The Information Rat Race

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Abstract

Information is a valuable good that requires scarce inputs, such as human talent, to produce. Competition among investors for these inputs creates an equilibrium channel that has not yet been modeled explicitly. This paper studies the dynamic implications of this channel for information choice, risk-taking, and welfare.

We study a dynamic portfolio choice problem with heterogeneous agents and endogenous information choice. Our central assumption is that investors compete for information in a market for information inputs. This creates a feedback loop in which relatively wealthy agents acquire more information, obtain superior portfolio performance, and get comparatively even wealthier. Two dynamic effects arise: First, interim losers anticipate their inability to acquire future information and take on more risk in an attempt to catch up with interim winners, while interim winners take on less risk to protect their lead. Second, acquiring information is a strategic complement. In addition, we perform a welfare analysis and discuss the model's implications for capital income inequality and delegated portfolio management.

Keywords: information acquisition, portfolio choice, dynamic equilibrium, trading frictions, inequality, delegated portfolio management

JEL Classification: D53, D63, G11

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1 Introduction

Information plays a central role in financial markets. Investors with superior information achieve higher returns on their portfolios. Information is, however, an expensive resource: French (2008) estimates that over 0.67% of aggregate value is spent each year on "the search for superior returns." Therefore, investors must allocate their wealth between purchasing financial assets and purchasing information.

In this paper, we adopt the perspective that, like other assets, information is in limited supply. This may be because the inputs needed for producing information (e.g., human capital, information technology) are also in limited supply. On that account, a financial market equilibrium is determined not only by financial asset prices, but also by a price for information. Specifically, we propose a dynamic model of financial markets in which heterogeneous agents compete for a limited supply of information regarding a risky asset. Agents cannot acquire this information directly, but buy resources needed for information production instead. We assume these "inputs" to be scarce, and impose market clearing to derive their prices in equilibrium.

We obtain the following results: First, a feedback loop arises in which wealthier agents obtain superior portfolio returns and thus increase their relative wealth. This creates relative performance concerns in which interim losers take on more risk to catch up with interim winners, while interim winners take on less risk to protect their lead. Second, these relative performance concerns generate complementarities in information acquisition. Third, welfare is lower than in a benchmark economy without competition for information. We relate these results to capital income inequality and delegated portfolio management.

We study a dynamic portfolio problem in which two ex-ante identical representative agents with constant relative risk aversion (CRRA) utility over final wealth must allocate their wealth between (i) a riskless asset, (ii) a risky asset, and (iii) information about the risky asset's short-term returns. Information cannot be acquired directly, but is produced with the use of scarce inputs. We assume CRRA preferences to exploit wealth effects. Richer agents with

CRRA utility buy more risky assets, and their valuation of information about these assets is thus higher.

We solve the model numerically and highlight its dynamic properties. Because of the limited supply of inputs, relative demand for information determines equilibrium allocations. Given the assumed CRRA preferences, the amount of information acquired is increasing in an agent's relative wealth. Indeed, information increases an investor's portfolio performance by allowing her to select superior investments. A dynamic feedback loop arises: The wealthier agent acquires more information and thereby achieves superior portfolio performance, which further increases their relative wealth advantage.

The intuition behind our main results is as follows: First, relative performance concerns arising in equilibrium create tournament-like risk-taking incentives. The agent whose portfolio performs relatively poorly has less wealth to buy information. This lowers the expected performance of her future portfolios even further. The wealth laggard tries to escape this vicious circle by increasing the riskiness of her portfolio, in an attempt to catch up with the wealth leader. In contrast, the leader benefits from the circle and "plays it safe" to protect her advantage.

The feedback loop responsible for these tournament-like risk-taking incentives is driven by future competition for information. The degree to which risk-taking is affected is, hence, an increasing function of an agent's horizon, and only arises in a dynamic economy. Therefore, models with myopic agents (e.g., Kacperczyk et al. (2014)) would not generate similar risk-taking behavior. Moreover, our feedback effect does not arise in models in which agents compete for scarce *consumption* goods only, as is the case in Demarzo et al. (2004, 2007, 2008). While in such an economy richer agents consume more, they do not achieve the superior portfolio returns needed to consume even more in future periods.

Second, information acquisition is a strategic complement for agents (i.e., agents purchase more information when others buy more information). Complementarities arise due to strategic risk-taking induced by the dynamic competition for information. In particular, an agent's information acquisition imposes a negative externality on other agents by reducing the expectation of their future relative wealth. To catch up, the other agent must take on more portfolio risk and increase their own information acquisition. Therefore, agents value information more when others are better informed.

This finding is in contrast to the commonly adopted notion that information acquisition is a substitute for agents (see Grossman and Stiglitz (1980) and Verrecchia (1982)). In their setting, equilibrium asset prices partially reflect private information. Thus, the ability of uninformed agents to free-ride on private information reflected in equilibrium asset prices decreases their incentive to buy information on their own behalf. Note that in our model, complementarities arise due to risk-taking rather than changes in price informativeness. As is the case with endogenous risk-taking, complementarities only arise in a dynamic setting, and their magnitude increases with the agent's trading horizon.

Third, we perform a welfare analysis by comparing the value function of ex-ante identical agents across horizons. As a benchmark, we consider an economy that is identical to ours but differs in the assumption that agents have access to separate information markets. Compared to the benchmark, our model implies both higher expected portfolio returns and higher ex-ante volatility of portfolio returns. Expected returns are higher because wealthier investors buy more information, so that the implied higher expected returns apply to more capital. Ex-ante return volatility is also higher because the feedback loop enlarges differences in wealth. We find that the negative effect of higher volatility exceeds the positive effect of higher expected returns.

We next apply our model to related issues. First, we make a connection between our findings and the documented increase in income inequality (Piketty (2003) and Atkinson et al. (2011)). Kacperczyk et al. (2014) argue that its root cause is the empirically observed increased dispersion in investor "skill." In that perspective, our paper can be viewed as endogenizing the dispersion of skill by linking it to information acquisition. To discuss skill dispersion, we enrich our model with trading frictions. Indeed, information is of value only if it can affect action. Therefore, trading frictions disproportionally hinder wealthier agents, since they take larger positions in risky securities and acquire more information. We argue that a decrease in trading frictions (e.g., lower transaction costs) causes a higher dispersion in investor skill/information, in turn leading to a higher level of capital income inequality.

Second, we note that the tournament-type risk-taking incentives highlighted in our analysis resemble those documented in the empirical literature on delegated portfolio management (e.g., Brown et al. (1996)). In our model, however, agents care about their own consumption, and tournament-like risk-taking arises even in the absence of incentive contracts.

In reality, the incentives of investors and portfolio managers are not identical. This may be because investors have long horizons, while portfolio managers are myopic. Our model states that in this case, reward schemes that induce tournament-like risk-taking are required to align incentives. This finding is in contrast to Basak and Makarov (2013), who find that clients are disadvantaged by relative performance concerns of mutual fund managers.

The paper proceeds as follows: Section 2 discusses the related literature. Sections 3 and 4 present the model choice and the solution method. We then solve the model and derive implications for portfolio performance (Section 5), risk-taking (Section 6), strategic information choice (Section 7), and welfare (Section 8). Last, we apply the model to study capital income inequality (Section 9), and delegated portfolio management (Sections 10). Section 11 discusses limitations and extensions.

2 Related literature

This paper is related to different branches of literature. In an attempt to most efficiently structure this section, we group related works by topic and outline the innovations of our paper accordingly.

Portfolio choice, information acquisition, and wealth

While there is a considerable literature on information choice in financial markets, only a handful of papers explore wealth effects using a CRRA utility function. Portfolio choice models with CRRA preferences are studied in Turmuhambetova (2005), Opp (2008), Peress (2003), and Breugem and Djordjevic (2014). Closely related to our paper is Peress (2003), who models a static CRRA agent equilibrium model with information acquisition in closed form.[1] Similar to our paper, he finds that wealthier households invest a larger share of their portfolio in risky assets and that the availability of costly information exacerbates wealth inequalities. We build on Peress (2003) by studying an economy with a limited supply of information and show that relative rather than absolute wealth determines information allocations. Moreover, while Peress (2003) studies a static economy, we focus on dynamic implications.[2] [3] Breugem and Djordjevic (2014) investigate the joint problem of information acquisition and portfolio choice of a *single* CRRA investor. Our paper focuses on the interaction between two CRRA agents through the market for information.

Income inequality

A growing stream of literature has identified an increase in income inequality across households (see e.g., Piketty (2003), Piketty and Saez (2003), Alvaredo et al. (2013), Autor et al. (2006), Atkinson et al. (2011)). Most commonly, the labor market is used to explain these inequalities (e.g., Acemoglu (1999), Acemoglu (2002), Katz and Autor (1999), Autor et al. (2006), Autor et al. (2008), and Autor and Dorn (2013)). Papers that explain (capital) income inequality with informational differences include Arrow (1987), Peress (2003), and Kacperczyk et al. (2014). Our paper contributes to this literature by considering a dynamic optimization model with ex-ante identical agents in which informational differences arise endogenously. These differences are more likely to emerge in economies with lower trading frictions. We contribute to the literature by showing that a decrease in transaction costs increases capital income inequality.

[1] An approximation is needed to obtain closed form results. Makarov and Schornick (2010) model a Verrecchia (1982) type economy with CARA utility, but with risk tolerance being a function of wealth. This allows them to do without the approximation made in Peress (2003)).

[2] Examples of dynamic models with endogenous information choice and *CARA* preferences are Wang (1993) and Massa (2002). Moreover, while Massa (2002) focuses on implications for the asset markets, we focus on consequences for risk-taking and strategic information choice.

[3] In a different framework, Kyle and Xiong (2001) demonstrate the importance of dynamic wealth effects of information for contagion in a 3-agent model.

Relative wealth and performance concerns

The notion that agents care about their relative consumption, in addition to their absolute consumption, has been widely documented (e.g., Veblen (1899), Schoenberg and Haruvy (2012)). The finance literature has utilized these concerns to improve the fit of asset pricing models. For example, Abel (1990) and Gali (1990) demonstrate that the inclusion of a utility for relative consumption can help explain the equity premium puzzle (see Mehra and Prescott (1985)) or can improve the fit of international asset pricing models (Gomez et al. (2009)). While in most of the literature relative wealth concerns are imposed exogenously, Demarzo et al. (2004, 2007, 2008) generate wealth preferences endogenously due to competition for scarce *consumption* goods. Our paper differs from their work by modeling a dynamic competition for scarce *information*. Specifically, we show that the competition for information dynamically generates a feedback loop that does not arise when agents compete for consumption alone.

Furthermore, our paper relates to the literature on status, which assumes that agents derive positive utility from their relative wealth position. Robson (1992) and Bakshi, Gurdip and Chen (1996) model economies with exogenously imposed preferences over status. Also related to our paper are Cole et al. (1992) and Cole et al. (2001), who study an economy in which agents are matched to partners with similar levels of status. Similar to what we show, is that status can be responsible for a persistence in portfolio performance. Our model differs from theirs, however, since we assume that agents can adjust their level of status—corresponding to information in our setting—at the cost of financial wealth. In addition, the main results of our paper are caused by dynamic optimization, which cannot be produced in the overlapping generations economy of Cole et al. (2001).

The delegated portfolio management literature studies relative *performance* rather than relative consumption or wealth concerns. Relative wealth concerns can arise due to compensation contracts/benchmarking (see, e.g., Brennan (1993), Carpenter (2000), Cuoco and Kaniel (2011), Basak and Pavlova (2013), and Basak and Makarov (2013)).[4] *Empirically,* Brown et al. (1996)

[4]Other sources of relative performance concerns typical to the delegated portfolio literature are (i) competition for fund flows (see, e.g., Chevalier and Ellison (1997) and Sirri and Tufano (1998), or (ii) career motives (see, e.g., Chevalier and Ellison (1999), Brown et al. (2001), and Kempf and Ruenzi (2007))

and Elton et al. (2003) show that compensation-induced incentive schemes in general can shift risk-taking such that interim losers gamble in an attempt to catch up with interim winners, and interim winners play it safe to protect their lead. Our paper show that even in the absence of such incentive schemes, investors would like to adopt a similar "tournament" type of risk-taking behavior.

Strategic information acquisition

Grossman and Stiglitz (1976), Grossman and Stiglitz (1980), and Hellwig (1980) show that in a rational expectations equilibrium in which agents acquire private information, equilibrium asset prices aggregate private information. This decreases the incentive of uniformed agents to acquire information, which makes acquiring information a substitute strategy.[5] Verrecchia (1982) shows that this substitution effect does not only apply to the investor's choice of whether to become informed, but also to the decision of how much information per investor should be acquired.

Acquiring information in a one asset framework can become a complement strategy, when some of the assumptions made in standard models such as Grossman and Stiglitz (1976) and Grossman and Stiglitz (1980) are relaxed. Examples of modifications in assumptions that yield strategic complementarities are presented in Ganguli and Yang (2009) and Avdis (2012) (change in learning) and Barlevy and Veronesi (2000), Mele and Sangiorgi (2011), and Garcia and Strobl (2010) (change in preferences).

Commonly in the literature, complementarities in information acquisition arise due to changes in price informativeness. In our paper, complementarities are driven by relative performance concerns. Most closely related to this paper is Garcia and Strobl (2010), who study a Grossman and Stiglitz economy in which agents have "keeping up with the Joneses" preferences and show

[5]In (perfectly) informationally efficient markets, the size of the substitution effect is so large that no agent would like to acquire information because uninformed agents obtain the same benefit of information but do not pay information acquisition costs. Hellwig (1980) and Diamond and Verrecchia (1981) show that when there is a source of noise in the economy, such as uncertain asset supply, the equilibrium asset prices do not perfectly aggregate information, and information has a positive value. Fully revealing equilibria due to imperfectly competitive or incomplete markets are presented in Jackson (1991) and Berk (1997).

that relative wealth concerns induce complementarities in information acquisition. In our paper, relative wealth preferences arise endogenously. Moreover, while in Garcia and Strobl (2010) agents can be either informed or uninformed, we allow agents to choose the desired amount of information quality on a continuous interval.[6]

3 Modeling Strategy

The focus of this paper is to analyze the equilibrium channel generated by dynamic competition for information. Despite the simplifying assumptions we impose in our model, the employment of CRRA preferences forces us to employ numerical solution techniques. To work out that our results are driven by competition for information, and not by other equilibrium effects, we adopt the following procedure.

First, we shut down all other equilibrium effects that do not contribute to communicating the main model mechanism. We do so by assuming that agents operate in separate production economies, and only interact via the market for information inputs. We refer the reader to Breugem (2014) for a discussion of other equilibrium effects. Second, we introduce two benchmark economies that are identical to our model with respect to asset markets, but have differently structured information markets. The comparison between our model and these benchmarks provides an additional source of identification of our results. We proceed by presenting our model and the two benchmark economies in detail.

3.1 Agents and assets

We consider a multi-period production economy with time $t \in \{1, .., T\}$. The economy is populated with two *ex-ante identical* agents who maximize their CRRA utility over their final

[6] Distinct to our model, complementarities can also arise in the form of "herding" investors choosing to acquire information about the same asset rather than buying information about different assets. Rational herding can arise because of short trading horizons (Froot et al. (1992)), characteristics of the production technology of information (Veldkamp (2006a) and Veldkamp (2006b)), returns to scale of information (Nieuwerburgh and Veldkamp (2010)) or relative wealth concerns (Niu (2013)).

period consumption $c_{i,T}$. There are two short-lived investment opportunities (assets) available to each agent. The first asset is risk-free and yields a unit payoff with certainty. The second asset is risky and produces a risky payoff $R_{i,t} \in \{R_H, R_L\}$.[7]

We let agents trade in isolation by assuming that each agent has access to a separate asset market. To fully isolate the agents' asset markets, we assume zero correlation between the risky asset of each agent.[8] The fraction of (individual) wealth invested in risky assets by agent i at time t is denoted by $\lambda^S_{i,t}$; the fraction invested in riskless assets is denoted by $\lambda^B_{i,t}$.

3.2 Information acquisition

3.2.1 Information quality

At every stage, each agent acquires an informative signal $y_{i,t} \in \{H, L\}$ about the next period return $R_{i,t+1}$. Agents only acquire information about the payoff process of their own market. The "quality" of the signal $y_{i,t}$ is denoted by $\rho_{i,t} \in [0, 1]$ and represents the correlation between the signal and the next period return on risky investments, specifically, $\rho_{i,t} = \mathrm{Corr}\,[y_{i,t}, R_{i,t+1}]$. Special cases include (i) $\rho_{i,t} = 0$ in which the signal is not informative and (ii) $\rho_{i,t} = 1$ which corresponds to a perfect foresight. Signals of higher quality are better predictors of future returns, but are more costly to acquire.

3.2.2 The production of information

Agents do not acquire information directly, but instead buy information *inputs* $\Lambda_{i,t}$.[9] These inputs are then used for the production of information. For example, we could refer to these

[7] Our assumption that assets are short-lived is only relevant when we introduce trading frictions (see Section 9). By using short term rather than long term assets, we eliminate two state variables. This delivers great numerical convenience, but rules out some dynamics generated by transaction costs discussed in Buss and Dumas (2014).

[8] Note that since agents trade in separate economies, the impact of the correlation between assets is very small and does not influence the sign of any of our results. Moreover, by setting asset correlations to zero, the interim performance effects generated by Basak and Makarov (2013) are entirely removed.

[9] Under CRRA assumptions, modeling the market for information directly would lead to uninteresting corner solutions (see e.g.: Romer (1990)). To focus on an interior solution, we model the market for information *inputs* instead. A direct market for information under CARA preferences is studied in Admati and Pfleiderer (1987).

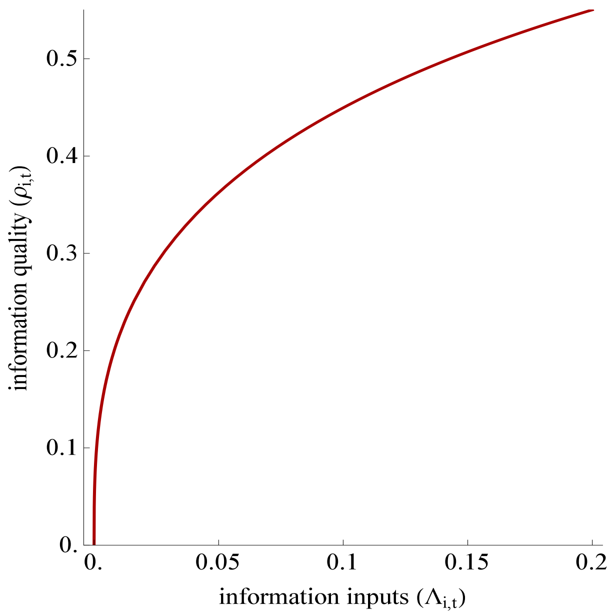

Figure 1: Quantity of inputs $\Lambda_{i,t}$ required to produce a certain information quality $\rho_{i,t}$ using the production function $\rho_{i,t} = \Psi(\Lambda_{i,t}) = \left(\frac{\Lambda_{i,t}}{\Lambda_{i,t}+1}\right)^{\frac{1}{\Gamma}}$. Note that the production of perfect information ($\rho_{i,t} = 1$) requires an infinite number of inputs.

inputs as the number of skilled portfolio managers. The production function Ψ_i is a one-to-one mapping between information quality and information inputs. Important assumptions are that (i) information of higher quality requires more inputs and that (ii) it is impossible to achieve perfect foresight ($\rho_{i,t} = 1$).[10] Throughout this paper, we employ the following information production function:

[10]More generally, Ψ must be invertible, increasing and strictly concave and must satisfy $\Psi'(0) = \infty$, $\Psi(0) = 0$ and $\Psi(\infty) = 1$. In this case its inverse Φ is increasing, strictly convex and satisfies $\Phi'(0) = 0$, $\Phi(0) = 0$ and $\Phi(1) = \infty$. These assumptions on the cost function are justified as follows:

1. The inverse function Φ can be referred to as the quantity of inputs required to produce a certain level of information. The assumption that Φ is convex loosely corresponds to assumptions in Verrecchia (1982) and Nieuwerburgh and Veldkamp (2010).
2. The assumption $\Psi'(0) = \infty$ or $\Phi'(0) = 0$ states that it is very easy to produce the first bit of information. This assures that even a poor agent (with low portfolio holdings and, therefore, a low benefit from information) still acquires a strictly positive amount of inputs. Mathematically this guarantees an interior solution. Economically, this can be justified that some information can be acquired at very low cost (for example by watching the news).
3. The assumption $\Psi(0) = 0$ or $\Phi(0) = 0$ states that without inputs, there is no information production. This is simply a normalization.
4. The assumption $\Psi(\infty) = 1$ or $\Phi(1) = \infty$ states that it is impossible to produce perfect foresight since it requires an infinite quantity of inputs to produce $\rho = 1$.

Appendix A discusses existence of a solution for a more general production function Ψ

$$\rho_{i,t} = \Psi\left(\Lambda_{i,t}\right) = \left(\frac{\Lambda_{i,t}}{\Lambda_{i,t}+1}\right)^{\frac{1}{\Gamma}} \tag{1}$$

Figure 1 shows the relationship between information quality $\rho_{i,t}$ and information inputs $\Lambda_{i,t}$ by plotting Ψ. By inverting the information cost function, we obtain the number of inputs required for the production of a particular level of information quality. The cost of information is then obtained by multiplying this level of inputs by their unit price Ξ_t:

$$\kappa\left(\rho_{i,t}\right) = \Xi_t \times \Lambda_{i,t} = \Xi_t \times \Psi^{-1}\left(\rho_{i,t}\right) = \Xi_t \times \frac{\rho_{i,t}^{\Gamma}}{1-\rho_{i,t}^{\Gamma}}$$

3.2.3 The market for information

A novel contribution of this paper is that input prices Ξ_t are derived endogenously, by imposing the following information input market clearing condition:

$$\underbrace{\Lambda_{1,t}\left(\Xi_t\right) + \Lambda_{2,t}\left(\Xi_t\right)}_{demand} = \underbrace{\bar{\Lambda}}_{supply} \tag{2}$$

We take the extreme position by assuming that the supply of inputs $\bar{\Lambda}$ is fixed. However, the main qualitative results of our model are robust to any supply function as long as the input supply is not perfectly elastic. We argue that this is a realistic assumption, since in general, the production of inputs (e.g., education of experts) is a time-consuming.

In our model, we adopt the notion that information cannot be traded after production, i.e., there is no market for second-hand information. This assumption could be justified by extreme adverse selection—i.e., agents do not trust that second-hand information is genuine—or the conception that information gets "outdated" quickly (see Allen (1986)). The study of a market for second-hand information is outside the scope of the present paper because it would require more than two agents to produce non-trivial results.[11]

[11] In a static model, Veldkamp (2006b) takes a different approach by assuming that information can be copied by initial producers of information.

3.3 Utility maximization

3.3.1 Timing

Within each period, each agent faces the joint problem of information acquisition and portfolio choice. *For each agent*, the timing within each period $t < T$ consists of four sub periods.

1. Investment returns $R_{i,t}$ are realized, and the investor's wealth $W_{i,t}$ is updated.
2. The agent decides upon the acquisition of $\Lambda_{i,t}$ and therefore chooses the quality $\rho_{i,t}$ of the signal $y_{i,t}$. Information expenditures are deducted from the investor's budget.
3. The signal $y_{i,t}$ is revealed, and the agent learns about $R_{i,t+1}$
4. The agent allocates her remaining wealth across risky and riskless assets.

In the last period, when $t = T$, agents consume their final wealth $c_{i,T} = W_{i,T}$.

3.3.2 Problem statement

Following the assumptions made in the previous paragraphs, the Hamilton-Jacobi-Bellman (HJB) equation of each agent is denoted by:

$$\begin{aligned}
V_{i,t}\left(W_{i,t}, W_{j,t}\right) = & \max_{\rho_{i,t}, \left\{\lambda^B_{i,t}\right\}, \left\{\lambda^S_{i,t}\right\}} \mathbb{E}_t\left[V_{i,t+1}\left(W_{i,t+1}, W_{j,t+1}\right)\right] \\
\text{subject to :} \quad & W_{i,t} \times \left(1 - \lambda^S_{i,t} - \lambda^B_{i,t}\right) = \Xi_t \times \Phi_i\left(\rho_{i,t}\right) \quad \forall y_{i,t} \\
& W_{i,t+1} = W_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \times R_{i,t+1}\right) \quad \forall y_{i,t} \\
& V_{i,T} = U\left(c_{i,T}\right) = \tfrac{W^{1-\gamma}_{i,T}}{1-\gamma} \\
& \lambda^S_{i,T} = \lambda^B_{i,T} = \rho_{i,T} = 0 \\
& W_{i,1} = 1/2
\end{aligned} \tag{3}$$

where γ is the coefficient of relative risk aversion and $\Phi = \Psi^{-1}$ is the inverse production function of information. We assume agents start with an identical level of initial wealth (equal to $\frac{1}{2}$). Notice that the value function has the wealth of the other agent as a second argument.[12] Both agents' wealth levels are relevant state variables since they jointly determine current and future prices of information inputs.[13]

3.3.3 First-order conditions

After substituting the definition of $W_{i,t+1}$ and rewriting probabilities explicitly, the Lagrangian for intermediate periods can be written as:

$$\begin{aligned} L_{i,t} = & \sum_{y_{i,t},y_{j,t},\eta_{i,t+1},\eta_{j,t+1}} \pi_1\left(\rho_{1,t}, y_{1,t}, R_{1,t+1}\right) \times \pi_2\left(\rho_{2,t}, y_{2,t}, R_{2,t+1}\right) \times \\ & V_{i,t+1}\left(W_{i,t} \times \left(\lambda_{i,t}^B + \lambda_{i,t}^S \times R_{i,t+1}\right), W_{j,t} \times \left(\lambda_{j,t}^B + \lambda_{j,t}^S \times R_{j,t+1}\right)\right) \\ & + \sum_{y_{i,t},y_{j,t}} \varphi_{i,t}\left(W_{i,t} \times \left(1 - \lambda_{i,t}^S - \lambda_{i,t}^B\right) - \Xi_t \times \Phi_i\left(\rho_{i,t}\right)\right) \end{aligned} \tag{4}$$

where $\varphi_{i,t}$ is the Lagrange multiplier and represents the shadow price of wealth. Note that the Lagrangian contains the portfolio decisions of other agents. In particular, portfolio decisions of others determine future wealth distributions and thereby future information input prices. The first-order conditions derived from (4) should thus be read as reaction curves. Likewise, the equilibrium that prevails from solving the system of first-order conditions is a Nash Equilibrium.

This strategic behavior does not contradict our assumption that agents represent a large group of small traders. Indeed, we still assume agents are price takers, both in the information input market and in the asset market. Therefore, agents in our model are strategic as in Verrecchia (1982), but not as in Vayanos (1999).

Probabilities in (4) are given by:

[12] A more precise, but more notationally intensive way to indicate dependence on signals is to write $\lambda_{i,t}^S\left(y_{i,t}\right)$, $\lambda_{i,t}^B\left(y_{i,t}\right)$ and $\varphi_{i,t}\left(y_{i,t}\right)$ instead of $\lambda_{i,t}^S$, $\lambda_{i,t}^B$ and $\varphi_{i,t}$.

[13] In an economy with many agents, the HJB equation is a function of the average wealth of the other agents in the economy and can be written in the form $V_{i,t}\left(W_{i,t}, \bar{W}_{-i,t}\right)$. Another way of writing the value function is by only using the argument $V_{i,t}\left(W_{i,t}, \{\Xi_t\}\right)$ or simply $V_{i,t}\left(W_{i,t}\right)$. To highlight the importance of relative wealth in the economy and to relate this paper with the literature on relative wealth concerns, we chose to explicitly mention $W_{j,t}$ in the value function, even though $W_{j,t}$ is not an argument in agents terminal utility. Similar notation is employed, for example, in Basak and Makarov (2013)

$$\pi_1\left(\rho_{i,t}, y_{1,t}, R_{1,t+1}\right) = \begin{cases} \frac{1+\rho_{i,t}}{2} & y_{1,t} = R_{1,t+1} \\ \frac{1-\rho_{i,t}}{2} & y_{1,t} \neq R_{1,t+1} \end{cases} \tag{5}$$

The higher the level of information quality $\rho_{i,t}$, the higher the probability that the signal corresponds to the next period state of nature. Reference cases include (i) $\rho_{i,t} = 0$, in which the the probability of getting a "correct" signal equals $\frac{1}{2}$, and (ii) $\rho_{i,t} = 1$, in which the signal always corresponds to the next period state of nature.

Each agent now determines the optimal information level and portfolio compositions. There are two first-order conditions with respect to $\lambda^B_{i,t}$:

$$\begin{aligned} &\sum_{y_{i,t}, y_{j,t}, \eta_{i,t+1}, \eta_{j,t+1}} \pi_1\left(\rho_{1,t}, y_{1,t}, R_{1,t+1}\right) \times \pi_2\left(\rho_{2,t}, y_{2,t}, R_{2,t+1}\right) \times \\ &\partial_1 V_{i,t+1}\left(W_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \times R_{i,t+1}\right), W_{j,t} \times \left(\lambda^B_{j,t} + \lambda^S_{j,t} \times R_{j,t+1}\right)\right) \quad = \varphi_{i,t} \quad \forall y_{i,t} \end{aligned} \tag{6}$$

There are two first-order conditions with respect to $\lambda^S_{i,t}$:

$$\begin{aligned} &\sum_{y_{i,t}, y_{j,t}, \eta_{i,t+1}, \eta_{j,t+1}} \pi_1\left(\rho_{1,t}, y_{1,t}, R_{1,t+1}\right) \times \pi_2\left(\rho_{2,t}, y_{2,t}, R_{2,t+1}\right) \times \\ &R_{i,t+1} \times \partial_1 V_{i,t+1}\left(W_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \times R_{i,t+1}\right), W_{j,t} \times \left(\lambda^B_{j,t} + \lambda^S_{j,t} \times R_{j,t+1}\right)\right) \quad = \varphi_{i,t} \quad \forall y_{i,t} \end{aligned} \tag{7}$$

There is one first-order conditions with respect to $\rho_{1,t}$[14]:

$$\begin{aligned} &\sum_{y_{i,t}, y_{j,t}, \eta_{i,t+1}, \eta_{j,t+1}} \partial\pi_1\left(y_{1,t}, R_{1,t+1}\right) \times \pi_2\left(\rho_{2,t}, y_{2,t}, R_{2,t+1}\right) \times \\ &V_{i,t+1}\left(W_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \times R_{i,t+1}\right), W_{j,t} \times \left(\lambda^B_{j,t} + \lambda^S_{j,t} \times R_{j,t+1}\right)\right) \quad = \Xi_t \times \Phi'_i\left(\rho_{i,t}\right) \times \varphi_{i,t} \end{aligned} \tag{8}$$

Equations 6 and 7 state that the expected marginal utility from investing an additional dollar in riskless or risky assets should be equal to the current shadow price of wealth. A similar condition arises for the marginal dollar invested in information: The left-hand side of (8) expresses the

[14] Where $\partial\pi_1\left(y_{1,t}, R_{1,t+1}\right) = \frac{\partial\pi_1\left(\rho_{i,t}, y_{1,t}, R_{1,t+1}\right)}{\partial\rho_{i,t}} = \begin{cases} \frac{1}{2} & y_{1,t} = \eta_{1,t+1} \\ -\frac{1}{2} & y_{1,t} \neq \eta_{1,t+1} \end{cases}$

marginal benefit that an agent enjoys for each additional unit of information quality $\rho_{1,t}$, and the right-hand side states the marginal costs.

Possession of information benefits the investor by providing a sharper filtration about future risky asset returns. The change in probability measure resulting from higher information quality is represented by $\partial\pi_1$ on the left-hand side of (8). The costs of information quality consist of the product of the number of inputs required to produce a certain level of quality and the input price. Since agents are price takers, the marginal costs of information quality equal the marginal productivity of inputs multiplied by the input price (see the right-hand side of equation 8).

Finally, there are two budget constraints:

$$W_{i,t} \times \left(1 - \lambda_{i,t}^S - \lambda_{i,t}^B\right) = \Xi_t \times \Phi_i\left(\rho_{i,t}\right) \quad \forall y_{i,t} \tag{9}$$

These equations simply state that the wealth not spent on risky or riskless investments is available for the purchase of information. Expenditures on information consists of the number of inputs required to produce a certain level of information quality multiplied by the unit input cost Ξ_t.

3.3.4 Overview of system

The entire system of equations now consists of 15 equations: 2×2 first-order conditions with respect to $\lambda_{i,t}^S$, 2×2 first-order conditions with respect to $\lambda_{i,t}^B$, 2×2 budget constraints, 2×1 first-order conditions with respect to $\rho_{1,t}$ and 1 information market clearing condition. There are 15 unknowns: 2×2 for $\lambda_{i,t}^S$, 2×2 for $\lambda_{i,t}^B$, 2×2 for $\varphi_{i,t}^S$, 2 for $\rho_{1,t}$ and Ξ_t.

3.4 Benchmark models

In this section, we introduce two benchmark economies to which we compare our model. The benchmark economies are identical to our model in terms of the asset market, but differ in

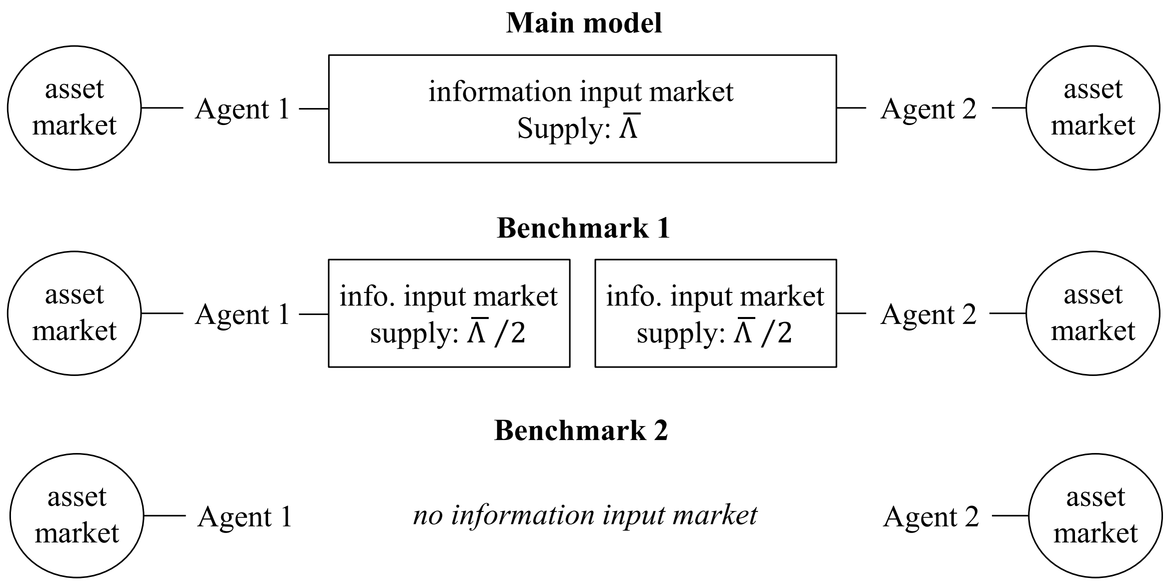

Figure 2: Overview of main model and two benchmark economies.

their assumptions regarding the information input market. Figure 2 shows these differences graphically.

In Benchmark 1, agents acquire information in the same way as in our main model, but do so in separated information markets. We assume the total supply of inputs is unchanged: thus, agent-specific market clearing conditions are:

$$\Lambda_{i,t} = \frac{\bar{\Lambda}}{2} \qquad \forall i \tag{10}$$

Market clearing information input prices $\Xi_{i,t}$ are thus agent-specific as well. While this benchmark preserves the information acquisition component, it eliminates the impact that other agents have on the equilibrium price of information, and thereby removes any strategic effect.

In Benchmark 2, we assume that agents cannot acquire information. This economy arises as a limit case from the main model and Benchmark 1 by setting $\bar{\Lambda} \to 0$. Agents in Benchmark 2 do not learn and act identically to agents in a discrete version of Merton (1971). The first-order conditions for each of the benchmark economies are derived in appendix B.

4 Solution method

This section briefly outlines our methodology. Due to our departure from the tractable CARA-normal framework, we are forced to employ numerical methods to tackle our system.

The model is solved using backwards induction. We start by solving the system of equations at the last node of the system over a set of values of state variables. Since computational time increases exponentially with the number of state variables used, we slightly reformulate our system. In particular, we define the variable relative wealth as $\omega_{i,t} = \frac{W_{1,t}}{W_{1,t}+W_{2,t}} \in (0,1)$. The resulting system, in which $\omega_{i,t}$ is the only state variable, is discussed in Appendix C.

We start solving the system of equations at the last node over a grid of values for $\omega_{i,t}$ and compute the value function at each of these points. The derivative of the value function is obtained with the use of the envelope theorem. We use the set of function values to construct interpolated functions.[15] Then, in the final last node, we use these function interpolants to solve the respective system of equations. We repeat this procedure and move backwards until the initial period of the model. At this initial node, we solve the system of equations for initial relative wealth level $\omega_{i,t} = \frac{1}{2}$, which completes the procedure.

The system is solved for the set of parameters displayed in Table 1. The values are chosen to facilitate the understanding of the main implications of this paper and are not intended as a calibration.[16] Therefore, the results presented in the following sections should be interpreted qualitatively rather than quantitatively.

5 Portfolio performance

This section is dedicated to the analysis of portfolio performance. We start by investigating static model implications and proceed by studying dynamic effects.

[15] We use a third order polynomial for interpolation

[16] An important step to calibrate the model is to increase the number of agents to allow for a more heterogeneous set of filtration. We leave this to future work as the introduction of more agents is computationally challenging.

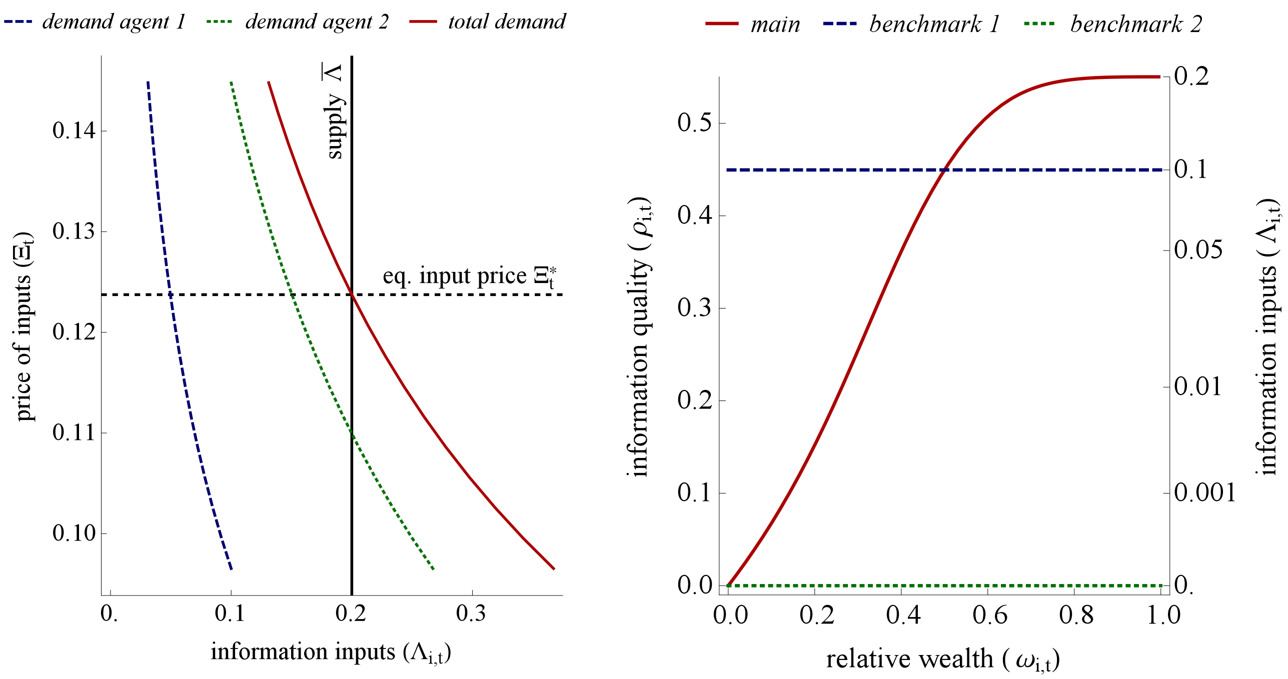

Figure 3: The market for information inputs. Left panel (a): (total) demand and supply for information inputs and equilibrium input price. Supply of inputs is fixed. Demand is downward sloping and increases with an agent's *absolute* wealth. We arbitrarily set $W_{1,t} = \frac{2}{5}$ and $W_{2,t} = \frac{3}{5}$ to illustrate this point. Right panel (b): Equilibrium information (input) allocation is an increasing function of *relative* wealth when agents compete for a limited supply of inputs. The double y-axis highlights the one-to-one mapping between information quality and information inputs (see Figure 1).

5.1 Static effects

A general feature of CRRA preferences is that agents who exhibit these preferences take on more *absolute* risk when they are wealthier. Consequently, deep-pocketed agents take larger speculative positions and thus value information about risky securities more. In addition, richer agents have a larger budgetary capacity to invest in information. These two effects contribute to the positive interdependence between information acquisition and *(absolute)* wealth as documented in Peress (2003).

Important in this paper is the assumption that information production requires inputs, that are in limited supply. Prices of inputs, being a function of aggregate demand, are are determined endogenously. The market for information inputs is depicted in Figure 3a. Richer agents value information more and "push up" the price of information inputs, while poorer agents value

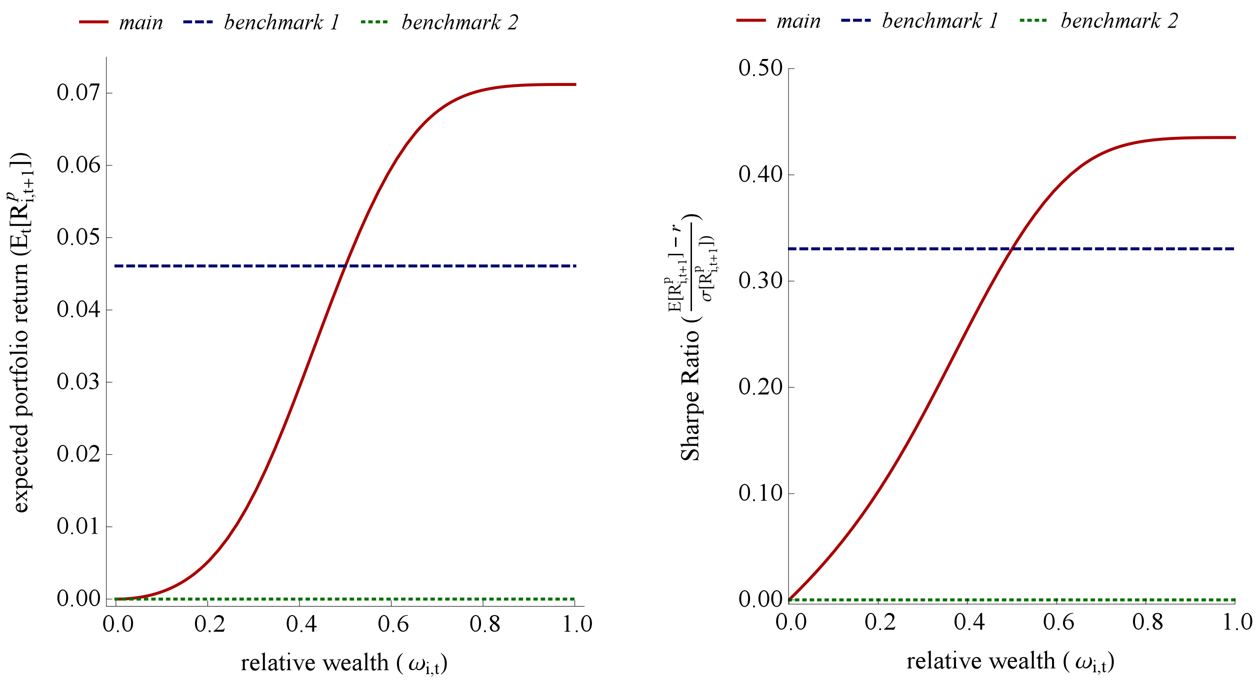

Figure 4: Portfolio performance. Left panel (a): expected returns and relative wealth. Right panel (b): Sharpe ratio and relative wealth. Wealthier agents achieve superior portfolio performance since they purchase more information.

information less and "pull down" this price. When equilibrium in the information input market prevails, information (input) allocation is an increasing function of *relative* wealth.

Proposition 1. *(information allocation): Information quality increases with an agent's relative wealth.*

Figure 3b shows the equilibrium input allocation as a function of relative wealth. We express the level of information allocation as a function of inputs on the right vertical axis. The corresponding level of information quality (see Figure 1) is displayed on the left axis. Note that in the two benchmark models, the level of relative wealth is irrelevant for the equilibrium information allocation. In particular, agents in these economies operate in autarky and acquire a fixed number of inputs (either $\frac{\bar{\Lambda}}{2}$ or 0 for Benchmarks 1 and 2, respectively) regardless of their relative position.

Information allows investors to allocate their wealth to the most profitable investment opportunities. Relatively prosperous agents acquire more information and therefore enjoy superior

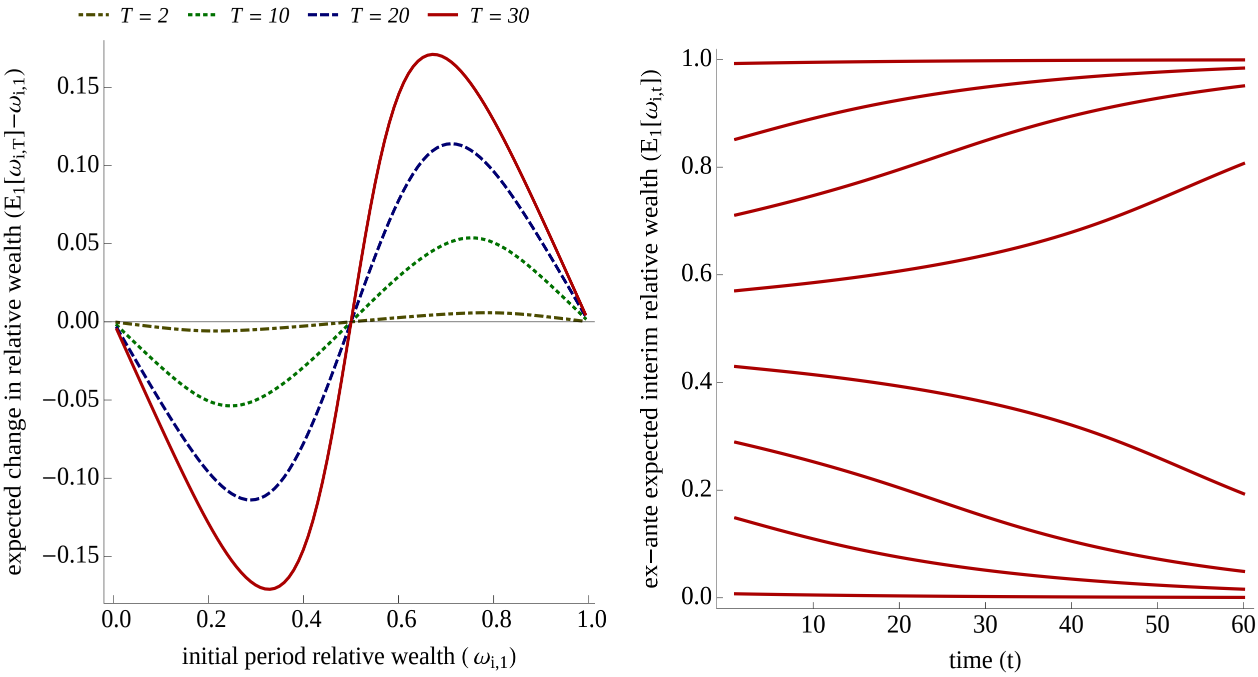

Figure 5: Inequality as a function of trading horizon and market efficiency. Top left panel (a): expected change in relative wealth and trading horizon. Top right panel (b): Expected relative wealth paths for various initial relative wealth levels for $T = 60$. Richer (poorer) agents are expected to become comparatively richer (poorer) over time.

portfolio performance. Figure 4 shows the expected portfolio returns and Sharpe ratios as a function of relative wealth.

Proposition 2. *(performance): Portfolio expected returns and Sharpe ratios increase with an agent's relative wealth.*

5.2 Dynamic effects

When agents trade sequentially, current gains from trade are added to the budget constraint of the next period. Agents who have performed well in the past periods have become wealthier and therefore increase their spending on information. This creates a feedback loop in which relatively wealthy agents acquire more information, obtain superior portfolio performance, and become relatively even wealthier. The situation is shown in Figure 5a. Agents who start off richer (poorer) than average will expect to face an increase (decrease) in relative wealth. The size of this effect increases in the number of trading periods.

Proposition 3. *(feedback effect): Agents who have performed well (badly) in the past, are expected to perform even better (worse) in future periods.*

The situation is furthermore clarified in Figure 5b, which shows several (ex-ante) expected relative wealth paths as a function of time. To better understand this graphic, consider the following example. Suppose an agent starts with 71% of total wealth. Then she is expected to possess 83% and 92% of total wealth by $t = 30$ and $T = 60$, respectively. Generally, when $T \to \infty$, the initially most (least) affluent agent expects to hold 100% (0%) of total wealth in the final period.[17]

In sum, this section has shown that competition for a limited supply of information creates a feedback loop that enlarges relative wealth differences. This generates a *way of life in which agents are caught up in a fiercely competitive struggle for wealth or information*, i.e., "The Information Rat Race."[18]

6 Relative performance concerns and risk-taking

This section demonstrates the way in which the competition for information generates relative performance concerns. We show how these performance concerns evolve over time and investigate their impact on investors' risk-taking.

6.1 Relative wealth concerns

Relative wealth concerns are caused by future competition for information inputs. Indeed, when agents start with identical initial wealth, their interim relative wealth level is determined by historical relative portfolio performance. Agents who have performed better in the past

[17] When we extend this model to multiple agents, this section literally predicts a "winner takes it all" equilibrium, in which one agent ends up with all wealth. In reality, however, agents can learn about multiple securities and may specialize in different securities (see Nieuwerburgh and Veldkamp (2010)), which would lead to multiple winners on different degrees of horizontal specialization.

[18] According to the Oxford Dictionary, a "rat race" is defined as "way of life in which agents are caught up in a fiercely competitive struggle for wealth or power."

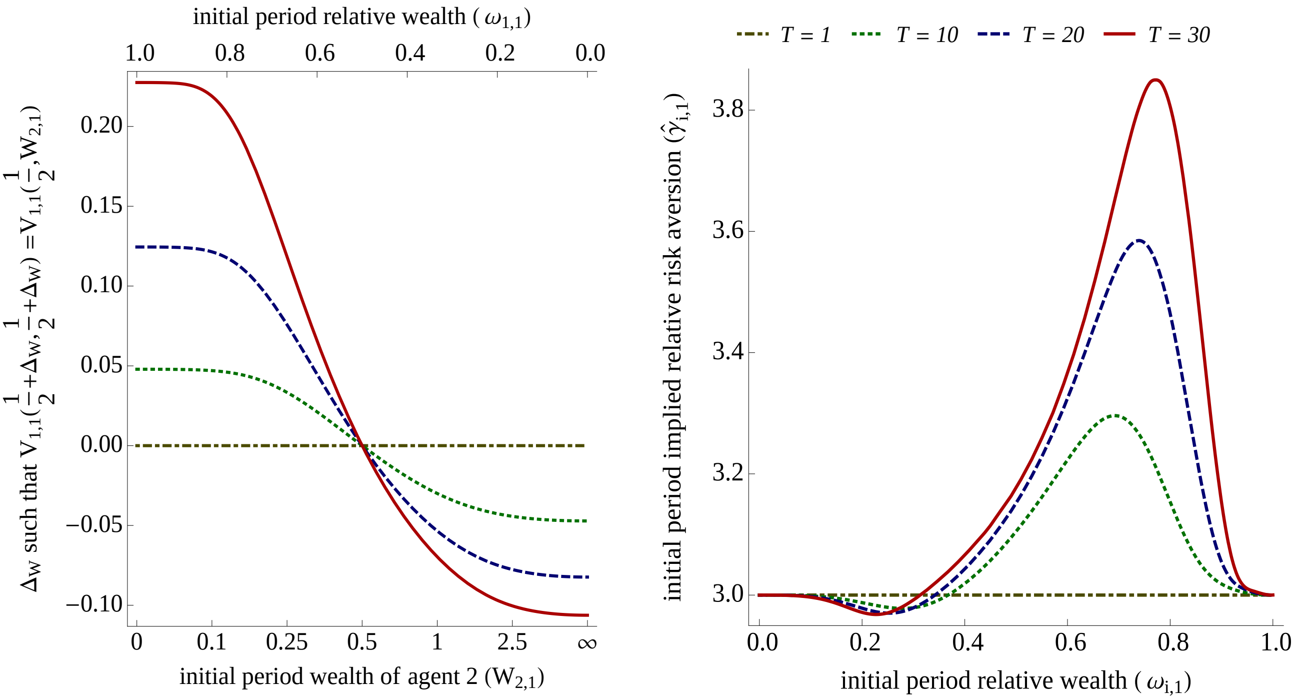

Figure 6: Relative performance preferences and risk-taking. Left panel (a): Relative wealth preferences: amount of wealth Δ_W that makes agent 1 between trading in an economy with symmetric wealth $W_1 = W_2 = \frac{1}{2} + \Delta_W$ and trading in an asymmetric economy with wealth $W_1 = \frac{1}{2}$ as a function of W_2. Right panel (b): Implied relative risk aversion at starting period as a function of (relative) starting wealth: $\hat{\gamma}_{i,t}\left(W_{i,t}, W_{j,t}\right) = -W_{i,t}\frac{\partial^2 V(W_{i,t},W_{j,t})/\partial W_{i,t}^2}{\partial V(W_{i,t},W_{j,t})/\partial W_{i,t}}$. Agents face lower utility and take on more (less) risk when others perform better (worse).

are presently more prosperous and impose a pecuniary externality by increasing the equilibrium price of information inputs. Following a similar line of argument, investors who have performed badly in the past impose negative price pressure. Thus, for a given level of their own performance, agents are better off when their peers perform worse.

Figure 6a shows the size of relative performance concerns as a function of trading horizon. Relative performance concerns are stronger when the horizon increases due to the larger number of future trading periods in which agents compete for information. Note that the feedback effect of Proposition 3 aggravates the effect by enlarging the relative wealth differences over time.

Agents in our model are worse off when others are wealthier, corresponding to the jealousy-type of preference denoted in Dupor and Liu (2003). Agents in our model, however, care about relative *wealth* rather than relative *consumption*. On that account, our agents behave as if they derive utility from "status" (see e.g., Bakshi, Gurdip and Chen (1996)). In contrast to this stream of literature, agents in our model are trapped in a feedback loop that aggravates wealth inequalities. Depending on their relative wealth position and trading horizon, agents modify their risk attitude in an attempt to weaken or strengthen this feedback effect. The next subsection studies this risk-shifting behavior of agents in detail.

6.2 Risk-taking

The shift in risk-taking is best explained by the endogenous desire of an investor to outperform her peers: Suppose agents start with identical levels of wealth and therefore buy the same quality of information. Since signals are imperfectly correlated across investors, realized portfolio returns may differ across agents. Unlucky agents have underperformed their peers, and thus face a lower relative wealth level in the next period. Due to the feedback effect described in Figure 5b, these agents are *expected* to underperform their peers even more in future periods.

We emphasize that the feedback loop only holds in expectation, because portfolio returns and realized signals are uncertain. Moreover, agents can act on their relative positions by adjusting their portfolio risk by buying more/less risky assets or purchasing more/less information. To quantify this risk-taking, we define the *implied relative risk aversion* $\hat{\gamma}_{i,t}$:

$$\hat{\gamma}_{i,t}\left(W_{i,t}, W_{j,t}\right) = -W_{i,t} \frac{\frac{\partial^2 V(W_{i,t}, W_{j,t})}{\partial W_{i,t}^2}}{\frac{\partial V(W_{i,t}, W_{j,t})}{\partial W_{i,t}}} \tag{11}$$

The implied *relative* risk aversion is relevant because it determines the *fraction* of wealth allocated to risky and riskless securities. Specifically, our measure computes the amount of relative risk an agent is willing to undertake: Agents with lower (higher) implied relative risk aversion adopt higher (lower) portfolio risk. Note that in the absence of risk-shifting effects—as is the case in Merton (1971)—$\hat{\gamma}_{i,t}$ is constant and corresponds to the coefficient for risk aversion imposed in the investor's utility function.

We plot the implied relative risk aversion for several values of relative wealth and horizons in Figure 6. Note that $\hat{\gamma}_{i,t} = \gamma_i$ for $T = 1$. Indeed, in a static model, there is no competition for future information, and hence, risk-taking is not affected. For larger horizons, however, the amount of risk an agent is willing to undertake depends on her relative wealth position. Interim losers fear their inability to acquire future information and take on more risk in an attempt to catch up with interim winners. In contrast, interim winners protect their lead and take on less risk. The larger the horizon, the stronger the feedback effect and therefore the larger the impact on interim risk-taking.

Proposition 4. *(interim risk-taking): Interim losers (winners) increase (decrease) risk-taking. The size of this effect increases in trading horizon.*

In the next section, we show how a similar type of risk-taking behavior affects strategic information choice. We resume the discussion about investor risk-taking in Section 10.

7 Complementarities in Information Acquisition

In this section, we show how relative performance concerns affect strategic information choice. The analysis of strategic information choice requires the computation of the slopes of the best response curves. Slopes cannot be obtained by solving our dynamic system, since its solution consists of a single fixed point only. To obtain the necessary comparative statics, we solve the model for various off-equilibrium values. This allows us to make statements about the slopes of the best reaction curves.

We demonstrate that complementarities in information acquisition can arise even in the absence of equilibrium asset prices. Our result contributes to the related literature in which complementarities typically arise due to changes in price informativeness (see Section 2). We break our line of reasoning into three successive arguments.

First, agents face a decrease in future expected relative wealth when others acquire more information. As a matter of fact, information acquisition increases *absolute* expected portfolio returns at the cost of higher *absolute* volatility. This translates into a higher expected but also more volatile *relative* wealth share for the acquiring agent. Consequently, the non-acquiring agent still faces a higher volatility in relative wealth, but a lower expected relative wealth level.

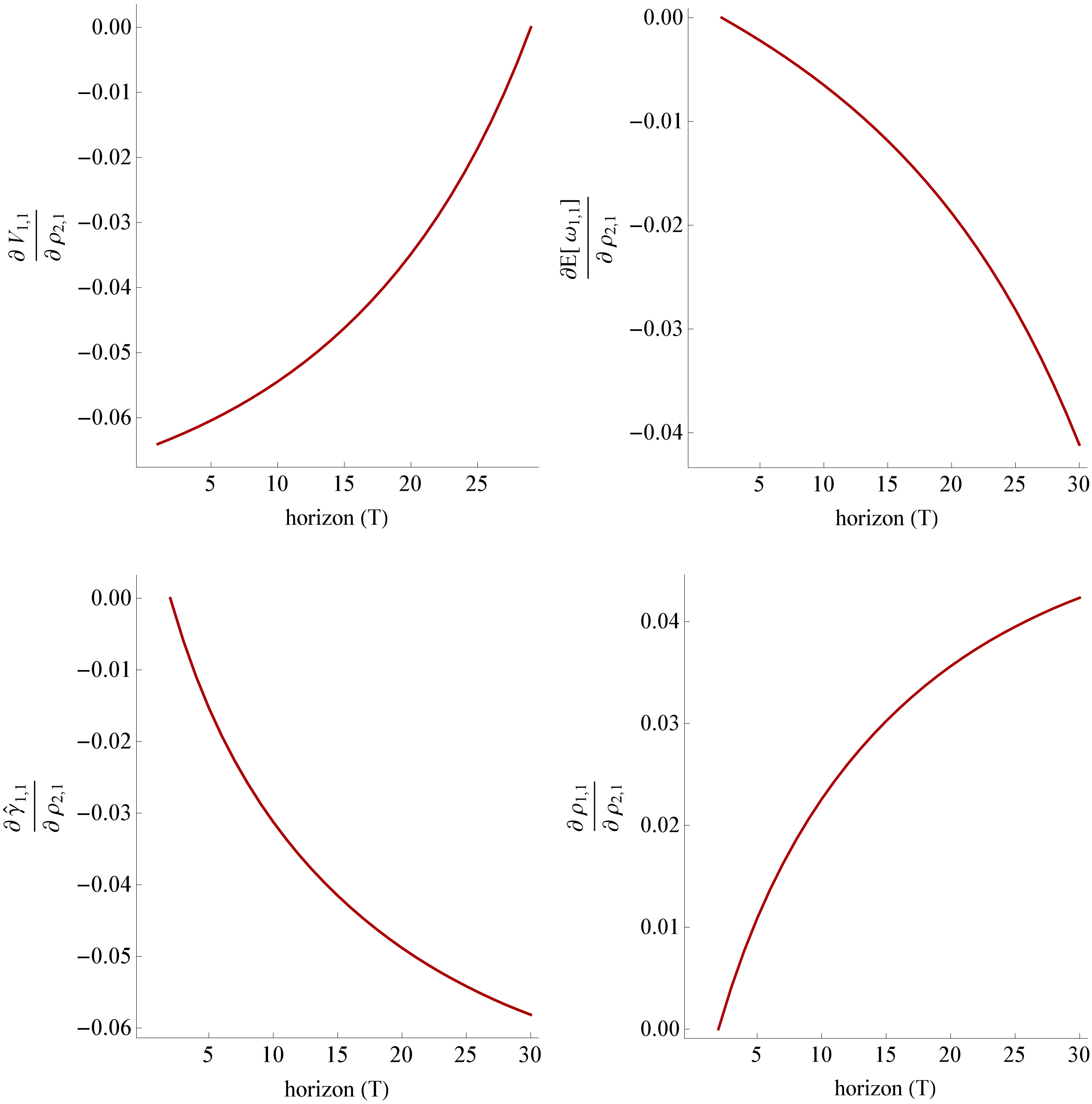

Figure 7: Strategic information acquisition: Top left panel (a): Impact of initial period information choice of other agent on (own) expected terminal wealth share. Top right panel (b): Impact of initial period information choice of other agent on (own) value. Bottom left panel (c): Impact of initial period information acquisition of other agent on (own) initial period implied relative risk aversion $\hat{\gamma}_{i,1}$, where $\hat{\gamma}_{i,t}\left(W_{i,t}, W_{j,t}\right) = -W_{i,t}\frac{\partial^2 V(W_{i,t},W_{j,t})/\partial W_{i,t}^2}{\partial V(W_{i,t},W_{j,t})/\partial W_{i,t}}$. Bottom right panel (d): Impact of initial period information acquisition of other agent on (own) information acquisition. The larger the trading horizon, the stronger the reinforcing (complementarity) effect of information purchase.

Due to the feedback effect of Proposition 3, the resulting differences in relative wealth are further enlarged in future periods. The longer the trading horizon, the larger the loss in expected wealth share and consequent loss in value of the non-acquiring agent. The situation is depicted in Figures 7a and 7b.

Second, agents take on more risk when their peers acquire more information. The argumentation is similar to that behind Proposition 4 and is constructed as follows: Suppose agents start with identical levels of wealth. Consistent with the previous paragraph, agent 1 faces a lower expected terminal wealth share when agent 2 buys more information.[19] This impact on wealth share increases in horizon as a result of the feedback effect. In an attempt to break this feedback effect, it is in agent 1's interest to take on more portfolio risk, implying a decreased risk aversion. Figure 7c shows the positive relationship between the amount of information acquired by agent 2 on the implied risk aversion of agent 1. Similar to the effect on expected relative wealth, the impact on risk-taking is increasing in horizon.

Last, the increase in risk tolerance of the non-acquiring agent is cause for larger speculative trading positions. As such, the non-acquiring agent values information about risky investments more and therefore acquires more information. We conclude that acquiring information is a reinforcing action that makes it a strategic complement. Note that the complementarity effect is exclusively driven by the feedback loop generated by future competition for information (see Figure 7d). Static or myopic models with similar settings (exogenous asset prices) cannot generate similar strategic effects due to the lack of future interaction between agents.

Proposition 5. *(complementarities in information acquisition): Information acquisition is a strategic complement for agents. The size of this effect increases in trading horizon.*

These results imply that complementarities in information acquisition can arise due to risk-taking incentives induced by dynamic competition for information. In contrast, the literature typically regards information as a strategic substitute for agents (e.g., Grossman and Stiglitz (1980) and Verrecchia (1982)). In their setting, equilibrium asset prices partially reflect private

[19] All comparisons are made with respect to the equilibrium point. Hence, to be specific, we investigate the case in which the agent decides to increase the level of information acquired from ρ^* to $\rho^* + \Delta_\rho$, where ρ^* is the equilibrium level of information for ex-ante identical agents at the initial period.

information. Thus, the ability of uninformed agents to free-ride on private information reflected in the equilibrium asset prices decreases their incentive to buy information on their own behalf. Papers that depart from this point of view provide modifications of Grossman and Stiglitz (1980), which cause asset prices to aggregate information differently (see Section 2). This can lead to situations in which information acquisition decreases the informativeness of the equilibrium price, stimulating others to buy more information. We contribute to this stream of work by showing that complementarities can arise even in the absence of equilibrium asset prices.

8 Welfare

In this section, we analyze the welfare generated by the rat race for information. Central to our welfare analysis is the role of the equilibrium effect between agents generated by a shared market for information inputs. To highlight the role of this effect, we contrast our our model against two benchmark economies (see Figure 2). In Benchmark 1, agents acquire information but operate entirely in isolation, since information markets are *separated* across agents. In Benchmark 2, the information market is *non-existent* and thus agents cannot acquire information.

The structure of this section is as follows: First, we introduce a measure of welfare that allows us to compare welfare across different relative wealth levels and trading horizons. Second, we perform a welfare analysis from the perspective of ex-ante identical agents as a function of trading horizon.

8.1 Welfare measure

This paragraph analyzes welfare effects as a function of trading horizon and relative wealth. The direct use of the value function is not an ideal measure of welfare: On the one hand, the value function is by definition higher for richer individuals, which prevents us from properly analyzing the incremental benefit of relative wealth across multiple wealth levels. On the other

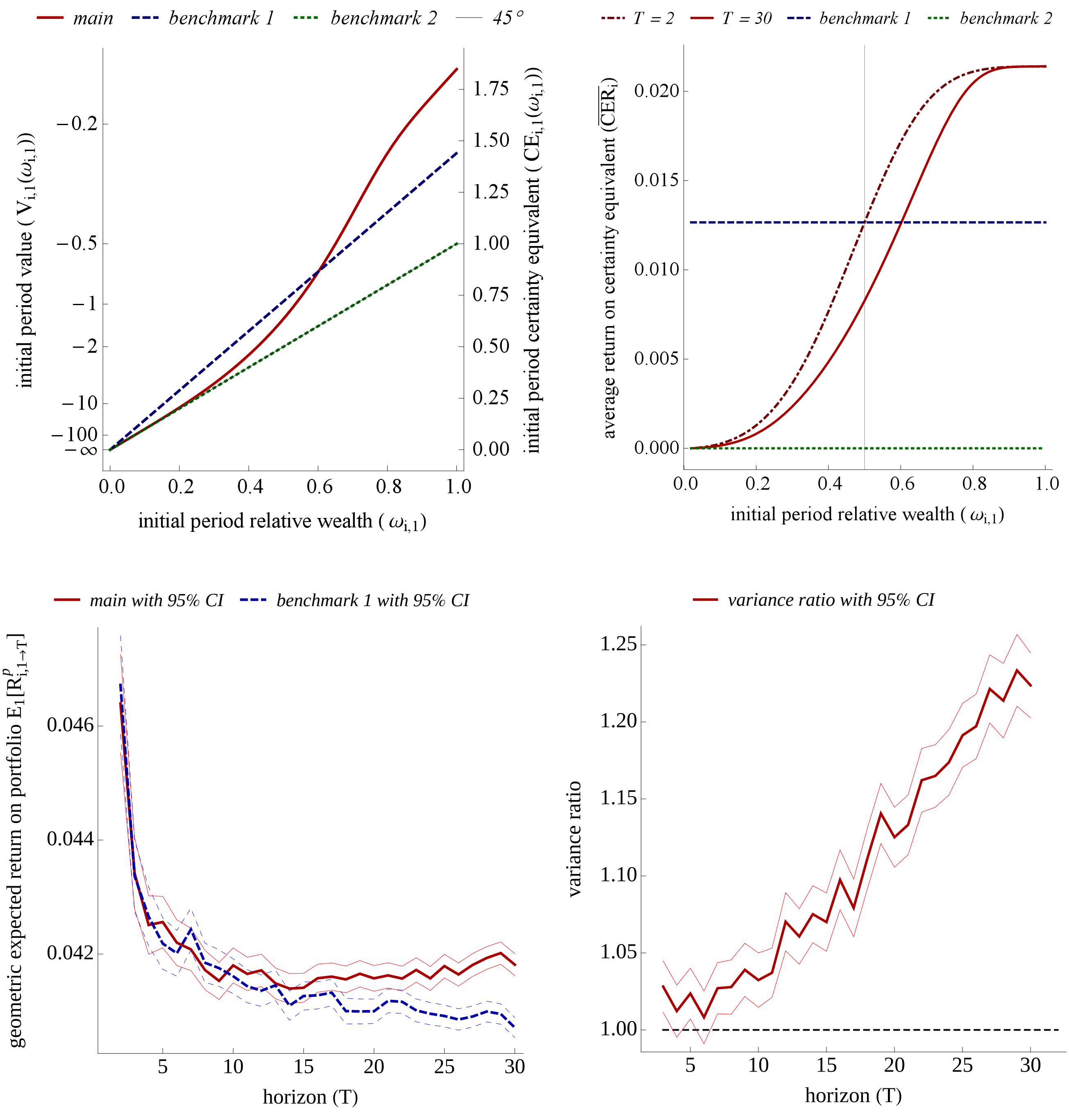

Figure 8: Welfare analysis: Top left panel (a): Welfare expressed in initial period value and certainty equivalent: $CE_{i,t} = U^{-1}(V_{i,t}(\omega_{i,t}))$. Top right panel (b): Welfare expressed in average return on certainty equivalent: $\overline{CER}_i = \left(\frac{U^{-1}(V(\omega_{i,1}))}{\omega_{i,1}}\right)^{\frac{1}{T}} - 1$. Bottom left panel (c): Expected return on portfolio (geometric average). Bottom right panel (d): Ratio of variance of main model terminal wealth and variance of Benchmark 1 terminal wealth. Our model with shared information markets yields higher expected, but more volatile portfolio performance. The total welfare creation is lower than in the first benchmark economy with separated information markets.

hand, the value function is increasing in trading horizon, which imposes a similar problem of properly comparing the value function across multiple horizons.

We construct our welfare measure in three steps. First, we transform the value function into certainty equivalent units. This measure corresponds to the number of consumption units that would make the agent indifferent between direct consumption or trading in the respective economy. We define the certainty equivalent (CE) as:

$$CE_{i,t}\left(\omega_{i,t}\right) = U^{-1}\left(V_{i,t}\left(\omega_{i,t}\right)\right) \tag{12}$$

Figure 8a shows the link between the value function and the CE. While the CE measures the amount of welfare in consumption units, it is still a function of relative wealth.[20] Our second step consists of dividing the CE by $\omega_{i,t}$, to allow fair comparisons across relative wealth levels. We define this ratio as the return on certainty equivalent (CER):

$$CER_{i,t}\left(\omega_{i,t}\right) = \frac{U^{-1}\left(V_{i,t}\left(\omega_{i,t}\right)\right)}{\omega_{i,t}} - 1 \tag{13}$$

Note that CER is naturally increasing in trading horizon. To compare welfare across horizons, we need a measure that is not affected by horizon directly. We address this issue—corresponding to the third step of our procedure—by using the geometric average of the CER[21]

$$\overline{CER}_{i}\left(\omega_{i,t}\right) = \left(\frac{U^{-1}\left(V_{i,1}\left(\omega_{i,1}\right)\right)}{\omega_{i,1}}\right)^{\frac{1}{T}} - 1 \tag{14}$$

Agents operating in any of the two benchmark economies always acquire a predetermined quality of information. Portfolio performance is in these economies independent of trading horizon or relative wealth level. Therefore, the $\overline{CER}$ of these economies is independent of relative wealth and time horizon as well. It now suffices to compare the $\overline{CER}$ of the main model to the constant levels of $\overline{CER}$ produced in the benchmark economies.

[20]Since aggregate initial wealth is fixed at 1, relative wealth and absolute initial wealth are identical

[21]In particular, for $T = 1$, we have $CER = \frac{U^{-1}(U(\omega_{i,1}))}{\omega_{i,1}} - 1 = 0$.

We next demonstrate the $\overline{CER}$ measure by comparing welfare across relative wealth levels. Figure 8b shows the impact of relative wealth on value for a fixed level of aggregate wealth. When comparing our main model with Benchmark 1, we find that richer agents benefit more from a shared market for information than poorer ones. Indeed, in line with our findings in section 6, agents in our main model are worse off when others are richer. Relative wealth concerns do not arise in Benchmark 1, since agents operate in autarky. Trivially, Benchmark 2 provides the lowest value due to the absence of information input markets.

8.2 Horizon effects

We next perform a welfare analysis from the perspective of the ex-ante identical agent. Since the value functions of the agents coincide at the initial node, no assumption on aggregation weights is needed to perform a welfare analysis. This greatly facilitates our welfare analysis compared to models with ex-ante heterogeneous agents.

Figure 8b shows the welfare impact expressed in $\overline{CER}$ for different horizons. Recall that the $\overline{CER}$ is constant for the two benchmark economies. Agents with ex-ante identical levels of wealth favor economies with *separate* information input markets (over *shared* markets) for larger trading horizons. Indeed, while for $T = 2$, identical agents are indifferent between the main model and Benchmark 1, for $T = 30$, the $\overline{CER}$ of our main model is significantly lower.

In an attempt to better understand this loss in welfare, we separately analyze expected returns and volatility generated by the main model and Benchmark 1. We simulate the economy and compute mean portfolio expected returns and volatility. Confidence intervals on these statistics are obtained using the bootstrapping methodology.

Figure 8c plots the expected returns per period.[22] We observe that for short horizons, the expected returns between the main model and the benchmark model are statistically indistinguishable. However, for longer periods, the main model yields significantly higher returns than Benchmark 1 does. This result is explained by the fact that richer agents are able to acquire

[22]geometric average

more information when information markets are shared. Consequently, larger shares of wealth attract more information and thus grow faster in expectation. Hence, from an ex-ante point of view, agents face higher expected returns on their portfolio in the main model.

Figure 8d plots the ratio of variances of terminal wealth in the main model to variances of terminal wealth in Benchmark 1. We observe that the main model produces a larger volatility of terminal wealth, which translates into a large ex-ante volatility on portfolio returns. Since agents prefer Benchmark 1 over the main model for large horizons, the disutility generated by the volatility of expected returns outweighs the benefit from a higher expected portfolio return.

Proposition 6. *(welfare): A shared market for information (main model) yields lower welfare than separated markets for information (Benchmark 1) .*

Proposition 7. *(risk and return trade-off): A shared market for information (main model) yields higher expected, but more volatile portfolio returns than separated markets for information (Benchmark 1) .*

In sum, a shared market for information leads to an increase in expected portfolio returns but also in portfolio volatility. Ex-ante identical agents fear the downside of the volatility effect more than they value the upside of larger expected returns. Therefore, the welfare generated in an economy with shared markets for information is lower than in an economy with separated markets for information.

9 Application I: Income inequality

In this section, we apply our model to the literature on capital income inequality. A growing literature has identified an increased income inequality across households. The increase in inequality has been documented by, among others, Piketty (2003) and Atkinson et al. (2011). Papers that explain capital income inequality with informational differences include Arrow (1987), Peress (2003), and Kacperczyk et al. (2014). Most closely related to our paper is

Kacperczyk et al. (2014), who explains capital inequality with an empirical increase in dispersion of investor "skill."

This paper contributes to the above literature by modeling an economy in which informational differences (or dispersion in "skill") arise endogenously. To explain why these differences are more prominent in financial markets currently than a few decades ago, we enrich the model with trading frictions. Jones (2002) shows that trading friction such as transaction fees and liquidity costs have diminished over time. We show that this reduction in frictions increases the dispersion of information allocation and hence increases capital income inequality.

We model trading frictions by a liquidity cost on speculative trade. As a consequence, the price of the risky security is adjusted as follows:[23]

$$S_{i,t} = 1 + \underbrace{\tau \times \lambda_{i,t}^{S} \times \omega_{i,t}}_{trading\, costs} \tag{15}$$

where τ represents the size of trading frictions or liquidity cost.[24] Trading frictions decrease the incentive to trade in risky securities. Rich agents take larger positions in risky securities and are therefore disproportionally affected by trading frictions. The reduction in speculative trading leads to a decreased value for information. This reduces the information acquisition gap between rich and poor agents (see Figure 9a).

Proposition 8. *(dispersion in skill): The dispersion in information quality is higher in markets with lower trading frictions.*

A reduced dispersion in information acquisition creates a decrease in the heterogeneity of portfolio performance. Indeed, Figure 9b shows that while trading frictions reduce the expected returns for most traders, poor agents perform slightly better due to the more equally allocated

[23] To avoid the inclusion of additional state variables, we normalize transaction costs by a factor $\bar{W}_t = W_{1,t} + W_{2,t}$.

[24] To correct for the reduction in wealth arising from trading frictions when computing our results, we refund the trading frictions to the agent who has incurred the costs. Importantly, agents do not anticipate this reimbursement when deriving their first-order conditions. This technique is similar to the one employed in Buss and Dumas (2014) and is consistent with our assumption that agents act as price takers.

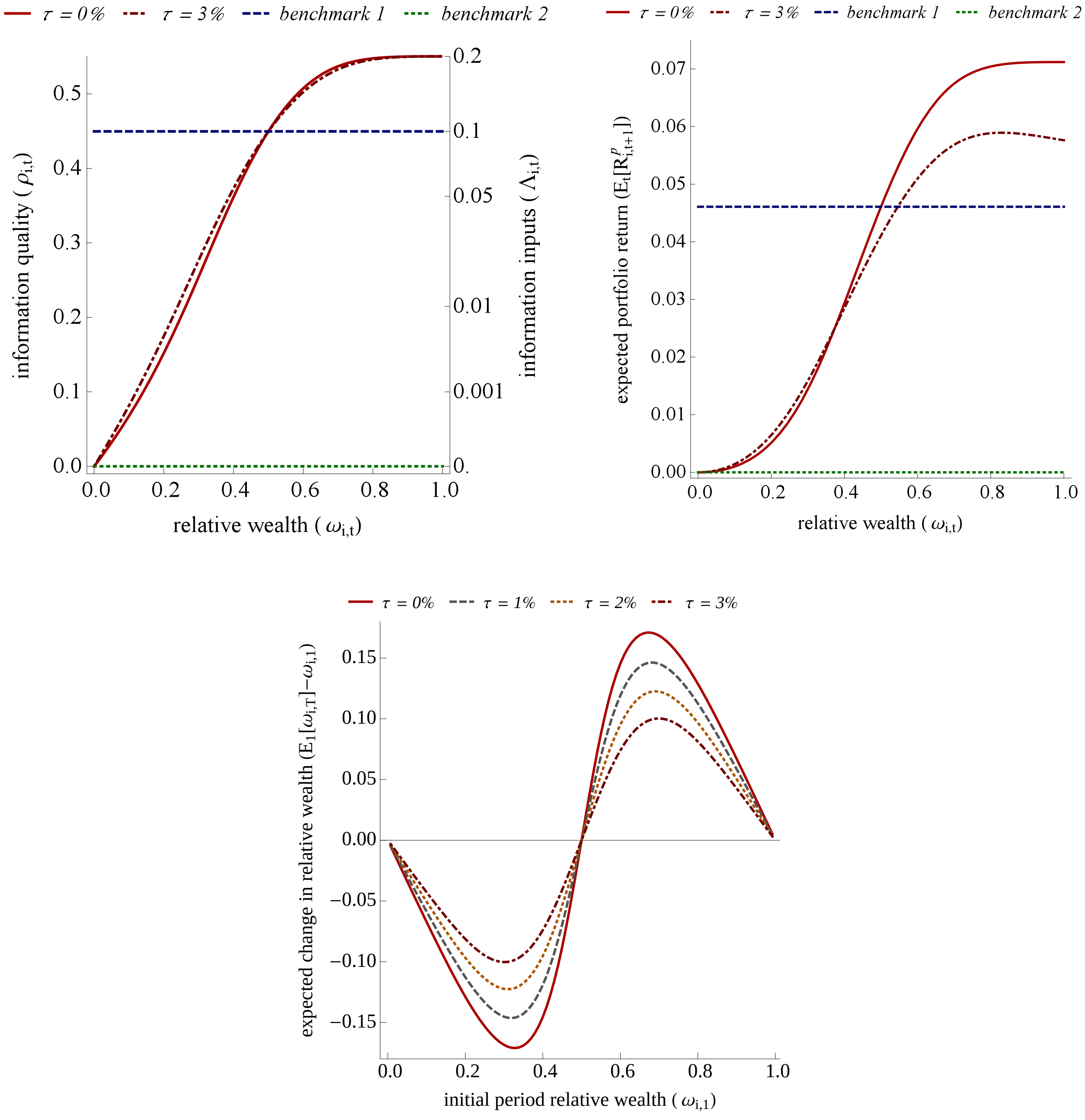

Figure 9: Information allocation, performance and trading frictions: Top left panel (a): Information allocation and trading frictions. Top right panel (b): Expected portfolio returns and trading frictions. Bottom panel (c): expected change in relative wealth and trading frictions. Trading frictions reduce the asymmetry of portfolio performance, and reduce capital income inequality.

information stock. Consequently, the feedback loop that amplifies relative wealth differences is weaker (see Figure 9c).

Proposition 9. *(capital income inequality): Expected capital income inequality is higher in markets with lower trading frictions.*

In sum, the empirically documented reduction in trading frictions contributes to an increased dispersion in information allocation or investor "skill." This increases the rate at which richer agents outperform poorer agents and helps explaining why capital income inequality has risen in the past few decades.

10 Application II: Delegated portfolio management

We next relate our model to the literature on delegated portfolio management. We proceed in two steps: First, we make predictions about the size of the actively managed industry. Second, we relate the risk-taking behavior documented in Section 6 to the empirical literature on manager risk-taking.

In this section, we assume that agents represent portfolio managers rather than individual investors. We simplify the analysis by assuming that there are no agency frictions and that managers' incentives are perfectly aligned with incentives of their clients. A more realistic model, which would include these agency frictions, is outside the scope of this paper.[25] [26]

10.1 Industry Size

While the size of the actively managed portfolio industry is declining as investors switch to passive strategies, the absolute size of the active industry is still very large (see French (2008)

[25]A recent model in which agency frictions stemming from delegated portfolio management are modeled from an equilibrium point of view is presented in Schumacher (2014).

[26]Nonetheless, by ignoring agency frictions, we demonstrate at the very least that our results are driven by the information rat race and not by imperfect incentive contracts.

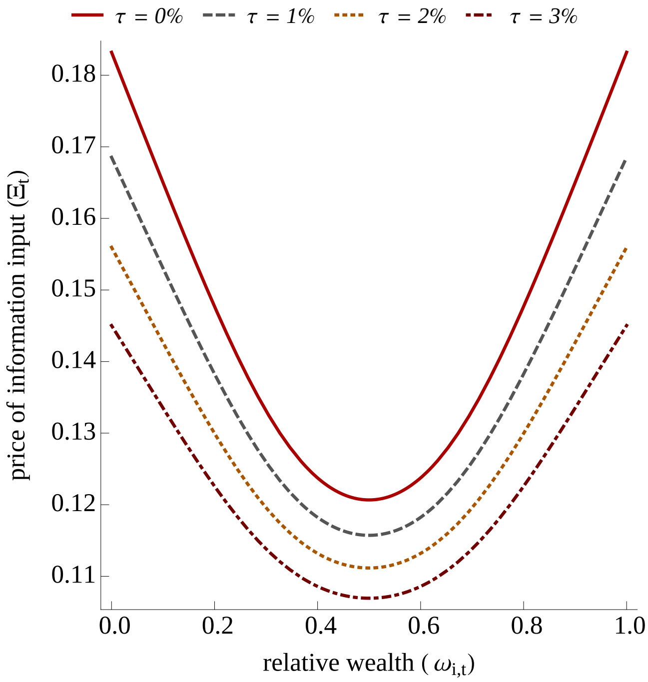

Figure 10: Equilibrium price of information inputs as a fraction of total wealth for various levels of trading frictions. Input prices are larger in economies with less trading frictions and higher wealth dispersion.

and Pastor et al. (2014)). We proceed by reporting the implications of our model for the size of this industry.

We proxy to the size of the active portfolio management industry by the fraction of total wealth spent on information. We show that there are two channels through which the empirically documented decrease in trading frictions affects the industry size.

First, notice that the presence of trading frictions decreases an investor's ability to exploit information. This trivially reduces the equilibrium price of information. The effect is displayed in Figure 10: For all levels of relative wealth, trading frictions reduce the equilibrium price of information.

The second channel is driven by the feedback effect of Proposition 3. Figure 10 shows that information has the highest price when agents have unequally distributed levels of wealth. The effect is driven by the increasing returns to scale for information that follow from the assumed CRRA preference structure (see Peress (2003)). A reduction in trading frictions increases the

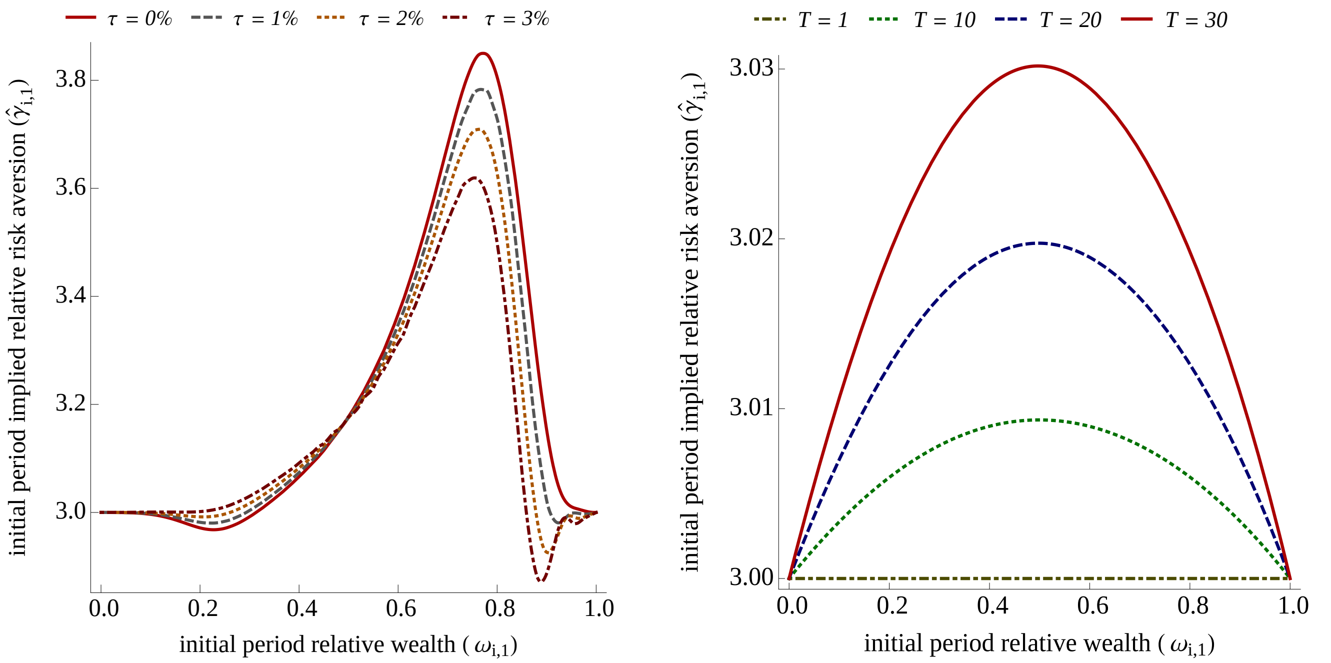

Figure 11: Relative performance preferences and risk-taking. Left panel (a): Implied relative risk aversion, relative wealth, and trading frictions. Trading frictions reduce the impact on risk-taking caused by dynamic competition for information. Right panel (b): implied relative risk aversion in case investors compete for consumption instead of information. Risk-taking incentives that arise from this exogenously imposed preference, with utility $U(W_{1,T}) = \frac{W_{1,T}}{W_{1,T}+W_{2,T}}$, are different to those generated by our model.

likelihood of the economy ending up with unequally distributed wealth levels (see Figure 9c). This further increases the total amount of wealth spent on information.

Proposition 10. *(size of industry): The fraction of wealth spent on information is larger when trading frictions are lower.*

10.2 Portfolio manager risk-taking

In Section 6, we found that—in an attempt to catch up with their peers—interim losers take on more risk, while interim winners protect their lead and take on less risk. This risk-taking behavior is consistent with the empirical findings of Brown et al. (1996) and Elton et al. (2003). In prior work, incentive contracts based on relative performance have been used to explain this conduct. In our model, however, we show that this "tournament-type" risk-taking arises not only for managers, but also for individual investors.

In addition, we argue that compensation-induced incentive schemes that trigger tournament-type risk-taking for managers are incentive aligning. Consider the case in which portfolio managers are myopic while their clients have long investment horizons. Figure 6b shows that risk-taking incentives for short-term managers are different than for long-term investors. In particular, long-term clients exhibit tournament risk-taking behavior as described in 4 while short-term managers do not change their risk-taking behavior as a function of relative performance. To align risk-taking incentives, clients need to stimulate their managers to take on more risk when they have performed poorly and to take on less risk when they have performed well.

Furthermore, Figure 11a shows that tournament-type risk-taking is more prominent in markets with lower trading frictions. This indicates that incentive schemes that align short-term managers' incentives with long-term clients' incentives, should be used more intensively in markets with low trading frictions.

Finally, we point out that the tournament-type risk-taking incentives do not coincide with those that arise when investors compete for consumption instead of information. Figure 11b shows the implicit risk aversion in an economy—without information markets—in which only one consumption unit is available. In contrast to our results, competition for consumption increases the implicit risk aversion over the entire cohort, with no difference between interim winners and interim losers.

11 Conclusion

We study a dynamic economy in which agents face the joint problem of information acquisition and portfolio choice. Our innovative assumption is that we impose market clearing conditions on inputs required to produce information and thereby derive information prices endogenously. Our analysis focuses on the dynamic implications for financial decision-making and welfare generated by competition for information inputs, i.e., *the information rat race.*

Our model yields several theoretical implications. We show that the repeated competition for information creates a feedback loop in which relatively wealthy agents acquire more information, obtain superior portfolio performance, and become comparatively even wealthier. Two dynamic effects arise: First, interim losers fear their inability to acquire future information and take on more risk in an attempt to catch up with interim winners, while interim winners take on less risk to protect their lead. In addition, we show that complementarities in information can arise in this setting. In contrast to the literature, complementarities are driven by endogenous relative performance concerns rather than changes in the level of price informativeness.

We then apply our model to related issues. First, we build on the literature on income inequality by linking this phenomenon with trading frictions. Next, we discuss the implications of our model for the size of the actively managed portfolio industry. Last, we discuss to what extent our model can explain empirically documented manager risk-taking and discuss some implications for portfolio manager risk-taking.

Our model contains numerous simplifications. Some modeling assumptions are imposed by the technical limitations of our method. For instance, the inclusion of additional agents or long-lived information requires an expanded set of state variables.[27] While our method does not rule out supplementary state variables, more powerful hardware would be required to deal with the resulting computational hurdles.

Other modeling assumptions are simplifications that serve to maintain focus on our core arguments. For instance, our paper does not model endogenous asset prices, intermediate consumption, or long-lived securities. While it is a non-trivial task to include these effects, we propose a method that deals with these issues in Breugem (2014).

We finalize this paper by discussing some extensions of the model that are currently on our research agenda. First, one could introduce a "technology" variable that describes the capability of an investor to transform inputs to information. This "technology" variable would serve as a TFP factor on the production function of information: The better the technology, the fewer inputs are needed to produce information.

[27]Note that we expect that the introduction of more agents increases the size of the complementary effect of figure 7d: With many agents, the best response curve will be subject to the (weighted) average of information acquisition of all agents. The size of the negative externality and its consequence on risk-taking should increase in the number of agents acquiring information (the price impact is larger). Hence, in an extended model with more agents, the complementarity effect will be potentially stronger than in the two agent case.

There are several ways such a variable could be modeled. For example, one could assume that the variable is a function of past investments in information. In this case, the technology variable would describe another dimension in which investors could outperform each other. This "double rat race" between investors potentially creates interesting spillovers to financial decision-making. Another way to model the technology variable is to allow agents to directly invest in technology. In this case, the agent solves a joint problem between (i) portfolio choice, (ii) information acquisition, and (iii) financial innovation. For an initial discussion regarding this extension, we refer the reader to Appendix D.

A second extension is the inclusion of more risky assets about which agents can learn. The setting would be ideally exploited with multiple agents. We expect that agents herein face an additional trade-off between hedging and specialization. Compared with existing papers in this field such as Nieuwerburgh and Veldkamp (2010), the limited supply of information inputs would most likely result in further specialization.

Finally, we could model an economy in which not only investors, but also firms compete for a common input. The rat race effect discussed in this paper would then influence the real side of the economy. For example, one could investigate the impact of a "too large" financial sector on the stability of the economic system. In such a model, the financial sector would not act as just a "side-show," as has been traditionally claimed by the macro-economic literature.

References

Abel, A. B., 1990. Asset Prices under Habit Formation and Catching Up with the Joneses. American Economic Review 80 (2), 38–42.

Acemoglu, D., 1999. Changes in Unemployment and Wage Inequality: an Alternative Theory and Some Evidence. American Economic Review 89, 1259–1278.

Acemoglu, D., 2002. Directed Technical Change. Review of Economic Studies 69, 781–809.

Admati, A. R., Pfleiderer, P., Oct. 1987. Viable allocations of information in financial markets. Journal of Economic Theory 43 (1), 76–115.

Allen, B., Jul. 1986. General Equilibrium with Information Sales. Theory and Decision 21 (1), 1–33.

Alvaredo, Facundo, Atkinson, Piketty, Saez, et al, 2013. The Top 1 Percent in International and Historical Perspective. Journal of Economic Perspectives 27, 1–21.

Arrow, K., 1987. The Demand for Information and the Distribution of Income. Probability in the Engineering and Informational Sciences 1 (3-13).

Atkinson, A., Piketty, T., Saez, E., 2011. Top Incomes over a Century or More. Journal of Economic Literature.

Autor, D., Dorn, D., 2013. The Top 1 Percent in International and Historical Perspective. American Economic Review 103, 1553–1597.

Autor, D., Katz, L., Kearney, M., 2006. The Polarization of the US Labor Market. Working Paper.

Autor, D., Katz, L., Kearney, M., 2008. Trends in US Wage Inequality: Revising the Revisionists. Review of Economics and Statistics 90, 300–323.

Avdis, E., Feb. 2012. Information Trade-Offs in Dynamic Financial Markets. SSRN Electronic Journal.

Bakshi, Gurdip and Chen, Z., 1996. The Spirit of Capitalism and Stock-Market Prices. American Economic Review 86 (1), 133–157.

Barlevy, G., Veronesi, P., 2000. Information Acquisition in Financial Markets. Review of Economic Studies 67 (1), 79–90.

Basak, S., Makarov, D., 2013. Competition among Portfolio Managers and Asset Specialization. Working Papers.

Basak, S., Pavlova, A., Aug. 2013. Asset Prices and Institutional Investors. American Economic Review 103 (5), 1728–1758.

Berk, J. B., Oct. 1997. The Acquisition of Information in a Dynamic Market. Economic Theory 9 (3), 441–451.

Brennan, M. J., 1993. Agency and Asset Pricing. Working Paper.

Breugem, M., 2014. Portfolio Choice with Endogenous Information: A Dynamic General Equilibrium Model with Transaction Costs. Working Paper.

Breugem, M., Djordjevic, M., 2014. Dynamic Portfolio Choice with Information Acquisition. Working Paper.

Brown, K. C., Harlow, W. V., Starks, L. T., Mar. 1996. Of Tournaments and Temptations: An Analysis of Managerial Incentives in the Mutual Fund Industry. The Journal of Finance 51 (1), 85–110.

Brown, S. J., Goetzmann, W. N., Park, J., Oct. 2001. Careers and Survival: Competition and Risk in the Hedge Fund and CTA Industry. The Journal of Finance 56 (5), 1869–1886.

Buss, A., Dumas, B., 2014. The Equilibrium Dynamics of Liquidity and Illiquid Asset Prices. Working Paper.

Carpenter, J. N., Oct. 2000. Does Option Compensation Increase Managerial Risk Appetite? The Journal of Finance 55 (5), 2311–2331.

Chevalier, J., Ellison, G., 1997. Risk Taking by Mutual Funds as a Response to Incentives. Journal of Political Economy 105 (6), 1167–1200.

Chevalier, J., Ellison, G., May 1999. Career Concerns of Mutual Fund Managers. The Quarterly Journal of Economics 114 (2), 389–432.

Cole, H. L., Mailath, G. J., Postlewaite, A., 1992. Social Norms, Savings Behavior, and Growth. Journal of Political Economy 100 (6), 1092–1125.

Cole, H. L., Mailath, G. J., Postlewaite, A., 2001. Investment and concern for relative position. Journal of Political Economy 100, 1092–125.

Cuoco, D., Kaniel, R., 2011. Equilibrium Prices in the Presence of Delegated Portfolio Management. Journal of Financial Economics 101 (2), 264–296.

DeMarzo, P., Kaniel, R., Kremer, I., 2007. Technological Innovation and Real Investment Booms and Busts. Journal of Financial Economics 85 (3), 735–754.

Demarzo, P. M., Kaniel, R., Kremer, I., 2004. Diversification as a Public Good: Community Effects in Portfolio Choice. Journal of Finance 59 (4), 1677–1716.

DeMarzo, P. M., Kaniel, R., Kremer, I., 2008. Relative Wealth Concerns and Financial Bubbles. Review of Financial Studies 21 (1), 19–50.

Diamond, D. W., Verrecchia, R. E., Sep. 1981. Information Aggregation in a Noisy Rational Expectations Economy. Journal of Financial Economics 9 (3), 221–235.

Dupor, B., Liu, W.-F., Mar. 2003. Jealousy and Equilibrium Overconsumption. American Economic Review 93 (1), 423–428.

Elton, E., Gruber, M., Blake, C., April 2003. Incentive fees and mutual funds. Journal of Finance 58 (2), 778–804.

French, K. R., Aug. 2008. Presidential Address: The Cost of Active Investing. The Journal of Finance 63 (4), 1537–1573.

Froot, K. A., Scharfstein, D. S., Stein, J. C., Sep. 1992. Herd on the Street: Informational Inefficiencies in a Market with Short-Term Speculation. The Journal of Finance 47 (4), 1461–1484.

Gali, J., 1990. Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices. Journal of Money, Credit and Banking 26, 1–8.

Ganguli, J. V., Yang, L., Mar. 2009. Complementarities, Multiplicity, and Supply Information. Journal of the European Economic Association 7 (1), 90–115.

Garcia, D., Strobl, G., Sep. 2010. Relative Wealth Concerns and Complementarities in Information Acquisition. Review of Financial Studies 24 (1), 169–207.

Gomez, J.-P., Priestly, R., Zapatero, F., Dec. 2009. Implications of Keeping-Up-with-the-Joneses Behavior for the Equilibrium Cross Section of Stock Returns: International Evidence. The Journal of Finance 64 (6), 2703–2737.

Grossman, S. J., Stiglitz, J. E., 1976. Information and Competitive Price Systems. American Economic Review 66 (2), 246–53.

Grossman, S. J., Stiglitz, J. E., 1980. On the Impossibility of Informationally Efficient Markets. American Economic Review 70 (3), 393–408.

Hellwig, M. F., 1980. On the Aggregation of Information in Competitive Markets. Journal of Economic Theory 22 (3), 477–498.

Jackson, M. O., Jan. 1991. Equilibrium, Price Formation, and the Value of Private Information. Review of Financial Studies 4 (1), 1–16.

Jones, C., 2002. A Century of Stock Market Liquidity and Trading Costs. Working Paper.

Kacperczyk, M., Nosal, J. B., Stevens, L., 2014. Investor Sophistication and Capital Income Inequality. Working Paper.

Katz, L., Autor, D., 1999. Changes in the Wage Structure and Earnings Inequality. Handbook of Labor Economics 3, 1463–1555.

Kempf, A., Ruenzi, S., Dec. 2007. Tournaments in Mutual-Fund Families. Review of Financial Studies 21 (2), 1013–1036.

Kyle, A. S., Xiong, W., Aug. 2001. Contagion as a Wealth Effect. The Journal of Finance 56 (4), 1401–1440.

Makarov, D., Schornick, A. V., Sep. 2010. A note on Wealth Effect Under CARA Utility. Finance Research Letters 7 (3), 170–177.

Massa, M., Apr. 2002. Financial Innovation and Information: The Role of Derivatives When a Market for Information Exists. Review of Financial Studies 15 (3), 927–957.

Mehra, R., Prescott, E. C., 1985. The Equity Premium: A Puzzle. Journal of Monetary Economics 15 (2), 145–161.

Mele, A., Sangiorgi, F., Sep. 2011. Uncertainty, Information Acquisition and Price Swings in Asset Markets. SSRN Electronic Journal.

Merton, R., 1971. Optimum Consumption and Portfolio Rules in a Continuous-Time Model. Journal of Economic Theory 3 (4), 373–413.

Nieuwerburgh, S. V., Veldkamp, L., 2010. Information Acquisition and Under-Diversification. Review of Economic Studies 77 (2), 779–805.

Niu, Z., Sep. 2013. Relative Performance Concern, Attention Allocation and Complementarities in Information Acquisition. SSRN Electronic Journal.

Opp, C. C. G., 2008. Intertemporal Information Acquisition and Asset Market Dynamics. Working Paper.

Pastor, L., Stambaugh, R., Taylor, L., 2014. Scale and Skill and Active Management. Working Paper.

Peress, J., Oct. 2003. Wealth, Information Acquisition, and Portfolio Choice. Review of Financial Studies 17 (3), 879–914.

Piketty, T., 2003. Income Inequality in France, 1901–1998. Journal of Political Economy 111, 1004–1042.

Piketty, T., Saez, E., 2003. Income Inequality in the United States, 1913–1998. The Quarterly Journal of Economics 118, 1–41.

Robson, A. J., 1992. Status, the Distribution of Wealth, Private and Social Attitudes to Risk. Econometrica 60 (4), 837–57.

Romer, P. M., 1990. Endogenous Technological Change. Journal of Political Economy 98 (5), S71–102.

Schoenberg, E. J., Haruvy, E., 2012. Relative performance information in asset markets: An experimental approach. Journal of Economic Psychology 33 (6), 1143–1155.

Schumacher, D., 2014. Contagion and Decoupling in Intermediated Financial Markets. Working Paper.

Sirri, E. R., Tufano, P., Oct. 1998. Costly Search and Mutual Fund Flows. The Journal of Finance 53 (5), 1589–1622.

Turmuhambetova, G., 2005. Decision Making in an Economy with Endogenous Information. PhD Dissertation.

Vayanos, D., 1999. Strategic Trading and Welfare in a Dynamic Market. Review of Economic Studies 66 (2), 219–54.

Veblen, T., 1899. The Theory of the Leisure Class: An Economic Study of Institutions. Macmillan.

Veldkamp, L. L., Jul. 2006a. Information Markets and the Comovement of Asset Prices. Review of Economic Studies 73 (3), 823–845.

Veldkamp, L. L., Jun. 2006b. Media Frenzies in Markets for Financial Information. American Economic Review 96 (3), 577–601.

Verrecchia, R. E., 1982. Information Acquisition in a Noisy Rational Expectations Economy. Econometrica 50 (6), 1415–30.

Wang, J., 1993. A Model of Intertemporal Asset Prices Under Asymmetric Information. Review of Economic Studies1 60, 249–282.

A Properties of the information production function

With the general assumptions stated in footnote 10, we can show existence of a solution for a more general information production function. First, denote

$$Z(\Lambda_{1,t}) = \frac{\Phi_1'(\Psi_1(\Lambda_{1,t}))}{\Phi_2'\left(\Psi_2\left(\bar{\Lambda} - \Lambda_{1,t}\right)\right)} \tag{A.16}$$

$$\Upsilon(\Lambda_{1,t}) = \frac{\varphi_{2,t}}{\varphi_{1,t}} \times \frac{LHS_1(\Lambda_{1,t})}{LHS_2(\Lambda_{1,t})} \tag{A.17}$$

where LHS_1 represents the left hand side of equation 2, using $\Lambda_{i,t} = \Phi(\rho_{i,t}) = \Psi^{-1}(\rho_{i,t})$.

Theorem. *If Ψ is invertible (continuous), increasing, strictly concave, and satisfies $\Psi'(0) = \infty$, $\Psi(0) = 0$ and $\Psi(\infty) = 1$, and if Υ is strictly positive and finite, there exists a value of $\Lambda_{1,t} \in \left(0, \bar{\Lambda}\right)$ for which $Z(\Lambda_{1,t}) = \Upsilon(\Lambda_{1,t})$.*

Corollary. In this case its inverse Φ is increasing, strictly convex and satisfies $\Phi'(0) = 0$, $\Phi(0) = 0$ and $\Phi(1) = \infty$.

Proof. The proof consist of several steps:

1. $\lim\limits_{\Lambda_{1,t}\to 0} \Psi_1(\Lambda_{1,t}) = 0$ and therefore $\lim\limits_{\Lambda_{1,t}\to 0} \Phi_1'(\Psi_1(\Lambda_{1,t})) = 0$
2. $\lim\limits_{\Lambda_{1,t}\to 0} \Psi_2\left(\bar{\Lambda} - \Lambda_{1,t}\right) > 0$ and therefore $\lim\limits_{\Lambda_{1,t}\to 0} \Phi_1'\left(\Psi_2\left(\bar{\Lambda} - \Lambda_{1,t}\right)\right) > 0$
3. By lines $1-2$, $\lim\limits_{\Lambda_{1,t}\to 0} Z(\Lambda_{1,t}) = \frac{\Phi_1'(\Psi_1(\Lambda_{1,t}))}{\Phi_2'\left(\Psi_2\left(\bar{\Lambda}-\Lambda_{1,t}\right)\right)} = 0$
4. $\lim\limits_{\Lambda_{1,t}\to \bar{\Lambda}} \Psi_1(\Lambda_{1,t}) > 0$ and therefore $\lim\limits_{\Lambda_{1,t}\to \bar{\Lambda}} \Phi_1'(\Psi_1(\Lambda_{1,t})) > 0$
5. $\lim\limits_{\Lambda_{1,t}\to \bar{\Lambda}} \Psi_2\left(\bar{\Lambda} - \Lambda_{1,t}\right) = 0$ and therefore $\lim\limits_{\Lambda_{1,t}\to \bar{\Lambda}} \Phi_1'\left(\Psi_2\left(\bar{\Lambda} - \Lambda_{1,t}\right)\right) = 0$
6. By lines $4-5$, $\lim\limits_{\Lambda_{1,t}\to \bar{\Lambda}} Z(\Lambda_{1,t}) = \frac{\Phi_1'(\Psi_1(\Lambda_{1,t}))}{\Phi_2'\left(\Psi_2\left(\bar{\Lambda}-\Lambda_{1,t}\right)\right)} = \infty$
7. Since $\Upsilon(\Lambda_{1,t})$ is finite and strictly positive for any value of $\Lambda_{1,t}$ and since $Z(\Lambda_{1,t})$ is continuous and satisfies lines 3 and 6, there exist a $\Lambda_{1,t} \in \left(0, \bar{\Lambda}\right)$ for which $Z(\Lambda_{1,t}) = \Upsilon(\Lambda_{1,t})$

□

B Benchmark economies

In this appendix we derive the first order conditions for the two benchmark models. For brevity, we only list the first order conditions and summarize the system for each of these economies.

Benchmark model 1: Separated market for information

The first benchmark allows agents to acquire information in the same way as in the main model, except that each agent faces a separate market of inputs. There are 8 equations for each agent to solve:

There are two F.O.C.'s with respect to $\lambda^B_{i,t}$:

$$\sum_{y_{i,t},\eta_{i,t+1}} \pi_i\left(\rho_{i,t}, y_{i,t}, R_{i,t+1}\right) \times \partial_1 V_{i,t+1}\left(W_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \times R_{i,t+1}\right)\right) = \varphi_{i,t} \quad \forall y_{i,t} \tag{B.1}$$

There are two F.O.C.'s with respect to $\lambda^S_{i,t}$:

$$\sum_{y_{i,t},\eta_{i,t+1}} \pi_i\left(\rho_{i,t}, y_{i,t}, R_{i,t+1}\right) \times R_{i,t+1} \times \partial_1 V_{i,t+1}\left(W_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \times R_{i,t+1}\right)\right) = \varphi_{i,t} \quad \forall y_{i,t} \tag{B.2}$$

There is one F.O.C. with respect to $\rho_{1,t}$[28]:

$$\sum_{y_{i,t},y_{j,t},\eta_{i,t+1},\eta_{i,t+1}} \partial\pi_1\left(y_{1,t}, R_{1,t+1}\right) \times V_{i,t+1}\left(W_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \times R_{i,t+1}\right)\right) = \Xi_{i,t} \times \Phi'_i\left(\rho_{i,t}\right) \times \varphi_{i,t} \tag{B.3}$$

There are two budget constraints:

$$W_{i,t} \times \left(1 - \lambda^S_{i,t} - \lambda^B_{i,t}\right) = \Xi_{i,t} \times \Phi_i\left(\rho_{i,t}\right) \quad \forall y_{i,t} \tag{B.4}$$

There is one market clearing equation for the individual information market:

[28]See footnote 14

$$\Phi_i(\rho_{i,t}) = \frac{\bar{\Lambda}}{2} \tag{B.5}$$

There are 8 unknowns: 2 for $\lambda^S_{i,t}$, 2 for $\lambda^B_{i,t}$, 2 for $\varphi^S_{i,t}$, 1 for $\rho_{1,t}$ and 1 for Ξ_t.

Benchmark model 2: No information

The second benchmark model assumes that no information acquisition is possible. For each agent, the system is defined as follows: There is one F.O.C.'s with respect to $\lambda^B_{i,t}$:

$$\frac{1}{2} \times \sum_{\eta_{i,t+1}} \partial_1 V_{i,t+1}\left(W_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \times R_{i,t+1}\right)\right) \quad = \varphi_{i,t} \tag{B.6}$$

There is one F.O.C.'s with respect to $\lambda^S_{i,t}$:

$$\frac{1}{2} \times \sum_{\eta_{i,t+1}} R_{i,t+1} \times \partial_1 V_{i,t+1}\left(W_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \times R_{i,t+1}\right)\right) \quad = \varphi_{i,t} \tag{B.7}$$

There is one budget constraints:

$$W_{i,t} \times \left(1 - \lambda^S_{i,t} - \lambda^B_{i,t}\right) = 0 \tag{B.8}$$

There are 3 unknowns: $\lambda^S_{i,t}$, $\lambda^B_{i,t}$ and $\varphi_{i,t}$.

C Wealth fraction as state variable

In this section we show how we use the bounded state variable $\omega_{i,t} = \frac{W_{1,t}}{W_{1,t}+W_{2,t}} \in (0,1)$ to solve the system. Denote by $\bar{W}_t = W_{1,t} + W_{2,t}$ the total wealth in the economy. We can rewrite equation system 3 as follows:

$$\begin{aligned}
& V_{i,t}\left(W_{i,t}, W_{j,t}\right) = \\
& \max_{\rho_{i,t},\{\lambda^B_{i,t}\},\{\lambda^S_{i,t}\}} \sum_{y_{i,t},y_{j,t},\eta_{i,t+1},\eta_{i,t+1}} \pi_1\left(\rho_{1,t}, y_{1,t}, R_{1,t+1}\right) \times \pi_2\left(\rho_{2,t}, y_{2,t}, R_{2,t+1}\right) \times V_{i,t+1}\left(W_{i,t+1}, W_{j,t+1}\right) \\
& \text{such that}: \quad W_{i,t} \times \left(1 - \lambda^S_{i,t} - \lambda^B_{i,t}\right) = \Xi_t \times \Phi_i\left(\rho_{i,t}\right) \quad \forall y_{i,t} \\
& \qquad W_{i,t+1} = W_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \times R_{i,t+1}\right) \quad \forall i, y_{i,t}
\end{aligned} \tag{C.1}$$

By replacing absolute wealth levels with $\bar{W}_t$ and $\omega_{i,t}$, we can write $V_{i,t+1}\left(W_{i,t+1}, W_{j,t+1}\right) = V_{i,t}\left(\omega_{i,t} \times \bar{W}_t, (1-\omega_{i,t}) \times \bar{W}_t\right)$. To keep notation as short as possible, define the function $H_{i,t}\left(\omega_{i,t}, \bar{W}_t\right) = V_{i,t}\left(\omega_{i,t} \times \bar{W}_t, (1-\omega_{i,t}) \times \bar{W}_t\right)$. Using the value function H and state variables $\bar{W}_t$ and $\omega_{i,t}$ instead of absolute wealth levels $W_{1,t}$ and $W_{2,t}$, the optimization problem can be restated as:

$$\begin{aligned}
& H_{i,t}\left(\omega_{i,t}, \bar{W}_t\right) = \\
& \max_{\rho_{i,t},\{\lambda^B_{i,t}\},\{\lambda^S_{i,t}\}} \sum_{y_{i,t},y_{j,t},\eta_{i,t+1},\eta_{i,t+1}} \pi_1\left(\rho_{1,t}, y_{1,t}, R_{1,t+1}\right) \times \pi_2\left(\rho_{2,t}, y_{2,t}, R_{2,t+1}\right) \times H_{i,t+1}\left(\omega_{i,t+1}, \bar{W}_{t+1}\right) \\
& \text{such that}: \quad \omega_{i,t} \times \bar{W}_t \times \left(1 - \lambda^S_{i,t} - \lambda^B_{i,t}\right) = \Xi_t \times \Phi_i\left(\rho_{i,t}\right) \quad \forall y_{i,t} \\
& \bar{W}_{t+1} = \bar{W}_t \times \left(\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right) + (1-\omega_{i,t}) \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right)\right) \quad \forall y_{i,t} \\
& \omega_{i,t+1} = \frac{\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right)}{\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right) + (1-\omega_{i,t}) \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right)} \quad \forall y_{i,t}
\end{aligned} \tag{C.2}$$

Theorem. *Define* $\Omega_i(x) = x^{1-\gamma} = (1-\gamma) \times U_i(x)$.

Now for all $t \in \{0,..,T-1\}$ *the value function* H *can be rewritten as:*

$$H_{i,t}\left(\omega_{i,t}, \bar{W}_t\right) = \Omega_i(\bar{W}_t) \times \hat{V}_{i,t}\left(\omega_{i,t}\right) \tag{C.3}$$

where the following functions are recursively defined:

$$\hat{V}_{i,t}(\omega_{i,t}) = \mathbb{E}_t\left[\Omega_i\left(\bar{G}_{t+1}(\omega_{i,t})\right) \times \hat{V}_{i,t+1}(\hat{\omega}_{i,t+1}(\omega_{i,t}))\right] \tag{C.4}$$

$$\hat{\omega}_{i,t+1}(\omega_{i,t}) = \frac{\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right)}{\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right) + (1-\omega_{i,t}) \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right)} \tag{C.5}$$

$$\bar{G}_{t+1}(\omega_{i,t}) = \frac{\bar{W}_{t+1}(\omega_{i,t})}{\bar{W}_t} = \omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right) + (1-\omega_{i,t}) \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right) \tag{C.6}$$

Proof. The proof is based on the induction argument. In the first step it is shown that the theorem holds for the last period $T-1$. The second step demonstrates that under the assumption that the theorem holds at time $t+1$, the theorem holds for time t as well.

Step 1: Theorem holds at time $T-1$

- Show that $\hat{\omega}_{i,T}$ and $\bar{G}_T$ solely depend on $\omega_{i,T-1}$ (not on $\bar{W}_{T-1}$)
  - Note that $H_{i,T}\left(\omega_{i,T}, \bar{W}_T\right) = U_i\left(\bar{W}_T \times \omega_{i,T}\right)$. After substituting definitions, the Lagrangian becomes:

$$\begin{aligned} L_{i,T-1} &= \mathbb{E}_{T-1}\left[U_i\left(\bar{W}_{T-1} \times \omega_{i,T-1} \times \left(\lambda^B_{i,T-1} + \lambda^S_{i,T-1} \cdot R_{i,T}\right)\right)\right] \\ &+ \sum_{y_{i,T-1}, y_{j,T-1}} \varphi_{i,T-1} \times \left(\omega_{i,T-1} \times \bar{W}_{T-1} \times \left(1 - \lambda^S_{i,T-1} - \lambda^B_{i,T-1}\right) - \Xi_{T-1} \times \Phi_i(\rho_{i,T-1})\right) \end{aligned} \tag{C.7}$$

  - We rewrite the Lagrangian as:

$$\begin{aligned} L_{i,T-1} &= \Omega_i(\bar{W}_{T-1}) \times \mathbb{E}_{T-1}\left[U_i\left(\omega_{i,T-1} \times \left(\lambda^B_{i,T-1} + \lambda^S_{i,T-1} \cdot R_{i,T}\right)\right)\right] \\ &+ \tfrac{\Omega_i(\bar{W}_{T-1})}{\Omega_i(\bar{W}_{T-1})} \times \bar{W}_{T-1} \times \sum_{y_{i,T-1}, y_{j,T-1}} \varphi_{i,T-1} \times \left(\omega_{i,T-1} \times \left(1 - \lambda^S_{i,T-1} - \lambda^B_{i,T-1}\right) - \tfrac{\Xi_{T-1}}{\bar{W}_{T-1}} \times \Phi_i(\rho_{i,T-1})\right) \end{aligned} \tag{C.8}$$

  - Now denote by $\xi_t = \frac{\Xi_{T-1}}{\bar{W}_{T-1}}$ the price of information inputs as a fraction of the total wealth in the economy and by $\phi_{i,t} = \varphi_{i,T-1} \times \frac{\bar{W}_{T-1}}{\Omega_i(\bar{W}_{T-1})} = \varphi_{i,T-1} \times \bar{W}^{\gamma}_{T-1}$ the transformed state prices. Now:

$$
\begin{aligned}
L_{i,T-1} &= \Omega_i\left(\bar{W}_{T-1}\right) \times \mathbb{E}_{T-1}\left[U_i\left(\omega_{i,T-1} \times \left(\lambda^B_{i,T-1} + \lambda^S_{i,T-1} \cdot R_{i,T}\right)\right)\right] \\
&+\Omega_i\left(\bar{W}_{T-1}\right) \times \sum_{y_{i,T-1},y_{j,T-1}} \phi_{i,t} \times \left(\omega_{i,T-1} \times \left(1 - \lambda^S_{i,T-1} - \lambda^B_{i,T-1}\right) - \xi_{T-1} \times \Phi_i\left(\rho_{i,T-1}\right)\right)
\end{aligned} \tag{C.9}
$$

– Since the term $\Omega_i\left(\bar{W}_{T-1}\right)$ enters solely as a multiplicative term, maximizing L_{T-1} is equivalent to maximizing $\hat{L}_{i,T-1} = \frac{L_{i,T-1}}{\Omega_i(\bar{W}_{T-1})}$

$$
\begin{aligned}
\hat{L}_{i,T-1} &= \mathbb{E}_{T-1}\left[U_i\left(\omega_{i,T-1} \times \left(\lambda^B_{i,T-1} + \lambda^S_{i,T-1} \cdot R_{i,T}\right)\right)\right] \\
&+ \sum_{y_{i,T-1},y_{j,T-1}} \phi_{i,t} \times \left(\omega_{i,T-1} \times \left(1 - \lambda^S_{i,T-1} - \lambda^B_{i,T-1}\right) - \xi_{T-1} \times \Phi_i\left(\rho_{i,T-1}\right)\right)
\end{aligned} \tag{C.10}
$$

– The entire system of equations now consists of the first order conditions of each agent plus the information market clearing condition $\Phi_1\left(\rho_{1,T-1}\right) + \Phi_2\left(\rho_{2,T-1}\right) = \bar{\Lambda}$. Since $\bar{W}_{T-1}$ does not enter the (modified) system, $\lambda^B_{i,T-1}, \lambda^S_{i,T-1}$ and $\rho_{i,T-1}$ can be obtained for every value of $\omega_{i,T-1}$.[29] As a direct consequence, $\hat{\omega}_{i,T}\left(\omega_{i,T-1}\right)$ and $\bar{G}_T\left(\omega_{i,T-1}\right)$ (being only functions of $\lambda^B_{i,T-1}, \lambda^S_{i,T-1}$ and $\omega_{i,T}$) only depend on relative wealth.

- Show that $\hat{V}_{i,T-1}$ solely depends on $\omega_{i,T-1}$ (not on $\bar{W}_{T-1}$)

  – First we decompose H:

$$
\begin{aligned}
H_{i,T-1}\left(\omega_{i,T-1}, \bar{W}_{T-1}\right) &= \mathbb{E}_{T-1}\left[H_{i,T}\left(\omega_{i,T}, \bar{W}_T\right)\right] \\
&= \mathbb{E}_{T-1}\left[U_i(\bar{W}_T \times \omega_{i,T})\right] \\
&= \mathbb{E}_{T-1}\left[\Omega_i(\bar{W}_T) \times U_i\left(\omega_{i,T}\right)\right]
\end{aligned} \tag{C.11}
$$

  – We use the fact that $\omega_{i,T}$ and $\bar{G}_T$ are functions of $\omega_{i,T-1}$ and use the definition of $\hat{V}_{i,T-1}\left(\omega_{i,T-1}\right)$

$$
\begin{aligned}
&= \mathbb{E}_{T-1}\left[\Omega_i(\tfrac{\bar{W}_{T-1}}{\bar{W}_{T-1}} \times \bar{W}_T) \times U_i\left(\hat{\omega}_{i,T}\left(\omega_{i,T-1}\right)\right)\right] \\
&= \mathbb{E}_{T-1}\left[\Omega_i(\bar{G}_T\left(\omega_{i,T-1}\right)) \times \Omega_i(\bar{W}_{T-1}) \times U_i\left(\hat{\omega}_{i,T}\left(\omega_{i,T-1}\right)\right)\right] \\
&= \Omega_i(\bar{W}_{T-1}) \times \hat{V}_{i,T-1}\left(\omega_{i,T-1}\right)
\end{aligned} \tag{C.12}
$$

[29] Indeed, $\varphi_{i,T-1}$ and Ξ_{T-1} do also depend on aggregate wealth. Hence, a simulation of the system is needed (or a backwards solution over a two state variable grid) in order to solve for these variables. Instead, we solve for ξ_t and $\phi_{i,t}$.

– Were we used $\hat{V}_{i,T-1}(\omega_{i,T-1}) = \mathbb{E}_{T-1}\left[\Omega_i\left(\bar{G}_T(\omega_{i,T-1})\right) \times \hat{V}_{i,T}(\hat{\omega}_{i,T}(\omega_{i,T-1}))\right]$

Step 2: Theorem holds at time t if it holds for time $t+1$.

- Assume

$$H_{i,t+1}\left(\omega_{i,t+1}, \bar{W}_{t+1}\right) = \Omega_i(\bar{W}_{t+1}) \times \hat{V}_{i,t+1}(\omega_{i,t+1}) \tag{C.13}$$

- Show that $\hat{\omega}_{i,t+1}$ and $\bar{G}_{t+1}$ solely depend on $\omega_{i,t}$ (not on $\bar{W}_t$)
  - Using the assumption $H_{i,t+1}\left(\omega_{i,t+1}, \bar{W}_{t+1}\right) = \Omega_i(\bar{W}_{t+1}) \times \hat{V}_{i,t+1}(\omega_{i,t+1})$. After substituting definitions, the Lagrangian is given by:

$$\begin{aligned} L_{i,t} = \mathbb{E}_t\Bigg[&\Omega_i(\bar{W}_t \times \omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right) + \bar{W}_t \times (1-\omega_{i,t}) \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right))..\\ &\times \hat{V}_{i,t+1}\left(\frac{\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right)}{\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right) + (1-\omega_{i,t}) \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right)}\right)\Bigg]\\ &+ \sum_{y_{i,t}, y_{j,t}} \varphi_{i,t} \times \left(\omega_{i,t} \times \bar{W}_t \times \left(1 - \lambda^S_{i,t} - \lambda^B_{i,t}\right) - \Xi_t \times \Phi_i(\rho_{i,t})\right) \end{aligned} \tag{C.14}$$

  - Rewrite the Lagrangian as:

$$\begin{aligned} L_{i,t} = \Omega_i\left(\bar{W}_{T-1}\right) \times \mathbb{E}_t\Bigg[&\Omega_i(\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right) + (1-\omega_{i,t}) \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right))..\\ &\times \hat{V}_{i,t+1}\left(\frac{\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right)}{\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right) + (1-\omega_{i,t}) \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right)}\right)\Bigg]\\ &+ \frac{\Omega_i\left(\bar{W}_{T-1}\right)}{\Omega_i\left(\bar{W}_{T-1}\right)} \times \bar{W}_{T-1} \times \sum_{y_{i,t}, y_{j,t}} \varphi_{i,t} \times \left(\omega_{i,t} \times \left(1 - \lambda^S_{i,t} - \lambda^B_{i,t}\right) - \frac{\Xi_{T-1}}{\bar{W}_{T-1}} \times \Phi_i(\rho_{i,t})\right) \end{aligned} \tag{C.15}$$

  - Use definitions of ξ_t, $\phi_{i,t}$ and $\hat{L}_{i,t}$

$$\begin{aligned} \hat{L}_{i,t} = \frac{L_{i,t}}{\Omega_i\left(\bar{W}_{T-1}\right)} = \mathbb{E}_t\Bigg[&\Omega_i(\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right) + (1-\omega_{i,t}) \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right))..\\ &\times \hat{V}_{i,t+1}\left(\frac{\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right)}{\omega_{i,t} \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right) + (1-\omega_{i,t}) \times \left(\lambda^B_{i,t} + \lambda^S_{i,t} \cdot R_{i,t+1}\right)}\right)\Bigg]\\ &+ \sum_{y_{i,t}, y_{j,t}} \phi_{i,t} \times \left(\omega_{i,t} \times \left(1 - \lambda^S_{i,t} - \lambda^B_{i,t}\right) - \xi_t \times \Phi_i(\rho_{i,t})\right) \end{aligned} \tag{C.16}$$

- The entire system of equations now consists of the first order conditions of each agent plus the information market clearing condition $\Phi_1(\rho_{1,t}) + \Phi_2(\rho_{2,t}) = \bar{\Lambda}$. Since $\bar{W}_t$ does not enter the (modified) system, $\lambda^B_{i,t}, \lambda^S_{i,t}$ and $\rho_{i,t}$ can be obtained for every value of $\omega_{i,t}$.[30] As a direct consequence, $\hat{\omega}_{i,t+1}(\omega_{i,t})$ and $\bar{G}_{t+1}(\omega_{i,t})$ (being only functions of $\lambda^B_{i,t}, \lambda^S_{i,t}$ and $\omega_{i,t+1}$) only depend on relative wealth.

- Show that $\hat{V}_{i,t}$ solely depends on $\omega_{i,t}$ (not on $\bar{W}_t$)
  - First we decompose H:

$$\begin{aligned} H_{i,t}\left(\omega_{i,t}, \bar{W}_t\right) &= \mathbb{E}_{T-1}\left[H_{i,t+1}\left(\omega_{i,t+1}, \bar{W}_{t+1}\right)\right] \\ &= \mathbb{E}_{T-1}\left[H_{i,t+1}\left(\omega_{i,t+1}, \bar{W}_{t+1}\right)\right] \end{aligned} \tag{C.17}$$

  - Use assumption:

$$\begin{aligned} &= \mathbb{E}_{T-1}\left[\Omega_i(\bar{W}_{t+1}) \times \hat{V}_{i,t+1}(\omega_{i,t+1})\right] \\ &= \mathbb{E}_{T-1}\left[\Omega_i(\tfrac{\bar{W}_t}{\bar{W}_t} \times \bar{W}_{t+1}) \times \hat{V}_{i,t+1}(\omega_{i,t+1})\right] \\ &= \Omega_i(\bar{W}_t) \times \mathbb{E}_{T-1}\left[\Omega_i(\bar{G}_{t+1}) \times \hat{V}_{i,t+1}(\omega_{i,t+1})\right] \end{aligned} \tag{C.18}$$

  - We use that $\omega_{i,T}$ and $\bar{G}_T$ are functions of $\omega_{i,T-1}$ and use the definition of $\hat{V}_{i,T-1}(\omega_{i,T-1})$

$$\begin{aligned} &= \Omega_i(\bar{W}_t) \times \mathbb{E}_{T-1}\left[\Omega_i\left(\bar{G}_{t+1}(\omega_{i,t})\right) \times \hat{V}_{i,t+1}(\hat{\omega}_{i,t+1}(\omega_{i,t}))\right] \\ &= \Omega_i(\bar{W}_t) \times \hat{V}_{i,t}(\omega_{i,t}) \end{aligned} \tag{C.19}$$

- Were we used $\hat{V}_{i,t}(\omega_{i,t}) = \mathbb{E}_t\left[\Omega_i\left(\bar{G}_{t+1}(\omega_{i,t})\right) \times \hat{V}_{i,t+1}(\hat{\omega}_{i,t+1}(\omega_{i,t}))\right]$

□

[30] Indeed, $\varphi_{i,T-1}$ and Ξ_{T-1} do also depend on aggregate wealth. Hence, a simulation of the system is needed (or a backwards solution over a two state variable grid) in order to solve for these variables. Instead, we solve for ξ_t and $\phi_{i,t}$.

D The market for financial innovations and welfare

The main assumption in our paper is the existence of a market for (short lived) information inputs. It is, however, a strong assumption that information production requires only short term information input. It would be more realistic to assume that the production of information requires long term technological investments such as automated news reading, large data analysis software and powerful hardware. These technologies would facilitate information production by complementing short term inputs such as labor. We refer to long term inputs as "financial innovations" or "technologies." In this section we briefly show how a monopolistic seller of innovations impacts investor's welfare.

On the one hand, innovations increase the productivity of information which should increase the demand for short-term inputs. On the other hand, they are costly and reduce the available wealth for investing in short-term inputs. We show that if the former effect dominates, that is, when financial innovation raises the demand for short-term inputs, then innovation can reduce welfare. The opposite is true when financial innovation lowers the demand for short-term inputs.

We assume a static version of the model described in section 3. The static nature of the model removes relative wealth preferences and eliminates strategic information acquisition. There is a monopolistic seller of one piece of innovation. For simplicity, it is assumed that the innovation is not divisible and, as such, the entire piece is sold to the highest bidder.

D.1 Externalities of innovation and welfare

Let i (without loss of generality) be the innovation-acquiring agent. For the moment, suppose that the innovation does not affect the welfare of agent j. The monopolistic seller extracts full surplus and sets the price of innovation $\hat{\varsigma}$ such that:

$$V_{i,1}^{\{i\}}\left(W_{i,1}-\hat{\varsigma}, W_{j,1}\right)=V_{j,1}^{\{\emptyset\}}\left(W_{i,1}, W_{j,1}\right) \tag{D.1}$$

Where superscripts indicate the set of agents that acquire the innovation. If, however, innovation affects the welfare of agent j, the seller of innovation extracts full surplus and the equilibrium price for innovation ς is given by:

$$V_{i,0}^{\{i\}}\left(W_{i,0}-\varsigma, W_{j,0}\right)=V_{j,0}^{\{j\}}\left(W_{i,0}, W_{j,0}\right) \tag{D.2}$$

The welfare generated by the sale of innovation denoted by Q is now given by:[31]

$$Q=V_{i,0}^{\{i\}}\left(W_{i,0}-\varsigma, W_{j,0}\right)-V_{i,0}^{\{\emptyset\}}\left(W_{i,0}, W_{j,0}\right)=V_{j,0}^{\{j\}}\left(W_{i,0}, W_{j,0}\right)-V_{j,0}^{\{\emptyset\}}\left(W_{i,0}, W_{j,0}\right) \tag{D.3}$$

In words, when the seller of innovation extracts full surplus, the total welfare created for investors corresponds to the size of the externality that the innovation imposes on the non-innovating agent.

D.2 The role of the information market

Innovation has an ambiguous impact of the demand for short-term inputs. On the one hand, it is assumed that innovation has a positive value since it increases the productivity of short term inputs (e.g.: financial experts).[32] On the other hand, the costly acquisition of the innovation reduces the remaining wealth available to spend on short term inputs. It is outside the scope of this appendix section to investigate the conditions under which the former effect dominates. Instead, we show the change in welfare as a function of the dominating effect: We find that innovation increases welfare if and only if innovation increases the demand for information inputs.

[31] The wealth equivalent of this change in welfare equals to $\hat{\tau}-\tau$

[32] For example, the innovation could represent a factor ν which increases the productivity of short-term inputs in the following way:

$$\rho_{i,t}=\Psi\left(\nu, \Lambda_{i,t}\right)=\left(\frac{\nu \times \Lambda_{i,t}}{\nu \times \Lambda_{i,t}+1}\right)^{\frac{1}{\Gamma}}$$

Denote by $\check{\Lambda}_i^Z(\Xi)$ the demand of agent i for short-term inputs when the set of agents Z acquires information. Assume that this demand function is downwards sloping and increasing in innovation:

$$\check{\Lambda}_i^{\{i\}}(\Xi) > \check{\Lambda}_i^{\{\emptyset\}}(\Xi) \qquad \forall \Xi \tag{D.4}$$

In a static model, there are no strategic information acquisition effects nor relative wealth concerns and the demand of the agent who does not acquire innovation is unaffected:

$$\check{\Lambda}_j^{\{i\}}(\Xi) = \check{\Lambda}_j^{\{\emptyset\}}(\Xi) \qquad \forall \Xi \tag{D.5}$$

Since supply of short term inputs is fixed at $\bar{\Lambda}$ and since demand is downwards sloping, the equilibrium price is higher when agent i innovates:[33]

$$\Xi^{\{i\}} > \Xi^{\{\emptyset\}} \tag{D.6}$$

By applying the envelope theorem to equation 4 one can show that a higher price of short-term inputs decreases welfare for agent j:

$$\frac{\partial V_j}{\partial \Xi} = \frac{\partial L_j}{\partial \Xi} = -\sum_{y_i, y_j} \varphi_i \times \Phi_i(\rho_i) < 0 \tag{D.7}$$

Hence, when innovation increases the of demand for short-term inputs, the equilibrium analyst price rises and the non-innovating agent faces a negative externality. Since the seller of innovation extracts the difference of the surplus of the innovating and the non-innovating agent, a reduction in investors' welfare takes place: $\mathcal{H} = V_{j,0}^{\{\emptyset\}}(W_{i,0}, W_{j,0}) - V_{j,0}^{\{j\}}(W_{i,0}, W_{j,0}) < 0$. In contrast, when innovation decreases the price of short-term inputs, the equilibrium price of short-term inputs decreases which imposes a positive externality on the non-acquiring agent leading to a higher level of welfare for investors.

[33] The superscripts indicate the set of agents that acquire the innovation.

Tables

Table 1: Default parameter values

Description	Parameter	Figures	Default Value
Risk aversion	γ_i	All	3
High return of risky asset	R_H	All	$6/5$
Low return of risky asset	R_L	All	$4/5$
Trading horizon	T	2-4, 9a+b, 10	2
		5-8, 9c, 11	30
Information production curvature	Γ	All	3
Total inputs of information production	$\bar{\Lambda}$	All	$1/5$
Total simulations		8	50000

Portfolio Choice with Endogenous Information: A Dynamic General Equilibrium Model with Frictions

Extension to "The Information Rat Race"

Matthijs Breugem*

November 6, 2014

PRELIMINARY AND INCOMPLETE

Abstract

We present a method capable of solving a dynamic equilibrium portfolio choice problem with CRRA preferences and endogenous information acquisition. The main difficulty in dealing with CRRA agents in economies with informational differences is modeling learning from equilibrium asset prices. Our method deals with this problem by assuming that agents have a private valuation for an asset's dividend stream. With this assumption, we do not need to assume that there are noisy traders in order to prevent the Grossman-Stiglitz paradox. Our method is capable of handling a dynamic economy with intermediate consumption, transaction costs and endogenous information costs.

Keywords: information acquisition, dynamic equilibrium, portfolio choice, asset prices

JEL Classification: D53, D63, G11

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1 Introduction

Information plays an important role in financial markets. Incorporating information into asset pricing models is, however, a non-trivial task. The main modeling challenge stems from the dual role that asset prices fulfill in the presence of informational asymmetries. Specifically, equilibrium prices not only serve as an allocation mechanism to clear the asset market, but also as a statistic aggregating private information. This dual role imposes a specific modeling difficulty by requiring the consistency of prices with an agent's beliefs.

Generally, equilibrium asset pricing papers employ the rational expectations method (see Muth (1961)) to tackle this issue.[1] The method consists of conjecturing a price function, which is to be verified in equilibrium. The method is most tractable when the price function is linear, which is the case when agents have constant absolute risk aversion (CARA) preferences and payoffs are normally distributed. Departure from this CARA-normal framework, however, leads to nonlinear price functions, which usually prevent the model from being solved, even when numerical methods are employed.[2]

While analytically tractable, the CARA-normal framework is not very realistic. For example, under CARA preferences, the optimal level of risky investments is independent of wealth. This implies that—in dollar terms—Warren Buffett should hold the same amount of risky investments as a typical US household. While asset pricing models without information choice have employed more realistic preference structures, models that employ rational expectations are practically imprisoned in the CARA-normal world. As a consequence, information choice in financial markets is has received little attention outside of this highly artificial environment.

This paper proposes a modeling method that can incorporate informational asymmetries in a setting with general preferences and non-normally distributed payoffs. The model is easily expandable to a multi-period setting and is capable of handling intermediate consumption and trading frictions such as transaction costs.

[1] Landmark models with perfectly competitive markets include Grossman and Stiglitz (1976, 1980); Hellwig (1980); Admati (1985). The case in which agents act strategically is discussed in Kyle (1985).

[2] Some departures from the CARA normal framework have tractable solutions: Nieuwerburgh and Veldkamp (2010) model an economy with CRRA preferences and log-normal payoffs. Barlevy and Veronesi (2000, 2007) model an economy with risk neutral investors. Our model allows for more general utility specifications.

The key assumption of our model is that agents value asset payoffs differently. This could be, for example, because agents derive dissimilar utility from perks or voting rights associated with the asset. Other sources of valuation differences could be transaction costs, taxes, or private income/endowment shocks. Because there are no noise traders in our model, equilibrium asset prices fully reveal private information. Therefore, investors' information sets are identical and we do not need to conjecture a price function.

Specifically, in our model, heterogeneous private valuations for the risky asset eliminate the need for noise traders. In classical CARA-normal models such as Hellwig (1980) and Verrecchia (1982), noise traders prevent the equilibrium price from perfectly aggregating private information. Consequently, the no-trade equilibrium of Milgrom and Stokey (1982) does not prevail and information has a strictly positive value. In our setting, agent-specific asset valuations ensure trade even when prices perfectly aggregate information. Hence, essentially, we mimic an economy with informational asymmetries in a perfectly revealing environment.

We note some limitations in our method that should be addressed in future research: First, the method does not allow for closed-form solutions and needs to be solved numerically. Second, our method is optimized for two representative agents. The inclusion of more agents is possible, but requires the introduction of more state variables, demanding significantly more computational power. Third, the type of "noise" introduced by private valuations is not identical to the noise supplied by liquidity traders. A profound understanding of the economic difference between these two sources of "noise" is at the top of our research agenda. Keeping in mind the above limitations, however, our method could help scholars explore the relatively unknown terrain of the asset pricing with endogenous information outside of the CARA-normal framework.

The remainder of this paper is organized as follows: Section 2 presents the proposed economic model and section 3 describes the solution technique. Section 4 shows implications for Breugem (2014). Section 5 concludes and provides avenues for future research.

2 Model

In this section, we present the model and derive a system of equations that determine the equilibrium of our economy. Our goal is to demonstrate the method's ability to incorporate features that are hard to model within the traditional rational expectation framework. We introduce a general version of the model and derive the respective system of equations. The use of subscripts and arguments is minimized where possible.

2.1 Agents and assets

We consider a multi-period economy with time $t \in \{1, .., T\}$. The economy is populated with two *ex-ante identical* representative agents who maximize their CRRA utility over their consumption stream $\{c_{i,t}\}$.

There are two investment opportunities (assets) available to each agent. The first asset—"the bond"—trades at price B_t and provides a riskless stream of unit payoffs (coupons). The second asset—"the stock"—trades at equilibrium price S_t and provides its owner with a risky stream of payoffs or dividends $D_{i,t} \in \{D_H, D_L\}$. Key to this section is that agents value these dividends differently, i.e., the risky asset has *agent-specific* dividend payoff trees. The correlation between these private dividend streams is given by $\mathrm{Corr}\left[D_{1,t}, D_{2,t}\right] = \alpha$. In the limit case of $\alpha = 1$, investors value dividends identically. Stock holdings of agent i at time t are denoted by $\theta^S_{i,t}$ while bond holdings are denoted by $\theta^B_{i,t}$.

The model allows for general specifications of the supply-side of assets. We assume constant elasticity of supply with elasticities $E_S, E_B \in [0, \infty]$. In the extreme case of $E_S = E_B = 0$, agents trade in an endowment economy. In the opposite extreme of $E_S = E_B = \infty$, agents operate in a pure production economy.

2.2 Information Acquisition

At every stage, each agent acquires an informative signal $y_{i,t} \in \{H, L\}$ about the next period dividend $D_{i,t+1}$. Each agent only acquires information about his own relevant payoff process.

The "quality" of the signal $y_{i,t}$ is denoted by $\rho_{i,t} \in [0,1]$ and represents the correlation between the signal and next period dividend or cash flow: $\rho_{i,t} = \text{Corr}\left[y_{i,t}, D_{i,t+1}\right]$. Special cases include (i) $\rho_{i,t} = 0$ in which the signal is not informative and (ii) $\rho_{i,t} = 1$ in which case the signal perfectly forecasts future cash flow.

We assume a general cost function, denoted by $\kappa\left(\rho_{i,t}\right)$, which is increasing in the quality of information. Agents face the following trade-off when deciding upon this quality level: On the one hand, purchasing a more precise signal increases the agent's ability to select superior investment opportunities. On the other hand, the acquisition of better information reduces the wealth available for investment and consumption.

2.3 Market Incompleteness

There are four sources of uncertainty in the economy (two for the dividend processes $D_{i,t+1}$ and two from private signals $Y_{i,t}$) and only two assets (the bond and the stock). For this reason, the market is (dynamically) incomplete. Table 3 provides an overview of the probabilities of each state of nature to occur. In the special case that $\alpha = 1$, the dividend process is identical among traders which eliminates one source of uncertainty. In this case, the number of sources of uncertainty is with three, which still greater than the number of investment opportunities. Note that markets are incomplete even when $\alpha = 1$.[3]

2.4 Learning

In this subsection, we describe how information choice affects the filtration of each agent in the economy. Essentially, we mimic an economy with asymmetric information in a perfectly revealing equilibrium. The privately valued asset payoffs create additional motives for trading that prevent the Grossman-Stiglitz paradox from arising.

[3]Moreover, in the special case that $\rho_{1,t} = \rho_{2,t} = 1$, the private signals are perfectly aligned with the dividend process, which reduces the sources of uncertainty with one (per agent). Markets are therefore complete when $\alpha = \rho_{1,t} = \rho_{2,t} = 1$. This case is, however, not of main interest since it results in a no-trade equilibrium.

2.4.1 Learning from Prices

In every period $t < T$, each agent learns about $D_{i,t+1}$ from two sources. First, the agent learns from the private signal $Y_{i,t}$. Second, the agent infers from equilibrium prices $S_{i,t}$ about the signal of the other agent. For a given level of information acquisition $\{\rho_{1,t}, \rho_{2,t}\}$, each agent i solves the model under the assumption that the equilibrium asset price S_t will reveal both her private signal $Y_{i,t}$ and the other agent's private signal $Y_{-i,t}$. Consequently, the agent submits a demand schedule conditional on the price realization, which implies that her orders are also conditional on $Y_{-i,t}$. Since both agents perform this same sequence of actions in equilibrium, each agent i will update her prior beliefs about $D_{i,t+1}$ through both $Y_{i,t}$ (own information acquisition) and $Y_{-i,t}$ (via prices from the other trader's information acquisition). Prior and posterior probabilities are displayed in Tables 3 and 4 respectively.

2.4.2 Noise trading and the Grossman-Stiglitz paradox

Information has a positive value in our economy despite the absence of noise traders. Two effects are responsible for this: First, since asset supply is imperfectly inelastic, production occurs and trading on information generates additional (total) surplus.

Second, imperfectly correlated valuations present the occurrence of a no-trade equilibrium. Only in an economy without private valuation ($\alpha = 1$), the equilibrium price reveals perfectly correlated valuations of $D_{i,t+1}$. If in addition the supply of assets is perfectly inelastic, there are no gains from trade, and the value of information decreases to zero. In contrast, when $\alpha < 1$, trading generates additional surplus and information has a strictly positive value.

2.5 Dynamic optimization

2.5.1 Timing

For every $t < T$, agents face the joint problem of information acquisition, consumption and portfolio choice. *For each agent*, The timing withing each period $t < T$ consists of four sub periods:

1. Dividends $D_{i,t}$ and coupons are realized and the investors' wealth is updated.

2. The agent decides upon the quality $\rho_{i,t}$ of the signal $y_{i,t}$. Information expenditures are deducted from the investor's budget.

3. The signal $y_{i,t}$ is revealed and the agent learns about $D_{i,t+1}$

4. The agent consumes and allocate her remaining wealth across risky and riskless assets.

2.5.2 General problem statement

The investor maximizes CRRA utility over a consumption stream subject to a budget constraint. Following the the assumptions made in the previous paragraphs, the agent's Hamilton-Jacobi-Bellman (HJB) equation is denoted by:

$$\begin{aligned} V_{i,t}\left(\theta^B_{i,t-1}, \theta^S_{i,t-1}\right) = & \\ \max_{\rho_{1,t}, \left\{\theta^S_{1,t}\right\}, \left\{\theta^B_{1,t}\right\}, \{c_{1,t}\}} & \quad \mathbf{1}_{ic} \times \sum_{y_{i,t}} \sum_{y_{-i,t}} \pi\left[y_{i,t}, y_{-i,t}, \cdot, \cdot\right] \times \frac{{c_{i,t}}^{1-\gamma}}{1-\gamma} \\ & + \sum_{y_{i,t}} \sum_{y_{-i,t}} \sum_{D_{i,t+1}} \sum_{D_{-i,t+1}} \pi\left[y_{i,t}, y_{-i,t}, D_{i,t+1}, D_{-i,t+1}\right] \times V_{i,t+1}\left(\theta^B_{i,t}, \theta^S_{i,t}\right) \\ \text{subject to } (\forall \mathrm{y}_1, \mathrm{y}_2) & \quad \theta^S_{1,t-1} \times \left(\mathbf{1}_{long} \times S_t + D_{i,t+1}\right) + \theta^B_{1,t-1} \times \left(\mathbf{1}_{long} \times B_t + 1\right) \\ & \quad = c_{1,t} + \kappa\left(\rho_{i,t}\right) + \theta^S_{1,t} \times S_t + \theta^B_{1,t} \times B_t \end{aligned} \tag{1}$$

Where $\mathbf{1}_{long}$ equals one if and only if assets are long-lived. $\mathbf{1}_{ic}$ equals one if and only if intermediate consumption takes place.[4] Short-hand notations are exhibited in Table 2. Note that the probabilities π are a function of $\{\rho_{i,t}\}$. To highlight the dependence of the probabilities π on $\{\rho_{i,t}\}$, see Figure 3. We denote the Lagrangian as follows:

$$\begin{aligned} \mathcal{L}_{1,t} = & \mathbf{1}_{ic} \times \sum_{y_{i,t}} \sum_{y_{-i,t}} \pi\left[y_{i,t}, y_{-i,t}, \cdot, \cdot\right] \times \frac{{c_{i,t}}^{1-\gamma}}{1-\gamma} \\ & + \sum_{y_{i,t}} \sum_{y_{-i,t}} \sum_{D_{i,t+1}} \sum_{D_{-i,t+1}} \pi\left[y_{i,t}, y_{-i,t}, D_{i,t+1}, D_{-i,t+1}\right] \times V_{i,t+1}\left(\theta^B_{i,t}, \theta^S_{i,t}\right) \\ & + \sum_{y_{i,t}} \sum_{y_{-i,t}} \pi\left[y_{i,t}, y_{-i,t}, \cdot, \cdot\right] \times \varphi_{i,t} \times \Big(\theta^S_{1,t-1} \times \left(\mathbf{1}_{long} \times S_t + D_{i,t+1}\right) \\ & + \theta^B_{1,t-1} \times \left(\mathbf{1}_{long} \times B_t + 1\right) - c_{1,t} - \kappa\left(\rho_{i,t}\right) - \theta^S_{1,t} \times S_t - \theta^B_{1,t} \times B_t\Big) \end{aligned} \tag{2}$$

[4] In the absence of intermediate consumption, the $\{c_{1,t}\}$ argument below the maximization sign should be removed.

2.5.3 First-order conditions

When deriving the first-order conditions, we denote the partial derivatives of probabilities with respect to information quality of agent i by $\partial_i \pi$ (see Table 5). We proceed by deriving the following 17 first order conditions and budget constraints:

- Four F.O.C.'s with respect to (conditional) consumption:[5]

$$c_{1,t}{}^{-\gamma} = \varphi_{1,t} \qquad \forall y_1, y_2 \tag{3}$$

- Four F.O.C.'s with respect to (conditional) stock holdings:

$$\sum_{y_{i,t}} \sum_{y_{-i,t}} \sum_{D_{i,t+1}} \sum_{D_{-i,t+1}} \pi\left[y_{i,t}, y_{-i,t}, D_{i,t+1}, D_{-i,t+1}\right] \times \tfrac{\partial}{\partial \theta^S_{i,t}} V_{i,t+1}\left(\theta^B_{i,t}, \theta^S_{i,t}\right) = \varphi_{1,t} \times S_t \qquad \forall y_1, y_2 \tag{4}$$

- Four F.O.C.'s with respect to (conditional) bond holdings:

$$\sum_{y_{i,t}} \sum_{y_{-i,t}} \sum_{D_{i,t+1}} \sum_{D_{-i,t+1}} \pi\left[y_{i,t}, y_{-i,t}, D_{i,t+1}, D_{-i,t+1}\right] \times \tfrac{\partial}{\partial \theta^B_{i,t}} V_{i,t+1}\left(\theta^B_{i,t}, \theta^S_{i,t}\right) = \varphi_{1,t} \times B_t \qquad \forall y_1, y_2 \tag{5}$$

- One F.O.C. with respect to information quality:

$$\begin{gathered} \sum_{y_{i,t}} \sum_{y_{-i,t}} \partial_i \pi\left[y_{i,t}, y_{-i,t}, \cdot, \cdot\right] \times \tfrac{c_{i,t}{}^{1-\gamma}}{1-\gamma} \\ + \sum_{y_{i,t}} \sum_{y_{-i,t}} \sum_{D_{i,t+1}} \sum_{D_{-i,t+1}} \partial_i \pi\left[y_{i,t}, y_{-i,t}, D_{i,t+1}, D_{-i,t+1}\right] \times V_{i,t+1}\left(\theta^B_{i,t}, \theta^S_{i,t}\right) \\ = \kappa'\left(\rho_{i,t}\right) \times \sum_{y_{i,t}} \sum_{y_{-i,t}} \varphi_{i,t} \end{gathered} \tag{6}$$

- Four budget constraints:

$$\begin{gathered} \theta^S_{1,t-1} \times \left(\mathbf{1}_{long} \times S_t + D_{i,t+1}\right) + \theta^B_{1,t-1} \times \left(\mathbf{1}_{long} \times B_t + 1\right) \\ = c_{1,t} + \kappa\left(\rho_{i,t}\right) + \theta^S_{1,t} \times S_t + \theta^B_{1,t} \times B_t \qquad \forall y_1, y_2 \end{gathered} \tag{7}$$

[5] These are absent in case $\mathbf{1}_{ic} = 0$

2.5.4 Market Clearing

In equilibrium, the bond and stock market clear. For a constant elasticity of asset supply, the corresponding market clearing conditions of the stock are given by:

$$\begin{array}{rclcc} 1 & = & S_t & E_s = \infty & \\ \theta_{1,t}^S + \theta_{2,t}^S & = & A\left(E_s\right) \cdot S_t^{E_S} & E_s \in (0, \infty) & \forall y_1, y_2 \\ \theta_{1,t}^S + \theta_{2,t}^S & = & 1 & E_s = 0 & \end{array} \tag{8}$$

Where $A\left(E_s\right)$ is a scaling factor. The market clearing conditions for the bond are similar and omitted for brevity. There are $2 \times 4 = 8$ market clearing equations in total.

2.6 System of Equations

The entire system of equations describing the equilibrium consists of both agents' first-order conditions and budget equations as well as the market clearing conditions. The system is to be solved at each node for each value of the state variables and consists of $2 \times 17 + 2 \times 4 = 42$ equations and 42 unknowns. In the absence of intermediate consumption, this system reduces to $2 \times 13 + 2 \times 4 = 34$ equations and 34 unknowns.

3 Solving method

In this section, we briefly outline our solution method. Due the our departure from the tractable CARA-normal framework, we are forced to employ numerical methods to tackle our system.

The model is solved using backwards induction. We start by solving the system of equations at the last node over a two-dimensional grid of values for $\left\{\theta_{i,t-1}^B, \theta_{i,t-1}^S\right\}$ and compute the value function at each of these points. The partial derivatives of the value function are obtained with the use of the envelope theorem. We use the set of function values to construct interpolated functions.[6] We next move to the final-last node and use these interpolants to

[6]We use a third-order polynomial for interpolation

solve the respective system of equations. We repeat this procedure and move backwards until the initial period of the model. At this initial node, we solve the system of equations for initial portfolio holdings $\left\{\theta_{i,1}^{B}, \theta_{i,1}^{S}\right\}$, which completes the procedure.

4 Applications to Breugem (2014)

In this section, we apply our method to *"The Information Rat Race"* presented in Breugem (2014). We adopt the notion that information is in limited supply and impose market clearing conditions on the information *input* market. To prevent the Grossman and Stiglitz (1976) paradox, we let agents have imperfectly correlated private valuations.

4.1 The market for information

Consistent with Breugem (2014), we assume that information production requires scarce inputs $\Lambda_{i,t}$. Investors buy these inputs at market price Ξ_t and produce information with the following quality:

$$\rho_{i,t} = \Psi\left(\Lambda_{i,t}\right) = \left(\frac{\Lambda_{i,t}}{\Lambda_{i,t}+1}\right)^{\frac{1}{\Gamma}} \tag{9}$$

Where Ψ represents the production function of information and Γ is the curvature of this function.[7] By inverting the information cost function, we obtain the number of required inputs for any level of information quality. The cost of information is then obtained by multiplying this level of inputs by their unit price Ξ_t:

$$\kappa\left(\rho_{i,t}\right) = \Xi_t \times \frac{\rho_{i,t}^{\Gamma}}{1-\rho_{i,t}^{\Gamma}} \tag{10}$$

The price of inputs is derived endogenously by imposing the following information input market clearing condition:

$$\underbrace{\Lambda_{1,t}\left(\Xi_t\right) + \Lambda_{2,t}\left(\Xi_t\right)}_{demand} = \underbrace{\bar{\Lambda}}_{supply} \tag{11}$$

[7] See Breugem (2014) for a discussion of the specific form of Ψ

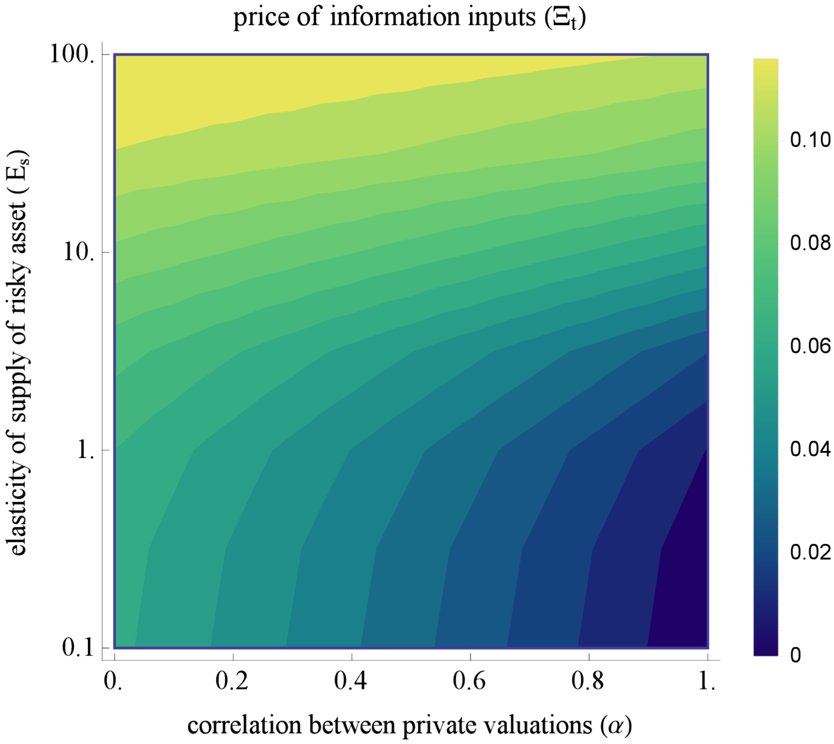

Figure 1: Equilibrium price of information inputs (Ξ_t) as a function of asset supply elasticity and correlation between preferences. The upper left corner approximates the setting in Breugem (2014) where $E_s = \infty$ and $\alpha = 0$. The lower right corner approaches a zero noise scenario ($E_s = 0$ and $\alpha = 1$). In the latter case, the Grossman-Stiglitz paradox arises and the price of information inputs is zero.

4.2 The value of information

We solve the model for the parameters specified in Table 1. Figure 1 shows the price of information inputs as a function of our two "alternatives" for noise traders: correlation between private valuations (α) and elasticity of supply of the risky asset (E_s).

In the first extreme case of $E_s = 0$ and $\alpha = 1$, there is no noise in the system. Information does not create gains from trade, and the price of information is zero (see Grossman and Stiglitz (1976)).

In the other extreme case, when there is infinite noise ($E_s = \infty$ and $\alpha = 0$), agents do not learn from prices and information is solely used to generate private benefits. This results in information with maximum value.

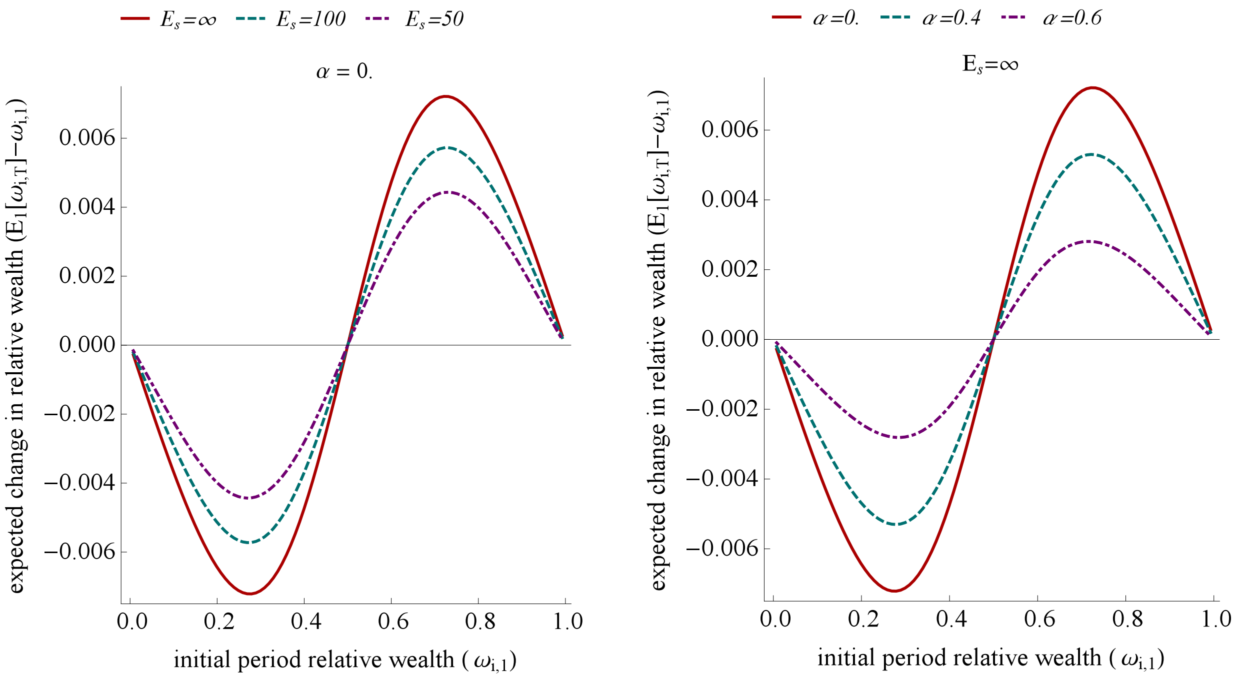

Figure 2: Changes in relative wealth $\omega_{1,t} = \frac{W_{1,t}}{W_{1,t}+W_{2,t}}$ as a function of elasticity of supply of the risky asset and correlation between private valuations. Equilibrium in the asset market decreases the size of the feedback effect responsible for the rat race documented in Breugem (2014).

Generally, we find that the equilibrium price increases in E_s and decreasing in α, which is consistent with the notion that these variables take the position of noise traders in classical models.

4.3 Portfolio performance

In this subsection, we analyze the impact of asset market clearing on the size of the feedback loop responsible for the rat race effect in Breugem (2014). That paper shows that competition for information creates a vicious cycle in which richer agents acquire more information, obtain higher returns on their portfolio and thus become comparatively even richer.

Figure 2 shows the size of the feedback loop by indicating the expected change in relative wealth as a function of current *relative* wealth. Although portfolio performance is increasing in relative wealth, the size of this effect is lower when (i) the supply of risky assets becomes less elastic and (ii) when agents' valuations are more correlated.

Proposition 1. *The dispersion in portfolio performance increases with the risky asset's elasticity of supply*

When the supply of the risky asset is less elastic, fewer gains from trade are generated by production. This generates a price impact effect similar to trading frictions. Since agents need to trade in order to exploit information, the price impact cost is particularly costly for rich agents. This decreases the performance gap between rich and poor agents (see Figure 2a).

Proposition 2. *The dispersion in portfolio performance decreases with the correlation agents' valuations of the risky asset*

An increase in the correlation between asset valuation increases the ability of agents to learn from equilibrium asset prices. Essentially, by increasing the price informativeness, agents are endowed with a larger fraction of the other agent's private information. Poor agents benefit more from this *free-riding effect* than rich agents do. This reduces the difference in portfolio performance (see Figure 2b).

5 Discussion

We present a method capable of solving a dynamic equilibrium portfolio choice problem with CRRA preferences and endogenous information acquisition. Our method assumes that agents have a private valuation for an asset's dividend stream. By doing so, we eliminate the need to have noise traders in our model to eliminate the Grossman-Stiglitz paradox.

We apply the method to Breugem (2014) and find that a lower elasticity of supply of risky assets and a higher correlation between private valuations reduce the rat-race effect and decrease equilibrium information input prices. In a pure exchange economy with identical valuations, the Grossman and Stiglitz (1976) paradox applies and the price of information inputs is zero.

A future version of this paper should include the range of parameters for which the rat-race effect of Breugem (2014) driving complementarities in information acquisition dominates the free-riding effect documented by Grossman and Stiglitz (1976). Moreover, we should provide an in-depth investigation of the relation between private valuations in our model and noise traders in classical CARA-normal models.

References

Admati, A. R., 1985. A Noisy Rational Expectations Equilibrium for Multi-asset Securities Markets. Econometrica 53 (3), 629–57.

Barlevy, G., Veronesi, P., 2000. Information Acquisition in Financial Markets. Review of Economic Studies 67 (1), 79–90.

Barlevy, G., Veronesi, P., 2007. Information Acquisition in Financial Markets: a Correction. Working Paper Series.

Breugem, M., 2014. The Information Rat Race. Working Paper.

Grossman, S. J., Stiglitz, J. E., 1976. Information and Competitive Price Systems. American Economic Review 66 (2), 246–53.

Grossman, S. J., Stiglitz, J. E., 1980. On the Impossibility of Informationally Efficient Markets. American Economic Review 70 (3), 393–408.

Hellwig, M. F., 1980. On the Aggregation of Information in Competitive Markets. Journal of Economic Theory 22 (3), 477–498.

Kyle, A. S., 1985. Continuous Auctions and Insider Trading. Econometrica 53 (6), 1315–1335.

Milgrom, P., Stokey, N., Feb. 1982. Information, Trade and Common Knowledge. Journal of Economic Theory 26 (1), 17–27.

Muth, 1961. Rational Expectations and the Theory of Price Movements. Econometrica 92 (3), 315–335.

Nieuwerburgh, S. V., Veldkamp, L., 2010. Information Acquisition and Under-Diversification. Review of Economic Studies 77 (2), 779–805.

Verrecchia, R. E., 1982. Information Acquisition in a Noisy Rational Expectations Economy. Econometrica 50 (6), 1415–30.

Tables

Table 1: Default parameter values

Description	Parameter	Default Value
Risk aversion	γ_i	3
High payoff of risky asset	D_H	$6/5$
Low payoff of risky asset	D_L	$4/5$
Information production curvature	Γ	3
Total inputs of information production	$\bar{\Lambda}$	$1/5$
Intermediate consumption	$\mathbf{1}_{ic}$	0
Long lived assets	$\mathbf{1}_{long}$	0
Elasticity of supply of riskless asset	E_B	∞
Elasticity of supply of risky asset	E_S	$\in [0, \infty]$
Correlation between valuations	α	$\in [0, 1]$

Table 2: Short hand notation and state dependence of key variables and the total number of occurrences per period, conditional on current period dividend realization.

variable	short notation	full notation	conditional occurrences
choice variables			
consumption	$c_{i,t}$	$c_{i,t}\left(y_{i,t}, y_{-i,t}\right)$	8
bond holdings	$\theta^B_{i,t}$	$\theta^B_{i,t}\left(y_{i,t}, y_{-i,t}\right)$	8
stock holdings	$\theta^S_{i,t}$	$\theta^S_{i,t}\left(y_{i,t}, y_{-i,t}\right)$	8
information quality	$\rho_{i,t}$	$\rho_{i,t}$	2
endogenous variables			
bond price	B_{t+1}	$B_{t+1}\left(y_{i,t}, y_{-i,t}\right)$	4
stock price	S_{t+1}	$S_{t+1}\left(y_{i,t}, y_{-i,t}\right)$	4
state price	$\varphi_{i,t}$	$\varphi_{i,t}\left(y_{i,t}, y_{-i,t}\right)$	8
(conditional) exogenous variables			
future dividends	$D_{i,t+1}$	$D_{i,t+1}$	4
private signal	$y_{i,t}$	$y_{i,t}$	4

Table 3: Overview of (prior) probabilities π for all states of nature.

	$y_{1,t}$	H	H	L	L	
	$y_{2,t}$	H	L	H	L	
$\pi\left[y_1, y_2, H, H\right]$		$\frac{(1+\rho_{1,t})(1+\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$\frac{(1+\rho_{1,t})(1-\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$\frac{(1-\rho_{1,t})(1+\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$\frac{(1-\rho_{1,t})(1-\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$\frac{1+\alpha}{4}$
$\pi\left[y_1, y_2, H, L\right]$		$\frac{(1+\rho_{1,t})(1-\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$\frac{(1+\rho_{1,t})(1+\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$\frac{(1-\rho_{1,t})(1-\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$\frac{(1-\rho_{1,t})(1+\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$\frac{1-\alpha}{4}$
$\pi\left[y_1, y_2, L, H\right]$		$\frac{(1-\rho_{1,t})(1+\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$\frac{(1-\rho_{1,t})(1-\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$\frac{(1+\rho_{1,t})(1+\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$\frac{(1+\rho_{1,t})(1-\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$\frac{1-\alpha}{4}$
$\pi\left[y_1, y_2, L, L\right]$		$\frac{(1-\rho_{1,t})(1-\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$\frac{(1-\rho_{1,t})(1+\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$\frac{(1+\rho_{1,t})(1-\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$\frac{(1+\rho_{1,t})(1+\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$\frac{1+\alpha}{4}$
$\pi\left[y_1, y_2, H, \cdot\right]$		$\frac{(1+\rho_{1,t})(1+\alpha\rho_{2,t})}{8}$	$\frac{(1+\rho_{1,t})(1-\alpha\rho_{2,t})}{8}$	$\frac{(1-\rho_{1,t})(1+\alpha\rho_{2,t})}{8}$	$\frac{(1-\rho_{1,t})(1-\alpha\rho_{2,t})}{8}$	$\frac{1}{2}$
$\pi\left[y_1, y_2, L, \cdot\right]$		$\frac{(1-\rho_{1,t})(1-\alpha\rho_{2,t})}{8}$	$\frac{(1-\rho_{1,t})(1+\alpha\rho_{2,t})}{8}$	$\frac{(1+\rho_{1,t})(1-\alpha\rho_{2,t})}{8}$	$\frac{(1+\rho_{1,t})(1+\alpha\rho_{2,t})}{8}$	$\frac{1}{2}$
$\pi\left[y_1, y_2, \cdot, H\right]$		$\frac{(1+\rho_{2,t})(1+\alpha\rho_{1,t})}{8}$	$\frac{(1+\rho_{2,t})(1-\alpha\rho_{1,t})}{8}$	$\frac{(1-\rho_{2,t})(1+\alpha\rho_{1,t})}{8}$	$\frac{(1-\rho_{2,t})(1-\alpha\rho_{1,t})}{8}$	$\frac{1}{2}$
$\pi\left[y_1, y_2, \cdot, L\right]$		$\frac{(1-\rho_{2,t})(1-\alpha\rho_{1,t})}{8}$	$\frac{(1-\rho_{2,t})(1+\alpha\rho_{1,t})}{8}$	$\frac{(1+\rho_{2,t})(1-\alpha\rho_{1,t})}{8}$	$\frac{(1+\rho_{2,t})(1+\alpha\rho_{1,t})}{8}$	$\frac{1}{2}$
$\pi\left[y_1, y_2, \cdot, \cdot\right]$		$\frac{1+\alpha\rho_{1,t}\rho_{2,t}}{4}$	$\frac{1-\alpha\rho_{1,t}\rho_{2,t}}{4}$	$\frac{1-\alpha\rho_{1,t}\rho_{2,t}}{4}$	$\frac{1+\alpha\rho_{1,t}\rho_{2,t}}{4}$	1

Table 4: (Posterior) probabilities of next period dividend realization conditional on current period signal. The willingness to trade depends on the difference between agents' valuations after learning, denoted by $\pi\left[y_1, y_2, H, \cdot\right] - \pi\left[y_1, y_2, \cdot, H\right]$

		$y_{1,t}$	H	H	L	L
		$y_{2,t}$	H	L	H	L
$D_{i,t+1}$	i					
H	1		$\frac{(1+\rho_{1,t})(1+\alpha\rho_{2,t})}{2(1+\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1+\rho_{1,t})(1-\alpha\rho_{2,t})}{2(1-\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1-\rho_{1,t})(1+\alpha\rho_{2,t})}{2(1-\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1-\rho_{1,t})(1-\alpha\rho_{2,t})}{2(1+\alpha\rho_{1,t}\rho_{2,t})}$
L	1		$\frac{(1-\rho_{1,t})(1-\alpha\rho_{2,t})}{2(1+\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1-\rho_{1,t})(1+\alpha\rho_{2,t})}{2(1-\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1+\rho_{1,t})(1-\alpha\rho_{2,t})}{2(1-\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1+\rho_{1,t})(1+\alpha\rho_{2,t})}{2(1+\alpha\rho_{1,t}\rho_{2,t})}$
H	2		$\frac{(1+\rho_{2,t})(1+\alpha\rho_{1,t})}{2(1+\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1+\rho_{2,t})(1-\alpha\rho_{1,t})}{2(1-\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1-\rho_{2,t})(1+\alpha\rho_{1,t})}{2(1-\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1-\rho_{2,t})(1-\alpha\rho_{1,t})}{2(1+\alpha\rho_{1,t}\rho_{2,t})}$
L	2		$\frac{(1-\rho_{2,t})(1-\alpha\rho_{1,t})}{2(1+\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1-\rho_{2,t})(1+\alpha\rho_{1,t})}{2(1-\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1+\rho_{2,t})(1-\alpha\rho_{1,t})}{2(1-\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1+\rho_{2,t})(1+\alpha\rho_{1,t})}{2(1+\alpha\rho_{1,t}\rho_{2,t})}$
difference in valuations $\pi\left[y_1, y_2, H, \cdot\right] - \pi\left[y_1, y_2, \cdot, H\right]$			$\frac{(1-\alpha)(\rho_{1,t}-\rho_{2,t})}{2(1+\alpha\rho_{1,t}\rho_{2,t})}$	$\frac{(1-\alpha)(\rho_{1,t}+\rho_{2,t})}{2(1-\alpha\rho_{1,t}\rho_{2,t})}$	$-\frac{(1-\alpha)(\rho_{1,t}+\rho_{2,t})}{2(1-\alpha\rho_{1,t}\rho_{2,t})}$	$-\frac{(1-\alpha)(\rho_{1,t}-\rho_{2,t})}{2(1+\alpha\rho_{1,t}\rho_{2,t})}$

Table 5: Overview of (prior) partial derivatives of probabilities π for all states of nature.

$y_{1,t}$	H	H	L	L
$y_{2,t}$	H	L	H	L
$\partial_1 \pi\left[y_1, y_2, H, H\right]$	$\frac{(1+\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$\frac{(1-\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$-\frac{(1+\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$-\frac{(1-\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$
$\partial_1 \pi\left[y_1, y_2, H, L\right]$	$\frac{(1-\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$\frac{(1+\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$-\frac{(1-\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$-\frac{(1+\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$
$\partial_1 \pi\left[y_1, y_2, L, H\right]$	$-\frac{(1+\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$-\frac{(1-\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$\frac{(1+\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$	$\frac{(1-\rho_{2,t})}{4} \times \frac{1-\alpha}{4}$
$\partial_1 \pi\left[y_1, y_2, L, L\right]$	$-\frac{(1-\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$-\frac{(1+\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$\frac{(1-\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$	$\frac{(1+\rho_{2,t})}{4} \times \frac{1+\alpha}{4}$