Is Imperfection Better?
Evidence from Predicting Stock and Bond Returns

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Abstract

We investigate the short-horizon stock and bond return predictability in a predictive regression and a predictive system using a Bayesian framework. In contrast to the predictive regression where the expected returns are modeled as a linear function of predictors, in the predictive system this assumption is relaxed, and predictors do not account for the entire variance in expected returns. We argue that a fair comparison of these two models has not been drawn yet. In our approach both models are estimated using the same Bayesian methodology and we carefully construct corresponding priors for both models. In particular, we focus on the prior beliefs about the coefficient of determination. By allowing for various distributions of this coefficient we account for different degrees of optimism about predictability. In our comparative study we take a look at the models from an investor's perspective. Therefore, we evaluate the out-of-sample performance of both models by calculating certainty equivalent returns implied by an asset allocation strategy. We show that relaxing the assumption of perfect predictors does not seem to pay off out-of-sample. Furthermore, we find that extreme optimism or pessimism about predictability decreases the performance of both models.

Keywords: return predictability, predictive system, predictive regression, Bayesian econometrics

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1 Introduction

The existence of return predictability is one of the most discussed questions in finance. Papers finding evidence in favor of predictability mostly use the predictive regression to assess the predictive power of different variables. Excess returns are regressed on various predictors, assuming a perfect correlation between the expected returns and predictors. On the other hand, the predictive system, proposed by Pastor and Stambaugh (2009), relaxes the assumption of perfect correlation. In their setup, the unobservable process of expected returns is (weakly) correlated with predictors, i.e. predictors are imperfect in the sense that they do not deliver full information about the expected return. Pastor and Stambaugh empirically show that this correlation between expected returns and predictors is far from being perfect. However, a comprehensive comparison of the predictive system and predictive regression as well as an assessment of the economic effects associated with their predictions is missing from the literature.

The comparison of these two models is not straightforward. As shown in Wachter and Warusawitharana (2009), different degrees of investor's skepticism about predictability (modeled as a prior distribution on R^2) are highly relevant for the performance of the predictive regression. For comparing the performance of the predictive system and the predictive regression we have to make sure that the priors used in these two models are comparable. To achieve this we propose an approach whereby priors on relevant parameters in the predictive system are chosen to match the prior on R^2 in the predictive regression.

The present paper investigates the predictive system in relation to a special case of the system, the predictive regression. As discussed in Pastor and Stambaugh (2009), the predictive system allows for imperfect predictors in terms of any degree of correlation between the predictors and expected returns. Instead of modeling the expected return as an explicit combination of predictors, they model each time series (returns as well as predictors) as AR(1) processes and let them interact through the covariance matrix of error terms. Therefore, the expected return does not depend only on the most recent value of the predictor, but on its lagged values as well. Furthermore, estimated residuals from predictive regressions are usually autocorrelated, but the associated complications are often ignored (see Stambaugh, 1999). An advantage of the predictive system is that it allows for modeling such serial correlation in residuals directly.

In contrast to Pastor and Stambaugh who use the system to analyse the effects of the correlation between the unexpected returns and innovations in expected returns on the properties of the estimates, we focus on the performance and asset allocation effects of different beliefs about the predictive power of the model. Different prior beliefs on the distribution of R^2, that corresponds to the fraction of explained variance, play a key role in our paper. This goodness-of-fit measure is often more intuitive for investors and easier to elicit than the individual coefficients of the models. Moreover, we analyse the out-of-sample properties of the models that are more relevant for an investor than the frequently presented in-sample evidence.

The main criterion for judging and comparing the economic performance of the predictive regression and the predictive system are certainty equivalent returns (henceforth CER). These are derived from the asset allocation implied by the out-of-sample predictions from both models. The goal of this paper is to compare the two models in a consistent way. Wachter and Warusawitharana (2009) use CERs derived from asset allocations to investigate the role of priors on R^2, but do this only for the predictive regression. Pastor and Stambaugh (2009) compare the predictive system to the predictive regression, but use a Bayesian approach to estimate the system, and OLS for the regression. Our comparison is based on estimating both models by the Bayesian MCMC technique. Therefore we accomplish to compare the characteristics and implications of the two models in a way which is unaffected by the estimation method.

While we emphasize the importance of prior beliefs we do not aim at analyzing several variables with potential predictive power. We only choose the most prominent predictors, namely the dividend-price ratio for stock returns and the yield spread for bond returns (Campbell and Shiller, 1988, 1991; Fama and French, 1988). In the empirical part we show that the predictive system does not outperform the predictive regression out-of-sample, i.e. allowing for imperfection in predictors does not improve the certainty equivalent returns.

The history of research on return predictability has brought more questions than answers. In the first models such as Samuelson (1965, 1969) and Merton (1969), excess returns were assumed to be unpredictable and investors should keep portfolio weights constant over time. However, the empirical literature in the 80s has found variables with predictive power to explain stock and bond returns (Keim and Stambaugh, 1986; Fama and French, 1989; Cochrane, 1992). After strong evidence in favor of return predictability on the aggregate level in the 90s and 00s, according to more recent evidence, return predictability is actually considered debatable or even illusory. In their comprehensive study with many different predictors Welch and Goyal (2008) show that although in-sample predictive power of the models might be significant, out-of-sample forecasts are poor. They argue that no variable has any significant predictive power. However, their conclusion is based on the OLS estimation of predictive regressions for various predictors, and is not robust with respect to different predictive models and estimation techniques. Therefore, the question whether there are variables containing some predictable components remains unsettled and still fascinates many researchers.

Attempts to deal with this question come from many areas of empirical finance. The first stream of literature tries to improve the forecasting performance by small refinements. Campbell and Thompson (2008) respond to Welch and Goyal (2008) by imposing constraints on the sign of the coefficients and return forecasts. Rapach et al. (2010) take combinations, i.e. means or medians, of predictions from different predictive regressions to obtain a better performance. Avramov (2002) adopts a Bayesian model averaging methodology to exploit the information from different predictors at once and finds both in-sample and out-of-sample predictability.

A second stream of literature attempts to explain the predictability phenomenon by various versions of time-variation like structural breaks, or time varying coefficients. Pesaran and Timmermann (2002) identify one structural break around 1991 after which predictability disappears. However, later studies differ quite considerably in terms of the timing of breaks and their number. Furthermore, the out-of-sample performance is found to be poor because breaks cannot be reliably detected in real-time (Lettau and Van Nieuwerburgh, 2008). Dangl and Halling (2012) assume the predictive regression with time-varying coefficients in a Bayesian framework, and provide a comprehensive look at the performance in this setup. They find that an investor following the optimal strategy implied by their model would be consistently better off than an investor using the historic mean.

The third way of looking at this phenomenon is by using regime-switching models. Most studies which find support for two regimes (Henkel et al., 2011), interpreted as recession and expansion, find a countercyclical pattern. While predictability during recessions is significantly better than the historical average, predictability during expansions is typically weaker, if at all. The intuition is simple. In bad times investors demand a higher risk premium. Furthermore, volatility is also higher. The prices are adjusted much more to discount rates per unit of price change. As a consequence, prices are more sensitive to a more volatile price-dividend ratio. Wachter and Warusawitharana (2011) assume an investor who distinguishes two states of the world, when returns are predictable and when returns are unpredictable, and assigns prior beliefs on the two states, i.e. the two models. They find strong support in favor of predictability.

Most of the recent papers in favor on predictability rely on the Bayesian estimation technique. This method allows an investor to incorporate her prior beliefs about the model to determine the optimal weights. The beginning of applying this approach in the asset allocation literature goes back to the paper by Kandel and Stambaugh (1996) and their simulation study. Although predictability seems to be weak in terms of frequentist statistical measures, they show that an investor observing the simulated data might significantly change her asset allocation and improve her performance.

This paper supplements the literature on predictability by elaborating on a fair comparison of the predictive regression and the predictive system in the Bayesian framework. It evaluates the out-of-sample performance of both models for different prior beliefs, compares the changes in asset allocation with respect to the model and the prior distribution. In the main comparison, we find that the predictive regression, a more parsimonious model with perfect predictors, turns out to perform better in terms of out-of-sample certainty equivalent returns.

The remainder of the paper is structured as follows. Section 2 presents the modeling framework and the estimation technique. We discuss both investigated models in detail and highlight their differences. In Section 3 we describe the data used for modeling, apply the suggested models to them and report our empirical findings. Section 4 concludes and provides suggestions for future research.

2 Econometric methodology

In this section we introduce the predictive regression and the predictive system, explain their main differences, and describe the estimation technique and criteria we use to evaluate the out-of-sample forecasts.

2.1 Model setup

In the most common way of modeling predictability by the predictive regression we assume that predictors, usually some financial ratios, provide full information about the expected returns. However, we might have some doubts whether these variables fully capture the actual market expectations. Therefore, we might search for an efficient estimate of unobservable expectations given the noisy proxies that are available. The predictive system offers one way to put some structure on the return process and model the noisy, a.k.a *imperfect*, predictors.

We can define the realized return r_{t+1} as a sum of the expected return μ_t and the unexpected return u_{t+1}

$$r_{t+1} = \mu_t + u_{t+1}.$$

The two models we consider in this paper differ with respect to the relation between the expected return μ_t and predictors. In the predictive regression, the expected return depends only on the recent value of predictors. The model is given by

$$
\begin{aligned}
r_{t+1} &= (1-\gamma)C + \gamma' \mathbf{x}_t + u^r_{t+1}, \\
\mathbf{x}_{t+1} &= (I-\delta)\mathbf{D} + \delta' \mathbf{x}_t + \mathbf{v}^r_{t+1},
\end{aligned}
$$

where $\mathbf{x}_t$ is a vector of predictors, γ, δ, C, $\mathbf{D}$ are regression coefficients and

$$
\begin{bmatrix} u^r_t \\ \mathbf{v}^r_t \end{bmatrix} \sim (\mathbf{0}, \Sigma^r)
$$

are identically distributed errors. Since the expected return is modeled as a linear function of the predictors, $\mu^r_t = (1-\gamma)C + \gamma' \mathbf{x}_t$, it implies perfect correlation between predictors and expected returns (not realized returns!). This means that the entire variance in the expected returns is explained by the current value of predictors. The predictive system proposed by Pastor and Stambaugh (2009, 2012) relaxes this assumption of perfect correlation. It takes the form

$$
\begin{aligned}
r_{t+1} &= \mu^s_t + u^s_{t+1}, \\
\mathbf{x}_{t+1} &= (I-\alpha)\mathbf{A} + \alpha \mathbf{x}_t + \mathbf{v}^s_{t+1}, \\
\mu^s_{t+1} &= (1-\beta)B + \beta \mu^s_t + w^s_{t+1},
\end{aligned}
$$

where μ^s_t is the unobservable expected excess return and α, β, $\mathbf{A}$, B are system coefficients. The error distribution is

$$
\begin{bmatrix} u^s_t \\ \mathbf{v}^s_t \\ w^s_t \end{bmatrix} \sim N(\mathbf{0}, \Sigma^s),
$$

and the errors are identically distributed. Both expected excess returns and predictors follow AR(1) processes. In the predictive system, the connection between the expected return and the predictors is not obvious. In fact, they are related through the error covariance matrix. In the case of a single predictor, the correlation between the expected return μ^s_t and the predictor x_t is determined by the correlation between the errors ρ_{vw} and the coefficients α and β

$$
\rho_{x\mu} = \rho_{vw} \sqrt{\frac{(1-\alpha^2)(1-\beta^2)}{(1-\alpha\beta)^2}}.
$$

Therefore, the predictive regression is a special case of the predictive system if $\alpha = \beta$ and ρ_{vw}=1.

In the predictive system, other than in the predictive regression, the current value of the predictor is not the only source of the information about the expected return. The additional information in the lagged realized returns and predictors is incorporated in a parsimonious way via the covariance structure.

The hypothesis that predictors are not perfectly correlated with the expected return is thoroughly discussed in Pastor and Stambaugh (2009). They argue that serial correlation of estimated residuals typically found in empirical studies of the predictive regression justifies using the predictive system. Moreover, different values of correlation ρ_{uw} allow for modeling various types of dependence of expected returns on lagged values of the predictor. As discussed in their paper, the bond price is purely driven by discount rate shocks, which implies the correlation between the innovations in expected returns and the unexpected return to be -1. For stocks, the analogous effect on the negative correlation might be weaker, but still present. Therefore, they investigate the role of more or less informative priors by varying the mass put on the negative values. They show that they get more precise in-sample estimates by assuming an informative prior.

The present paper contributes to the analysis of the system by looking at its out-of-sample properties that are important for an investor. Besides the prior distribution on the correlation ρ_{uw} that is analysed in Pastor and Stambaugh (2009) we stress the importance of the priors on parameters relevant for the implied prior distribution on R^2. Moreover, we provide a comparison of the predictions from the predictive system to the predictive regression, where both models are estimated in a Bayesian framework with the same prior beliefs.

2.2 Estimation technique and prior distribution

As shown in Wachter and Warusawitharana (2009) the estimation technique has an effect on the performance. In their paper the predictive regression estimated by OLS exhibits poor performance (in terms of CER) compared to the same model estimated by the Bayesian technique. This dependence of results on the estimation technique justifies using the MCMC method for estimating the predictive system as well. Furthermore, a Bayesian approach allows us to consider different investor's beliefs about the predictive power of the system before knowing the data.

Economically significant effects of different priors are also stressed in Shanken and Tamayo (2012). In contrast to this paper and the paper by Wachter and Warusawitharana (2009) with the focus on the prior about the coefficient of determination, they analyse a larger set of parameters and their prior distributions. However, it is not so obvious for an investor to form prior beliefs about many individual parameters. We offer a more parsimonious approach where an investor might have an opinion about the R^2, but the prior beliefs about the rest of the parameters are non-informative.

For the estimation of the predictive regression we adopt the framework proposed by Wachter and Warusawitharana (2009). They allow for different prior beliefs on R^2, which is usually interpreted as an indicator of predictive power. In this paper we develop a similar approach for the predictive system that allows us to compare the two models and analyse the effects of the priors.

In the predictive system, we need to estimate the unobservable series of expected returns μ_t^s and several parameters: α, β, $\mathbf{A}$, B, and a covariance matrix Σ^s. As discussed in Pastor and Stambaugh (2009), the system is not fully identified only by the data. In the Bayesian framework they impose an additional structure on the covariance matrix Σ^s that guarantees the identification of all parameters.

The Bayesian setup allows us to put an informative prior distribution on the parameters about which we have some intuition, and non-informative priors otherwise. Our main focus is on the parameters that have an impact on R^2 of the first equation in the system which is defined as

$$R^2 = 1 - \frac{\sigma_u^2}{\sigma_r^2} = 1 - \frac{\sigma_u^2}{\frac{\sigma_w^2}{1-\beta^2} + \sigma_u^2}. \tag{1}$$

We implement different priors on R^2 by imposing restrictions on the distributions of σ_u^2, σ_w^2 and β. We model the prior on R^2 indirectly by imposing a specific structure on the prior distribution of the error covariance matrix Σ^s. We choose the informative prior on the error covariance submatrix Σ_{11}

$$\Sigma_{11} = \begin{bmatrix} \sigma_u^2 & \sigma_{uw} \\ \sigma_{uw} & \sigma_w^2 \end{bmatrix},$$

but the non-informative prior about the other elements of the error covariance matrix Σ. Stambaugh (1997) argues that such a prior can be modeled as a posterior of Σ with a non-informative prior and an alternative hypothetical sample of T_1 observations of v and T_2 observations of (u, w), where $T_1 << T_2 << T$. As Pastor and Stambaugh (2009) we use $T_1 = K + 3$ and $T_2 = T/5$, where K is the number of predictors. Therefore, the informative prior on the submatrix Σ_{11} has an inverted Wishart distribution

$$\Sigma_{11} \sim IW(T_2\hat{\Sigma}_{11}, T_2 - K).$$

The prior mean $E(\Sigma_{11})$ is defined in order to get different distributions of R^2. In contrast to Pastor and Stambaugh (2009) we do not analyse the prior on ρ_{uw}, but we focus on the

Table 1: Relation of the expected value on prior R^2 and the parameter k

Parameter k	50%	10%	5%	1%	0.5%	0.1%
$\mathrm{E}(R^2)$	–	10.356%	5.081%	1.003%	0.508%	0.100%

The mean of R^2 is calculated by using a formula for the ratio of two inverted gamma distributions (Ali et al., 2007). The reported values are based on the length of the time-series used in the empirical part. For a longer time-series, the expected value of R^2 would be even closer to the parameter k.

diagonal elements σ_u^2 and σ_w^2. By using the relation

$$\sigma_\mu^2 = \frac{\sigma_w^2}{1-\beta^2}$$

in equation (1), we see that these variances together with the value of β determine the value of R^2. As a starting point we investigate an investor with the same priors as used by Pastor and Stambaugh (2009). We set the average variance of the unexpected return σ_u^2 to 95% of the sample return variance σ_r^2. The average variance of the shocks in the expected returns σ_w^2 are chosen in the way that combined with $\beta = 0.97$ delivers the variance of the expected return σ_μ^2 to 5% of the sample return variance. Pastor and Stambaugh argue that these values imply a plausible prior on R^2. We go one step further and analyse a richer set of priors to see the effect of different beliefs about predictability. An investor with the priors used in Pastor and Stambaugh (2009) is our benchmark investor and with the prior expected value of 5% for the coefficient of determination he is considered to be modest (compared to the other types of investors). Moreover, we consider a few more types of investors with

$$\mathrm{E}(\sigma_\mu^2) = k\sigma_r^2 \quad \text{and} \quad \mathrm{E}(\sigma_u^2) = (1-k)\sigma_r^2$$

where k is the fraction of explained variance and $1-k$ is the fraction of unexplained variance. The explained variance is lower for skeptical investors, more specifically we consider $k = 1\%, 0.5\%, 0.1\%$. We choose these values to obtain priors comparable to those in Wachter and Warusawitharana (2009). Finally, we also consider more optimistic priors with $k = 50\%$ and 10%.

To derive a closed-form solution for the relation between $\mathrm{E}(R^2)$ and k is not possible. However, we can sketch this relation by using the formula derived in Ali et al. (2007). They consider two independent random variables X and Y with inverted gamma distribution (one-dimensional Wishart distribution). They are able to derive the formula for the ratio $X/(X+Y)$ by using the gamma function. In our case, X corresponds to the variance of μ and Y is the variance of u. (However, they do no have to be independent

Table 2: Comparison of parameters in the predictive system and the predictive regression

	Fraction of explained variance					
	50%	10%	5%	1%	0.5%	0.1%
Predictive system, k	0.5	0.1	0.05	0.01	0.005	0.001
Predictive regression, σ_η	1.60	0.37	0.24	0.10	0.07	0.03

in general in our model.) With this one-dimensional simplification we can calculate the expected value of R^2 for all k except $k = 50\%$ where no formula exists. From Table 1 we see that $\mathrm{E}(R^2)$ and k are practically the same and thus, for ease of exposition, in the rest of the paper we will refer to $\mathrm{E}(R^2)$ and k as the same parameter.

The framework described above allows us to estimate the predictive system with different means of the prior distribution on R^2. Moreover, the framework derived in Wachter and Warusawitharana (2009) enables us to model different means of the prior on R^2 in the predictive regression. By defining the normalized variable η

$$\eta = \frac{\sigma_x \delta}{\sigma_u}$$

with the normally distributed prior $\eta \sim N(0, \sigma_\eta^2)$, Wachter and Warusawitharana show that

$$R^2 = \frac{\eta^2}{\eta^2 + 1}.$$

Therefore, by choosing the corresponding pairs of parameters σ_η in the predictive regression and k in the predictive system, we can estimate the models for investors with various beliefs about predictability.

Table 2 reports which values of σ_η^2 from Wachter and Warusawitharana (2009) correspond to the parameter k for the predictive system in our framework. In the comparison of the models we thus assume that the prior distribution on R^2 in the system and the regression has the same mean. However, we have to point out that the distributions are not identical and differ in higher moments. Although we have looked carefully at the prior distributions implied by different values of model parameters that have an impact on R^2, we are not able to match the higher moments.

As we do not focus on the correlation ρ_{uw} between the unexpected return and innovations in the expected return, we estimate the model for the same three priors on ρ_{uw} that are used in Pastor and Stambaugh (2009). They argue that this correlation is negative and thus informative priors reflect this belief. Non-informative priors with the same mass for positive and negative values, less informative with a positive mass only on negative values, and more informative that have most mass on negative values close to -1. However, we argue based on the empirical results below that this prior does not have a strong effect on the out-of-sample performance. All priors on the other parameters of the system are the same as in Pastor and Stambaugh (2009).

Thus, we have defined a framework that allows for a comparison of the predictive system and the predictive regression. Both models are estimated by the Bayesian approach. Therefore, we accomplish to compare the characteristics and implications of the two models in a way which is unaffected by the estimation technique.

2.3 Out-of-sample performance

Our focus is to investigate the out-of-sample performance of the predictive system and the predictive regression given different investor's beliefs. Although many papers find strong in-sample predictability, the results in real-time evaluations of the models are mostly much weaker. In this paper we compare the performance of the two suggested models estimated by the Bayesian framework in real time. We now describe the evaluation procedure in more detail.

For measuring out-of-sample performance we use an expanding window strategy. Following Wachter and Warusawitharana (2009) we estimate the models after observing at least 20 years of quarterly data. By simulating $200,000$ ($75,000$) draws, dropping the first $50,000$ ($1,000$) as a burn-in phase and taking every third draw from the rest to decrease the serial correlation we obtain the posterior distributions for the predictive regression (predictive system). Both models are re-estimated every four quarters. Predictions for $t+1$ are computed every quarter, holding estimates fixed throughout a year, but using observed predictors lagged by one quarter.

In the optimal portfolio choice problem we consider an investor who holds stocks, bonds and a risk-free asset in her portfolio. She maximizes expected utility in the next period $t+1$ conditional on the information available now (period t). Although we look at the one-period static asset allocation problem, it would be possible to predict more periods ahead and look at the effects of the dynamic asset allocation problem. However, this is beyond the scope of this paper which provides a first step in comparing the performance of the models in static setup.

The investor solves the one-period portfolio choice problem

$$\max \mathrm{E}_t(U(W_{t+1})),$$

where $U(\cdot)$ is a utility function, W_t is the wealth at time t, and the expected value is calculated conditional on all available information through time t. We consider an investor with a mean-variance utility function to make our results comparable to other studies (Wachter and Warusawitharana, 2009; Dangl and Halling, 2012). Therefore, the stock and bond weights from time t to $t+1$ are given by

$$\omega_t^i = \frac{1}{A} \frac{\mathrm{E}_t(r_{t+1}^i)}{\mathrm{Var}_t(r_{t+1}^i)}, \tag{2}$$

where A is a risk aversion coefficient, i is the index of risky assets, and $\mathrm{E}_t(r_{t+1}^i)$ and $\mathrm{Var}_t(r_{t+1}^i)$ are the first two moments of the posterior distribution of the returns at time $t+1$ conditional on the information at time t. Both moments are determined for each model separately. As it stands, the covariance among assets in the predictive regression and the predictive system cannot be easily modeled. Therefore, the asset weights in equation (2) are based on an asset correlation of zero. To investigate the role of covariance, we additionally use the correlation between the risky assets from the historical data (i.e. the sample correlation) as a robustness check in the empirical part.

The final wealth is given by

$$W_{t+1} = W_t \underbrace{\left(\sum_i \omega_t^i r_{t+1}^i + r_{f,t+1} \right)}_{r_{t+1}^p},$$

where r_t^i is the realized return on the risky asset i, $r_{f,t}$ is the realized return on the risk-free asset, and r_t^p is the portfolio return.

For assessing the out-of-sample performance it is important for an investor how predictability is mapped into gains and losses of her strategy. As in the optimal asset allocation problem an investor takes into account the first and the second moment of the return distribution, it is a natural choice to take these moments also into account when evaluating the model performance. Therefore, we measure the performance in terms of the certainty equivalent return (CER) that evaluates the model in economic terms, adjusting for risk. For the portfolio over the considered out-of-sample time period we define CER as

$$\mathrm{CER} = \bar{r}^p - \frac{A}{2} v_r^p \qquad \bar{r}^p = \frac{1}{n} \sum_t r_t^p \qquad v_r^p = \frac{1}{n-1} \sum_t \left(r_t^p - \bar{r}^p \right)^2,$$

where t is the index of all quarters in the out-of-sample period 1972–2011, $\bar{r}^p$ is the average expected portfolio return over the out-of-sample period, and v_r^p is the portfolio variance over the out-of-sample period. In the tables below CER is multiplied by 400 to express them as an annual percentage.

To assess the significance of the out-of-sample performance is not easy, because the parameters are re-estimated sequentially, and the usual tests known from an in-sample analysis cannot be applied (i.e. transferred) to an out-of-sample context. Dangl and Halling (2012) use daily data to estimate the monthly variance of the portfolio from stocks and the risk-free asset. However, as we do not have the daily data for bond prices, we cannot repeat their test. Furthermore, the simulation exercise done by Wachter and Warusawitharana (2009) in the predictive regression is infeasible for the predictive system due to time constraints. Therefore, for each certainty equivalent return we calculate the bootstrap standard error (Efron and Gong, 1983) that reflects the variability of the mean of CER.

3 Data and empirical findings

We conduct our analysis on the quarterly data spanning the first quarter of 1952 until the last quarter of 2011. Following other studies we begin our sample after 1951 when the Fed was allowed to pursue an independent monetary policy.

All financial data are obtained from the Center for Research in Security Prices (CRSP). Excess stock returns are defined as the quarterly returns on the NYSE-AMEX-NASDAQ index in excess of the three-month Treasury bill. Similarly, the excess bond returns are constructed as the quarterly returns on ten-year Treasury bond minus the three-month Treasury bill. The dividend-price ratio, used as a predictor of the stock returns (Campbell and Shiller, 1988; Fama and French, 1988), is constructed as a sum of total dividends paid over the previous 12 months divided by the current price. Dividends are calculated from monthly stock returns including and excluding dividends on the value-weighted NYSE-AMEX-NASDAQ index. The yield spread, used as a predictor of the bond returns (Campbell and Shiller, 1991), is constructed as the yield on the five-year bond minus the yield on the three-month bond.

3.1 Results

This section describes the results obtained from the estimation of both considered models, the predictive regression and the predictive system. First, we evaluate the out-of-sample performance for the entire sample period and then we conduct several robustness checks.

We start with the analysis of the effects of the prior distribution for R^2 on the in-sample model performance. Table 3 shows the mean and standard deviation of the posterior distribution of R^2 for the predictive system and predictive regression for both risky assets. A similar analysis has been already done for stock returns in Pastor and Stambaugh (2009). However, they do not distinguish different priors on R^2, but only use one prior labeled P&S in our tables. Moreover, the predictive regression in their paper is estimated by OLS which makes a comparison to the Bayesian estimates obtained for the predictive system difficult. Nevertheless, our more comprehensive results are consistent with their conclusion on the R^2 for stock returns. For every column (i.e. prior on the fraction of the explained variance) the mean of in-sample R^2 for the predictive system is higher than for the predictive regression. However, the results for bond returns are less clear. For the most optimistic prior, the predictive system yields a higher posterior mean for R^2 than the predictive regression. For all other priors the results are opposite. In any case, given the high standard deviations, deriving strong conclusions may be problematic, and we take these results only as a first indication of the model performance.

Table 4 reports our main results, presenting certainty equivalent returns of asset allocation strategies obtained from the entire out-of-sample period 1972-2011. We report CER calculated for both models with different prior distributions on R^2 for different priors on the correlation ρ_{uw} in the predictive system. We consider investors with two different risk aversion coefficients $A = 2$ and $A = 5$. For both degrees of risk aversion the results are qualitatively very similar.

First, we discuss the results for the risk aversion coefficient $A = 2$. By comparing the predictive system to the predictive regression for the same prior on R^2 (i.e. comparing across rows in Table 4) the CER for the predictive regression is higher for any prior on ρ_{uw}. In other words, relaxing the assumption of a perfect correlation between expected returns and the predictor does not seem to pay off. Regarding the behavior of CER with respect to the fraction of explained variance we find an inverted U- or J-shape. Extreme investors on both tails, an optimistic investor with a high expected value of prior R^2 and a skeptical investor with a low expected value, tend to perform worse that investors with a modest prior distribution. However, the most optimistic investor in our study is in a worse position than the most skeptical investor for both models For $A = 5$, the difference between the models is slightly weaker, but the predictive regression still exhibits superior performance to the predictive system. Similarly as for $A = 2$, the CER for $A = 5$ varies with the prior beliefs about predictability (R^2) and exhibits a U-shape. On the other hand, the performance is not so sensitive to the prior on the correlation ρ_{uw}.

We now investigate the asset allocation weights. The effects of different levels of optimism on the stock weights for the predictive system can be seen in Figure 1, Panel A. We plot estimated weights given by equation (2) for different priors on predictive power, while

Table 3: In-sample performance: Posterior R^2

Posterior R^2 (%), 1952 – 2011						
	Fraction of explained variance					
	50% W&W optimistic	10% ←	5% P&S	1% W&W	0.5% W&W →	0.1% W&W pessimistic
Stocks						
Predictive system	8.70	5.55	4.63	3.00	2.38	1.17
	(3.28)	(4.61)	(5.17)	(6.23)	(5.42)	(4.13)
Predictive regression	1.36	1.30	1.26	1.02	0.84	0.16
	(1.57)	(1.06)	(0.83)	(0.65)	(0.59)	(0.20)
Bonds						
Predictive system	8.62	2.65	1.82	0.92	0.59	0.18
	(2.97)	(2.05)	(2.82)	(3.50)	(2.79)	(1.43)
Predictive regression	5.54	5.22	4.78	2.75	1.65	0.23
	(3.00)	(2.85)	(2.64)	(1.69)	(1.14)	(0.24)

The table reports means and standard deviations (in parentheses) of posterior R^2 for the predictive system and predictive regression. The predictor is the dividend-price ratio for the stock returns and the yield spread for the bond returns. Different beliefs about the prior distribution on R^2 are considered, characterized by the mean of the prior distribution. Lower values represent more skeptical investors. For the predictive system we report the results for the more informative prior on the correlation between the expected and unexpected returns. Data are quarterly and span from 1952 to 2011.

choosing the prior on ρ_{uw} to be non-informative. More optimism about predictability is mirrored in more volatile weights. The more pessimistic an investor is, the less weight (in absolute terms) he puts in the risky assets. While the weights for the most optimistic investor vary from -200% to 200%, the weights for the most pessimistic investor are almost constant at about 40%. These results are consistent with findings of Wachter and Warusawitharana (2009) for the predictive regression. Furthermore, the dynamics of weights over time reflect an unfavorable situation for stocks in the 90s, when the holdings for stocks are mostly negative. The spikes each year are caused by the fact that we estimate the model on a yearly basis and keep the estimated parameters constant for all quarters in that year.

Panel B of Figure 1 shows the sensitivity to the prior discussed in Pastor and Stambaugh (2009). We fix the prior on R^2 to 5% and plot the stock weights for different priors on ρ_{uw}. There is no clear monotonicity or any clear pattern in the weights when changing

Table 4: Out-of-sample performance: Certainty equivalent returns

Panel A: Risk aversion $A = 2$**,** $1972 - 2011$

		Fraction of explained variance					
		50% W&W optimistic	10% ⟵	5% P&S	1% W&W	0.5% W&W ⟶	0.1% W&W pessimistic
System	Prior on ρ_{uw}						
	More informative	3.45	7.49	8.31	8.37	6.83	6.08
		(0.38)	(0.29)	(0.27)	(0.19)	(0.17)	(0.15)
	Less informative	5.25	8.18	8.65	7.49	7.67	8.39
		(0.41)	(0.32)	(0.30)	(0.25)	(0.22)	(0.16)
	Non-informative	4.54	7.94	8.73	8.08	8.07	9.19
		(0.45)	(0.33)	(0.30)	(0.26)	(0.24)	(0.20)
Regression		8.12	8.64	8.95	10.76	10.80	9.77
		(0.47)	(0.44)	(0.40)	(0.28)	(0.22)	(0.16)

Panel B: Risk aversion $A = 5$**,** $1972 - 2011$

		Fraction of explained variance					
		50% W&W optimistic	10% ⟵	5% P&S	1% W&W	0.5% W&W ⟶	0.1% W&W pessimistic
System	Prior on ρ_{uw}						
	More informative	4.91	6.46	6.79	6.81	6.19	5.87
		(0.15)	(0.12)	(0.11)	(0.08)	(0.07)	(0.06)
	Less informative	5.59	6.73	6.91	6.44	6.52	6.83
		(0.16)	(0.13)	(0.12)	(0.10)	(0.09)	(0.06)
	Non-informative	5.30	6.62	6.94	6.69	6.70	7.17
		(0.18)	(0.13)	(0.12)	(0.10)	(0.09)	(0.08)
Regression		6.76	6.96	7.08	7.79	7.81	7.40
		(0.18)	(0.17)	(0.16)	(0.11)	(0.09)	(0.07)

Certainty equivalent returns are calculated for the predictive system and predictive regression. The portfolio consists of stocks, bonds and a risk-free asset. The predictor is the dividend-price ratio for the stock returns and the yield spread for the bond returns. Optimal weights are calculated for a mean-variance investor with risk aversion coefficient $A = 2$ (Panel A) and $A = 5$ (Panel B). At the beginning of each year, starting in 1972, we estimate the model and use the estimated parameters for calculating the optimal portfolio for each quarter in this year. Different beliefs about the prior distribution on R^2 are considered, characterized by the expected value of the prior distribution. Lower values represent more skeptical investors. For the predictive system we report the results for different priors on the correlation between the expected and unexpected returns as in Pastor and Stambaugh (2009). The non-informative prior is flat on most of the $(-1, 1)$ interval, the less informative implies most mass below zero and the more informative imposes most mass below -0.7. Data are quarterly and span from 1952 to 2011. Numbers in parenthesis are bootstrap standard errors.

the prior. A similar pattern holds for bond holdings. This indicates that the prior on the correlation between expected and unexpected returns in the predictive system does not play a key role for the investor.

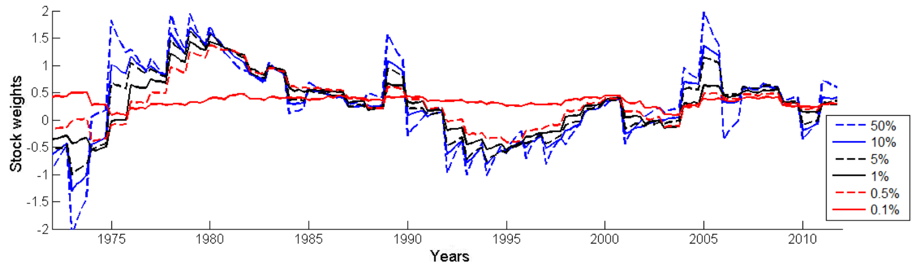
Finally, in Panel C of Figure 1 we compare the stock weights for both the predictive regression (PR in the legend) and the predictive system (PS in the legend) with the same prior beliefs about the predictability power, R^2. As in Panel B, the prior on R^2 is fixed to 5%. The weights from the predictive regression are slightly less volatile than the weights from the predictive system for any prior on ρ_{uw}. The higher volatility for the predictive system might be explained by higher precision of the estimated parameters documented in Pastor and Stambaugh (2009). As the parameter uncertainty is lower it increases the weights a mean-variance investor is willing to allocate.

3.2 Robustness checks

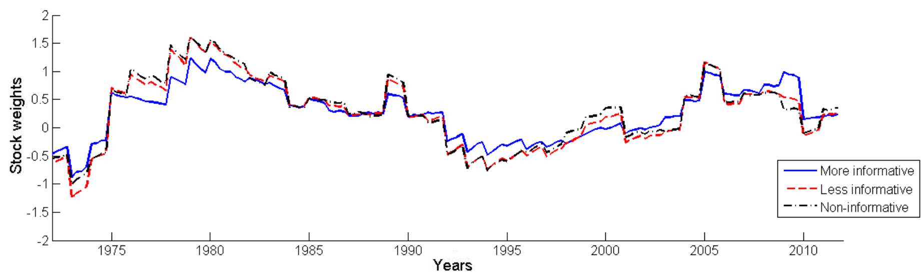
To investigate the robustness of our results, we conduct several additional checks. As a first robustness check we explore the role of correlation between the stock and bond returns. As we are able to estimate stock and bond returns for each model only separately, we have assumed zero correlation among risky asset so far. To show that the results are not driven by this assumption we use the correlation between the stock and bond returns obtained from historical data to calculate the optimal weights. The CERs accounting for this correlation are reported in Table 5. The absolute values are lower compared to the main analysis. However, the comparative advantage of the predictive regression is still there. As the correlation from historical data might not be the same as implied by the predictive models, we further calculate CER for a constant correlation, in particular 10% and 20% (chosen to be of a similar magnitude as time-varying correlations from the historical data). Since the results are qualitatively the same, we do not report them in a separate table.

Second, we consider different subsamples. As there seems to be a persistent decline in expected returns and an increase in the steady-state growth rate of the economy at the beginning of 90s (Lettau and Van Nieuwerburgh, 2008; Pesaran and Timmermann, 2002), we split the out-of-sample period into two halves: $1972-1991$ and $1992-2011$. Tables 6 and 7 report CER for these subsamples and the same degrees of risk aversion as reported for the entire period. The absolute CER in the first sample is higher than in the second. This is consistent with the evidence in the literature indicating a weaker degree of or no predictability starting in the nineties. In the first period there is no clear preference for either model. For a not too optimistic investor ($k = 5\%$ or $k = 10\%$) the predictive system outperforms the predictive regression. However, this is the only case in our robustness exercises when the predictive system pays off compared to the predictive regression. For

Panel A: Sensitivity to the prior on R^2 for the predictive system using a non-informative prior on ρ_{uw}



Panel B: Sensitivity to the prior on ρ_{uw} for a prior on R^2 fixed at 5%



Panel C: Sensitivity to the model choice for a prior on R^2 fixed at 5%

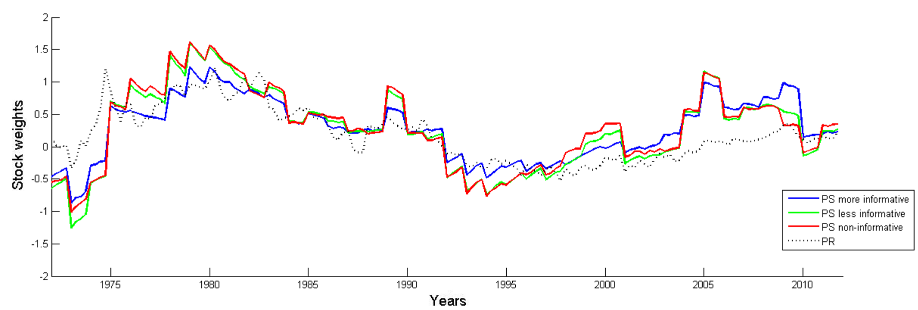


Figure 1: Stock weights for investors with different beliefs in predictability

pessimistic prior beliefs, the predictive regression outperforms all other models. For less averse investors with $A = 5$, the results are less volatile and less sensitive to the prior on R^2. In the second period, the absolute returns are lower. Nevertheless, if we compare the predictive regression to the predictive system, the predictive regression always outperforms the system. Thus, the main conclusion does not change when considering these subsamples.

In addition to splitting the sample we limit the asset weights of risky assets to a range between 0% and 150% of the overall portfolio as in Dangl and Halling (2012). From Table 8 we can derive that the absolute performance increases for all models. In terms of relative performance, the results indicate the same pattern as in the analysis without constraints, but now they are even more pronounced. The predictive regression exhibits consistently better performance than the predictive system. Finally, we check the performance by using a rolling window of 20 years instead of expanding window for the estimation. Since the results are almost the same as in the main analysis we do not report them in a separate table.

To sum up, we find that the prior beliefs matter for the model performance. Although in-sample forecasts indicate better results for the predictive system, out-of-sample performance that is more relevant for an investor, is more in favor of the predictive regression. Overall, an investor following predictions from the predictive regression estimated by the MCMC framework obtains superior out-of-sample performance relative to the predictive system.

4 Conclusion

This paper proposes an approach that allows for comparing the predictive regression and the predictive system on the basis of similar prior distributions. We consider optimistic and pessimistic investors who differ in their beliefs about predictability. We compare the models in terms of the out-of-sample certainty equivalent returns from the asset allocation strategy.

The present paper thus contributes to the literature in two ways: First, we elaborate the setup under which the predictive regression and the predictive system are comparable. This involves a framework for defining priors in the predictive system which allow for matching the mean of the prior on R^2 in the predictive regression, and using the same MCMC technique for estimating both models. Second, we investigate the out-of-sample performance that tends to correlate poorly with the in-sample results. For the post-war data on bond and stock returns, our results cast doubt on the ability of the predictive system to increase investor's profit and indicate that relaxing the assumption of a perfect predictor does not improve the performance out-of-sample. Our results support the current stream of literature that more complex models do not necessarily improve the performance (Feldhütter et al., 2013; Sarno et al., 2013). Furthermore, we explore the role of investor's beliefs about predictability. Our results for the predictive system are consistent with findings in Wachter and Warusawitharana (2009) for the predictive regression that the beliefs about predictability matters and it pays off to have a modest prior. When

Table 5: Out-of-sample performance: Certainty equivalent returns, correlation included

Panel A: Risk aversion $A = 2$, $1972 - 2011$

		Fraction of explained variance					
		50%	10%	5%	1%	0.5%	0.1%
				P&S	W&W	W&W	W&W
		optimistic	⟵			⟶	pessimistic
System	Prior on ρ_{uw}						
	More informative	2.56	6.52	7.20	7.43	5.85	5.31
		(0.37)	(0.30)	(0.27)	(0.18)	(0.17)	(0.14)
	Less informative	3.76	7.14	7.72	7.39	6.85	8.02
		(0.41)	(0.33)	(0.31)	(0.26)	(0.23)	(0.14)
	Non-informative	3.22	6.81	7.86	7.23	7.49	8.99
		(0.45)	(0.34)	(0.30)	(0.27)	(0.25)	(0.19)
Regression		7.66	8.15	8.68	10.16	10.61	8.77
		(0.44)	(0.43)	(0.38)	(0.28)	(0.22)	(0.21)

Panel B: Risk aversion $A = 5$, $1972 - 2011$

		Fraction of explained variance					
		50%	10%	5%	1%	0.5%	0.1%
		W&W		P&S	W&W	W&W	W&W
		optimistic	⟵			⟶	pessimistic
System	Prior on ρ_{uw}						
	More informative	4.54	6.07	6.34	6.43	5.79	5.56
		(0.15)	(0.12)	(0.11)	(0.07)	(0.07)	(0.06)
	Less informative	4.97	6.30	6.53	6.34	6.19	6.68
		(0.17)	(0.12)	(0.12)	(0.10)	(0.09)	(0.06)
	Non-informative	4.76	6.17	6.58	6.34	6.46	7.09
		(0.17)	(0.13)	(0.13)	(0.10)	(0.09)	(0.08)
Regression		6.58	6.76	6.98	7.55	7.73	7.26
		(0.17)	(0.16)	(0.16)	(0.11)	(0.09)	(0.09)

Certainty equivalent returns are calculated for the predictive system and predictive regression. The portfolio consists of stocks, bonds and a risk-free asset. The predictor is the dividend-price ratio for the stock returns and the yield spread for the bond returns. The correlation between the risky assets is calculated from the historical average and used for both models. Optimal weights are calculated for a mean-variance investor with risk aversion coefficient $A = 2$ (Panel A) and $A = 5$ (Panel B). At the beginning of each year, starting in 1972, we estimate the model and use the estimated parameters for calculating the optimal portfolio for each quarter in this year. Different beliefs about the prior distribution on R^2 are considered, characterized by the expected value of the prior distribution. Lower values represent more skeptical investors. For the predictive system we report the results for different priors on the correlation between the expected and unexpected returns as in Pastor and Stambaugh (2009). The non-informative prior is flat on most of the $(-1, 1)$ interval, the less informative implies most mass below zero and the more informative imposes most mass below -0.7. Data are quarterly and span from 1952 to 2011. Numbers in parenthesis are bootstrap standard errors.

Table 6: Out-of-sample performance: Certainty equivalent returns, subsamples

Panel A: Risk aversion $A = 2$, 1972 − 1991

		Fraction of explained variance					
		50% W&W optimistic	10% ⟵	5% P&S	1% W&W	0.5% W&W ⟶	0.1% W&W pessimistic
System	Prior on ρ_{uw}						
	More informative	9.42	14.45	14.53	10.99	9.18	8.24
		(0.71)	(0.63)	(0.60)	(0.34)	(0.29)	(0.24)
	Less informative	10.56	15.29	15.27	12.56	11.15	8.45
		(0.94)	(0.77)	(0.74)	(0.64)	(0.57)	(0.31)
	Non-informative	8.63	15.69	15.44	12.14	10.68	7.80
		(1.08)	(0.77)	(0.73)	(0.65)	(0.60)	(0.42)
Regression		11.65	12.69	13.40	15.45	13.99	9.56
		(1.20)	(1.10)	(0.99)	(0.65)	(0.51)	(0.40)

Panel B: Risk aversion $A = 5$, 1972 − 1991

		Fraction of explained variance					
		50% W&W optimistic	10% ⟵	5% P&S	1% W&W	0.5% W&W ⟶	0.1% W&W pessimistic
System	Prior on ρ_{uw}						
	More informative	8.86	10.80	10.83	9.37	8.62	8.22
		(0.28)	(0.25)	(0.24)	(0.13)	(0.12)	(0.10)
	Less informative	9.32	11.14	11.12	10.01	9.43	8.34
		(0.37)	(0.31)	(0.29)	(0.25)	(0.22)	(0.13)
	Non-informative	8.55	11.29	11.18	9.83	9.25	8.09
		(0.43)	(0.31)	(0.29)	(0.26)	(0.24)	(0.17)
Regression		9.71	10.12	10.40	11.19	10.59	8.80
		(0.46)	(0.43)	(0.39)	(0.25)	(0.20)	(0.16)

Certainty equivalent returns are calculated for the predictive system and predictive regression. The portfolio consists of stocks, bonds and a risk-free asset. The predictor is the dividend-price ratio for the stock returns and the yield spread for the bond returns. Optimal weights are calculated for a mean-variance investor with risk aversion coefficient $A = 2$ (Panel A) and $A = 5$ (Panel B). At the beginning of each year, starting in 1972 and ending in 1991, we estimate the model and use the estimated parameters for calculating the optimal portfolio for each quarter in this year. Different beliefs about the prior distribution on R^2 are considered, characterized by the expected value of the prior distribution. Lower values represent more skeptical investors. For the predictive system we report the results for different priors on the correlation between the expected and unexpected returns as in Pastor and Stambaugh (2009). The non-informative prior is flat on most of the $(-1, 1)$ interval, the less informative implies most mass below zero and the more informative imposes most mass below -0.7. Data are quarterly and span from 1952 to 2011. Numbers in parenthesis are bootstrap standard errors.

Table 7: Out-of-sample performance: Certainty equivalent returns, subsamples

Panel A: Risk aversion $A = 2$**,** $1992 - 2011$

		Fraction of explained variance					
		50% W&W optimistic	10% ⟵	5% P&S	1% W&W	0.5% W&W ⟶	0.1% W&W pessimistic
System	Prior on ρ_{uw}						
	More informative	−2.38	0.80	2.30	5.76	4.48	3.92
		(0.79)	(0.51)	(0.46)	(0.39)	(0.38)	(0.34)
	Less informative	0.51	1.52	2.43	2.67	4.31	8.31
		(0.70)	(0.50)	(0.43)	(0.32)	(0.29)	(0.32)
	Non-informative	1.06	0.70	2.42	4.19	5.54	10.55
		(0.71)	(0.53)	(0.43)	(0.35)	(0.33)	(0.37)
Regression		4.74	4.73	4.65	6.17	7.64	9.97
		(0.60)	(0.59)	(0.57)	(0.47)	(0.37)	(0.25)

Panel B: Risk aversion $A = 5$**,** $1992 - 2011$

		Fraction of explained variance					
		50% W&W optimistic	10% ⟵	5% P&S	1% W&W	0.5% W&W ⟶	0.1% W&W pessimistic
System	Prior on ρ_{uw}						
	More informative	1.15	2.39	2.97	4.32	3.81	3.58
		(0.32)	(0.20)	(0.19)	(0.16)	(0.15)	(0.14)
	Less informative	2.29	2.68	3.03	3.11	3.76	5.34
		(0.28)	(0.20)	(0.18)	(0.13)	(0.12)	(0.13)
	Non-informative	2.50	2.35	3.03	3.73	4.27	6.25
		(0.29)	(0.22)	(0.17)	(0.14)	(0.13)	(0.15)
Regression		4.01	4.00	3.96	4.55	5.12	6.02
		(0.24)	(0.24)	(0.23)	(0.19)	(0.15)	(0.10)

Certainty equivalent returns are calculated for the predictive system and predictive regression. The portfolio consists of stocks, bonds and a risk-free asset. The predictor is the dividend-price ratio for the stock returns and the yield spread for the bond returns. Optimal weights are calculated for a mean-variance investor with risk aversion coefficient $A = 2$ (Panel A) and $A = 5$ (Panel B). At the beginning of each year, starting in 1992 and ending in 2011, we estimate the model and use the estimated parameters for calculating the optimal portfolio for each quarter in this year. Different beliefs about the prior distribution on R^2 are considered, characterized by the expected value of the prior distribution. Lower values represent more skeptical investors. For the predictive system we report the results for different priors on the correlation between the expected and unexpected returns as in Pastor and Stambaugh (2009). The non-informative prior is flat on most of the $(-1, 1)$ interval, the less informative implies most mass below zero and the more informative imposes most mass below -0.7. Data are quarterly and span from 1952 to 2011. Numbers in parenthesis are bootstrap standard errors.

Table 8: Out-of-sample performance: Certainty equivalent returns, constraints on weights

Panel A: Risk aversion $A = 2$**,** $1972 - 2011$

		Fraction of explained variance					
		50% W&W optimistic	10% ⟵	5% P&S	1% W&W	0.5% W&W ⟶	0.1% W&W pessimistic
System	Prior on ρ_{uw}						
	More informative	9.60	10.13	10.17	9.20	8.17	7.65
		(0.22)	(0.21)	(0.21)	(0.16)	(0.13)	(0.13)
	Less informative	10.15	10.01	9.89	9.03	9.04	9.08
		(0.22)	(0.21)	(0.21)	(0.20)	(0.19)	(0.15)
	Non-informative	9.91	9.48	9.60	9.14	9.07	8.95
		(0.22)	(0.21)	(0.21)	(0.20)	(0.19)	(0.19)
Regression		10.45	10.38	10.36	10.39	10.03	9.70
		(0.23)	(0.23)	(0.23)	(0.22)	(0.20)	(0.17)

Panel B: Risk aversion $A = 5$**,** $1972 - 2011$

		Fraction of explained variance					
		50% W&W optimistic	10% ⟵	5% P&S	1% W&W	0.5% W&W ⟶	0.1% W&W pessimistic
System	Prior on ρ_{uw}						
	More informative	6.92	7.16	7.07	6.85	6.39	6.31
		(0.13)	(0.12)	(0.11)	(0.07)	(0.06)	(0.06)
	Less informative	7.58	7.35	7.27	6.58	6.62	6.97
		(0.15)	(0.13)	(0.12)	(0.11)	(0.10)	(0.06)
	Non-informative	7.46	7.15	7.10	6.81	6.86	7.28
		(0.15)	(0.13)	(0.12)	(0.11)	(0.11)	(0.08)
Regression		8.39	8.43	8.44	8.42	8.03	8.35
		(0.13)	(0.13)	(0.13)	(0.10)	(0.09)	(0.07)

Certainty equivalent returns are calculated for the predictive system and predictive regression. The portfolio consists of stocks, bonds and a risk-free asset. The predictor is the dividend-price ratio for the stock returns and the yield spread for the bond returns. Optimal weights are calculated for a mean-variance investor with risk aversion coefficient $A = 2$ (Panel A) and $A = 5$ (Panel B). We limit the weights to lie between 0 and 1.5 for each risky asset. At the beginning of each year, starting in 1972, we estimate the model and use the estimated parameters for calculating the optimal portfolio for each quarter in this year. Different beliefs about the prior distribution on R^2 are considered, characterized by the expected value of the prior distribution. Higher values represent more skeptical investors. For the predictive system we report the results for different priors on the correlation between the expected and unexpected returns as in Pastor and Stambaugh (2009). The non-informative prior is flat on most of the $(-1, 1)$ interval, the less informative implies most mass below zero and the more informative imposes most mass below -0.7. Data are quarterly and span from 1952 to 2011. Numbers in parenthesis are bootstrap standard errors.

estimating the model, reasonable prior beliefs help to achieve higher profit for an investor.

To check the robustness of our results we conduct several exercises. First, we explore the effect of different levels of correlation among risky assets. Second, we look at different subsamples, before and after 1991, to investigate the effect of a possible structural break in that year. Third, we constrain the weights to lie between 0 and 150%, and fourth, we use a rolling window of 20 years instead of an expanding window for the estimation. Finally, we look at the sensitivity of results to the prior discussed in Pastor and Stambaugh (2009). The pattern in all robustness checks is similar to that in the main analysis.

The paper could be extended along several dimensions. In the future, we would like to look at a richer set of predictors, and investigate whether the predictive regression outperforms the predictive system for other predictors as well. Furthermore, we are going to explore possibilities how to estimate correlation among several assets in the context of the predictive regression and the predictive system. Another interesting extension would be to consider the multi-period asset allocation problem that has been studied in the simpler setup (Barberis, 1999) or in-sample in the predictive system (Pastor and Stambaugh, 2012).

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