, as follows:

$$\overline{V_{VC}} = \omega_3 \times \overline{V_A} + (1 - \omega_3) \frac{\int_{V^*}^{V^{**}} V dZ_V(V)}{(Z_V(V^{**}) - Z_V(V^*))}, \quad \text{(A-23)}$$

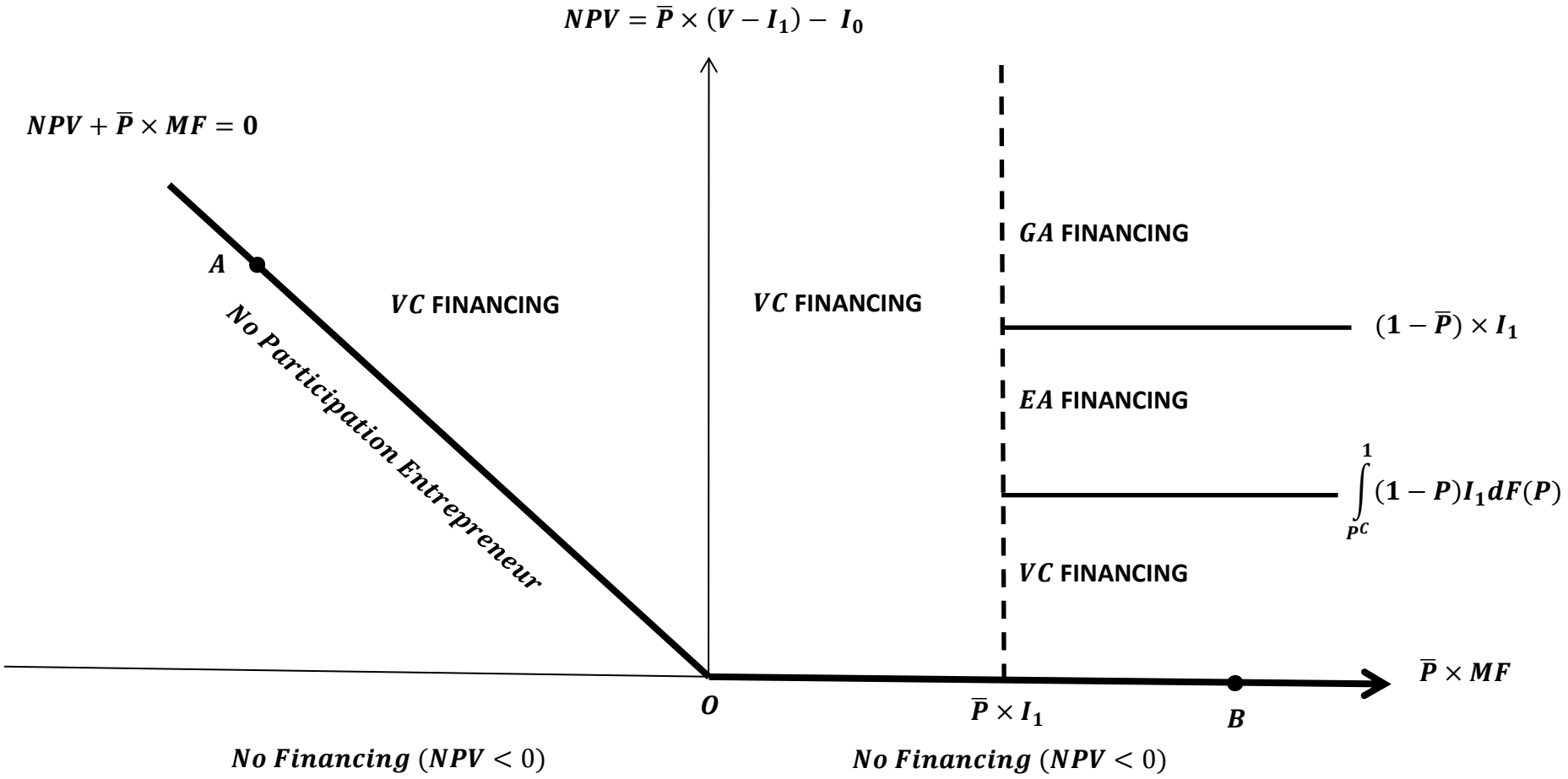
   where $\omega_3 = \frac{(1-Z_V(V^{**}))\times Q(I_1)}{(1-Z_V(V^{**}))\times Q(I_1)+(Z_V(V^{**})-Z_V(V^*))} \in (0,1)$.

   Again, the weighted average on the right-hand side of Equation (A-23) is below its highest argument, $\overline{V_A}$.

Q.E.D

Figure 1

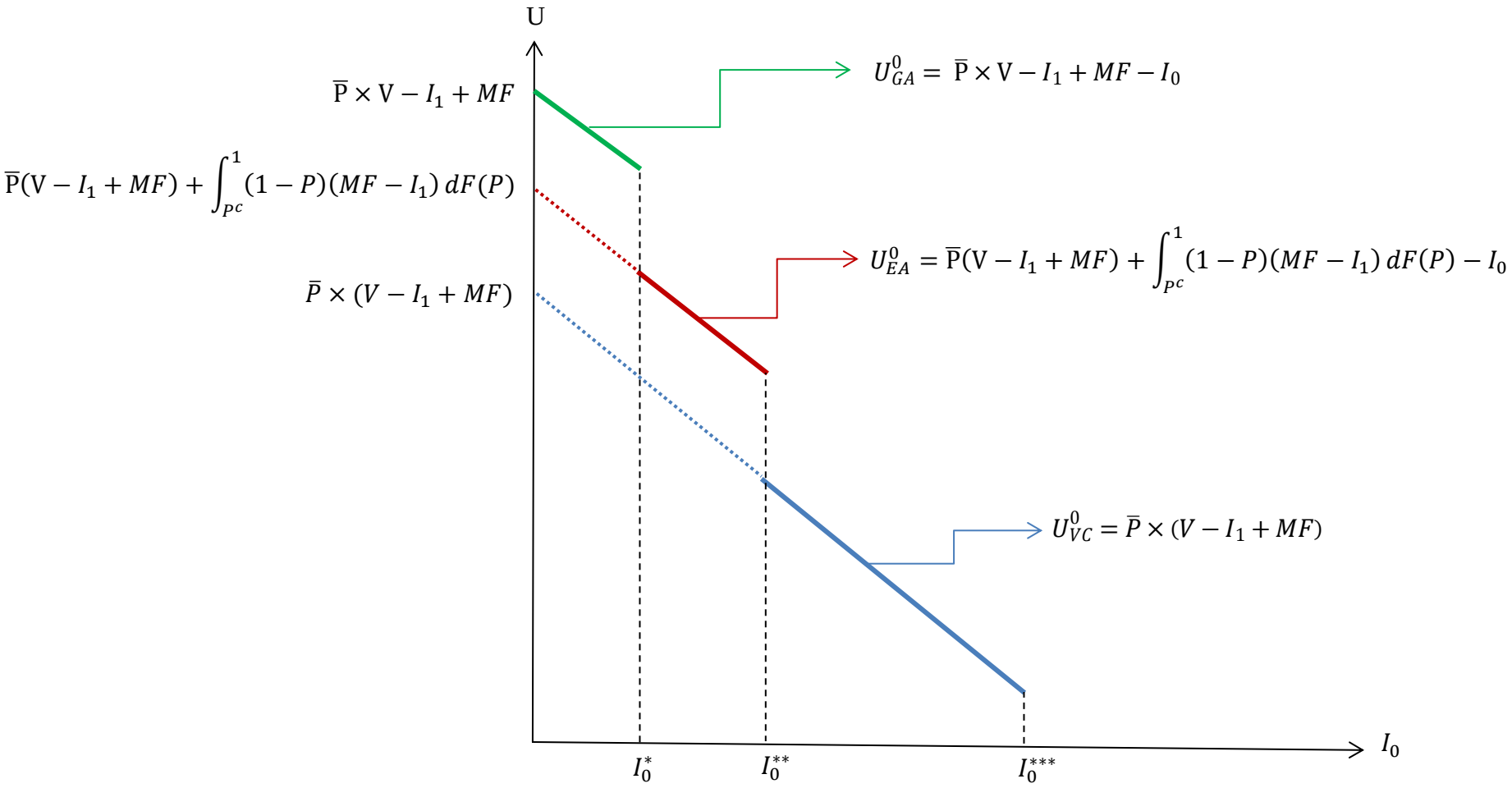
The Equilibrium of the Financing Game at $t = 0$



In the area above the heavy lines AO and OB, the entrepreneur brings ventures to the market and investors are willing to fund the ventures. In the area to the left of the dotted line ($MF = I_1$), the entrepreneur prefers and obtains VC financing. In the area to the right of the dotted line, the entrepreneur prefers GA financing. When the venture's NPV exceeds $(1 - \bar{P}) \times I_1$, he secures GA financing. If the NPV is lower but exceeds $\int_{P^C}^{1} (1 - P) dF(P)$, he secures EA financing; Otherwise he secures VC financing.

Figure 2

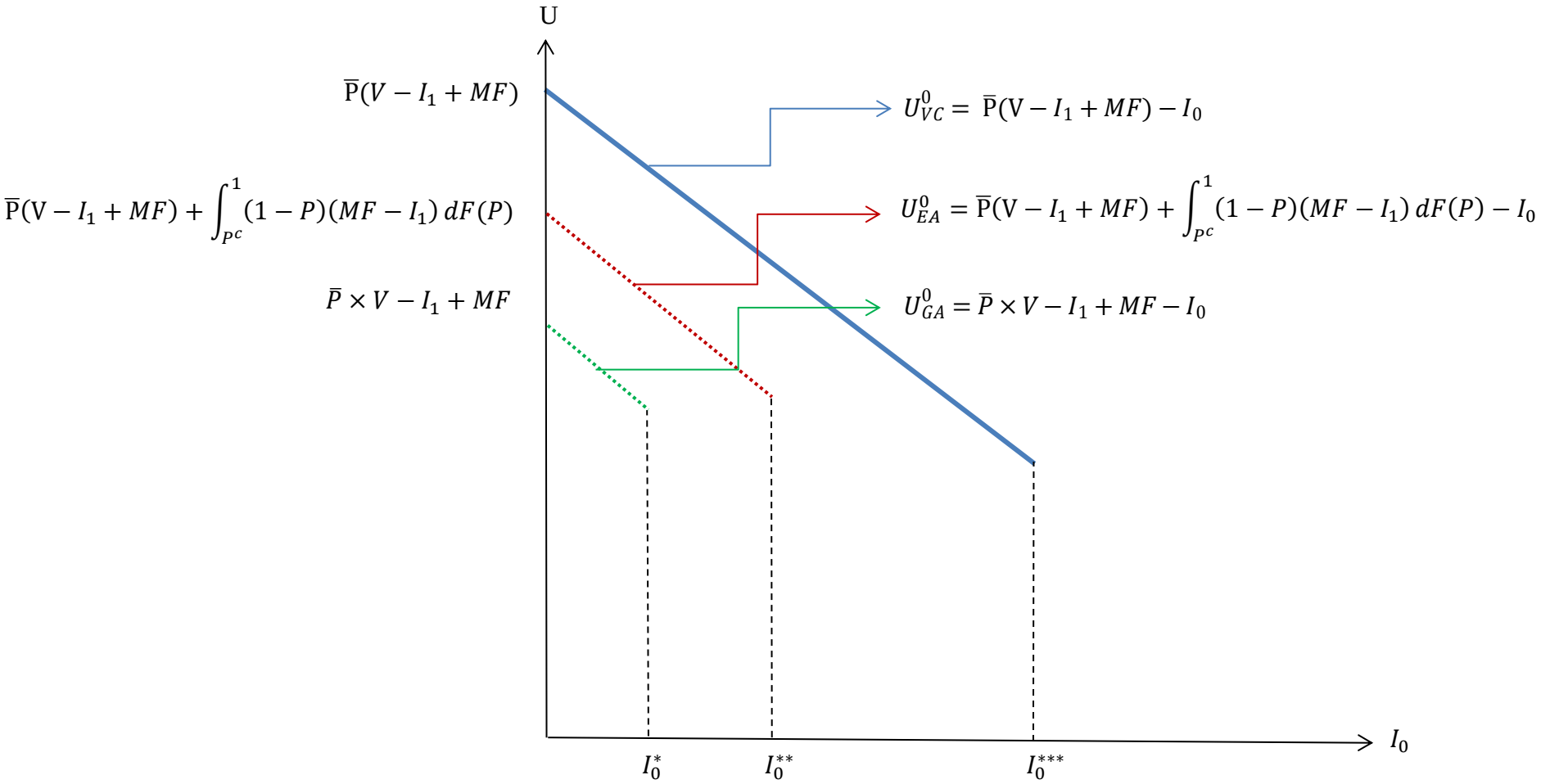
Venture Financing at $t = 0$ for Different I_0 when $MF - I_1 > 0$



I_0^*, I_0^{**}, and I_0^{***}, given by Equations (A-3), (A-4), and (A-5), are the highest initial investment levels that *GA*, *EA* and *VC* investors are willing to fund, respectively. The solid lines describe the entrepreneur's financing preferences. Since $MF - I_1 > 0$, the entrepreneur prefers *GA* financing. When $I_0 \leq I_0^*$, all investors are willing to provide financing at $t = 0$. The entrepreneur secures *GA* financing. When $I_0^{**} \geq I_0 \geq I_0^*$, only *EA* and *VC* investors are willing to invest. The entrepreneur secures *EA* financing. When $I_0^{***} \geq I_0 \geq I_0^{**}$, only *VC* investors are willing to provide financing. The entrepreneur secures *VC* financing. When $I_0 \geq I_0^{***}$, the venture is not fundable.

Figure 3

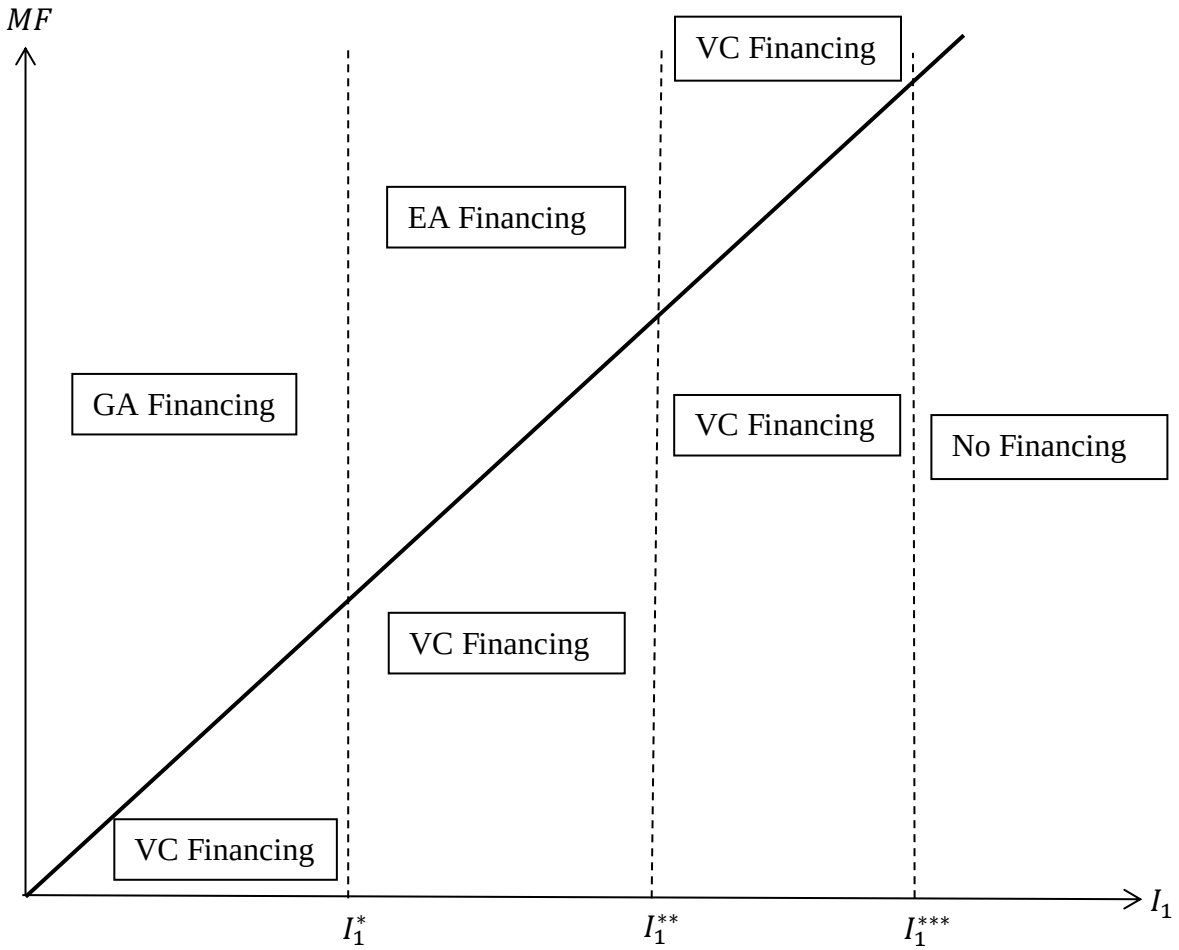
Venture Financing at $t = 0$ for Different I_0 when $MF - I_1 < 0$



I_0^*, I_0^{**}, and I_0^{***}, given by Equations (A-3), (A-4), and (A-5), are the highest initial investment levels that *GA*, *EA*, and *VC* investors are willing to fund, respectively. The solid line describes the entrepreneur's financing preferences. When $I_0 \leq I_0^{***}$, the entrepreneur prefers *VC* financing and V*C* investors are willing to finance the venture, resulting in *VC* financing. When $I_0 \geq I_0^{***}$, I_0^{***} the venture is not fundable.

Figure 4

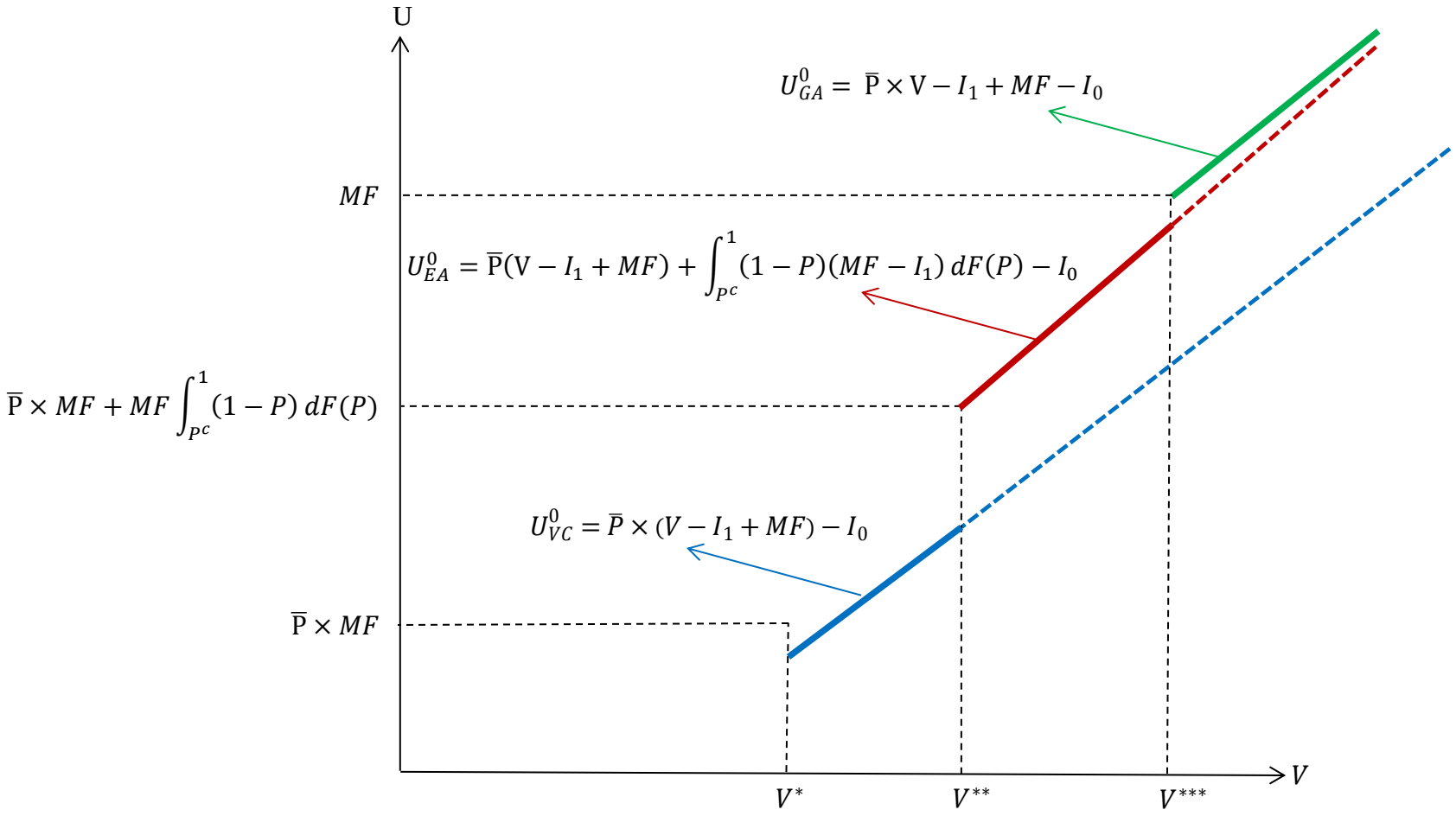
Venture Financing at $t = 0$ for Different Investments I_1



I_1^*, I_1^{**}, and I_1^{***}, given by Equations (A-12), (A-13), and (A-14), are the highest follow-up investment levels that *GA*, *EA*, and *VC* investors are willing to fund, respectively. The solid 45-degree line describes the entrepreneur's financing preferences. Above the line, $MF > I_1$ and the entrepreneur prefers GA financing. When $I_1^* > I_1$, all investors are willing to provide financing and the entrepreneur secures GA financing. When $I_1^{**} > I_1 > I_1^* I_1^*$ only *EA* and *VC* investors are willing to invest. The entrepreneur secures *EA* financing. When $I_1^{***} > I_1 > I_1^{**}$, only *VC* investors are willing to provide financing. The entrepreneur secures VC financing. Below the 45-degree line, $MF < I_1$ and the entrepreneur prefers VC financing. *VC* investors are willing to provide financing when $I_1^{***} > I_1$, and the entrepreneur secures *VC* financing. When $I_1^{***} < I_1$, the venture is not fundable (below and above the 45-degree line).

Figure 5

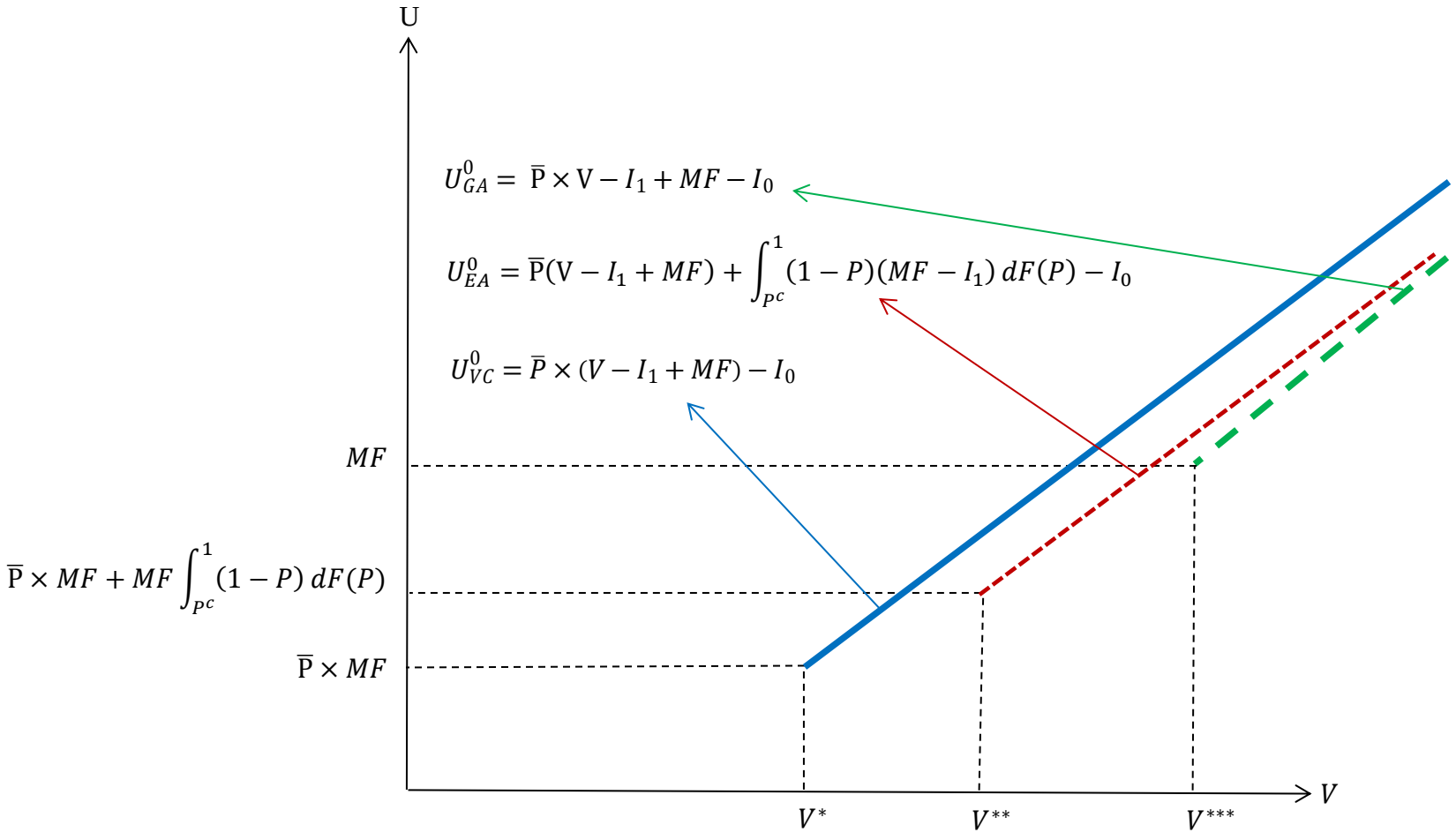
Venture Financing at $t = 0$ for Different V when $MF - I_1 > 0$



V^*, V^{**}, and V^{***}, given by equations (A-15), (A-16), and (A-17), are the lowest exit values of ventures in which *VC*, *EA*, and *GA* investors are willing to provide funding, respectively. The solid lines describe the entrepreneur's financing preferences. When $V \geq V^{***}$, all investors are willing to provide financing at $t = 0$. The entrepreneur secures *GA* financing. When $V^{***} > V \geq V^{**}$, only *EA* and *VC* investors are willing to finance the venture. The entrepreneur secures *EA* financing. When $V^{**} > V \geq V^{*}$, only *VC* investors are willing to provide financing. The entrepreneur secures VC financing. When $I_0 \geq I_0^{***}$, the venture is not fundable.

Figure 6

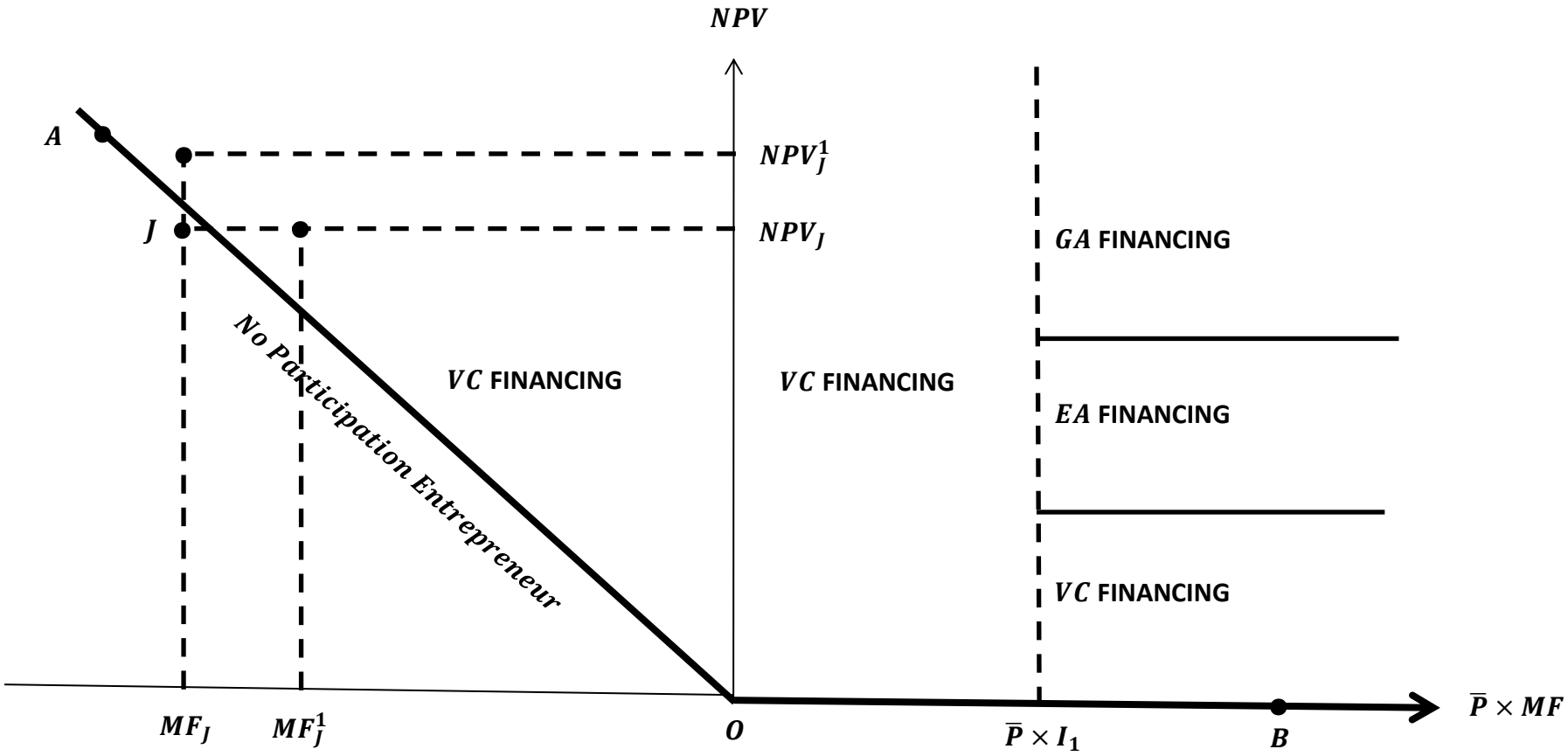
Venture Financing at $t = 0$ for Different V when $MF - I_1 < 0$



V^*, V^{**}, and V^{***} , given by equations (A-15), (A-16), and (A-17), are the lowest exit values of ventures in which *VC*, *EA*, and *GA* investors are willing to provide funding, respectively. The solid lines describe the entrepreneur's financing preferences. When $V \geq V^*$, the entrepreneur prefers *VC* financing and V*C* investors are willing to finance the venture, resulting in *VC* financing. When $V^* > V$ the venture is not fundable.

Figure 7

Policy Implications



Entrepreneur J has motivational factors MF_J and a venture with NPV_J. The entrepreneur does not bring the venture to the market. The government can subsidize the venture by $NPV_J^1 - NPV_J$ or enhance the entrepreneur's motivational factor by $MF_J^1 - MF_J$. This will entice the entrepreneur to bring the venture to the market, resulting in more new venture activity.