

Bank Capital, Liquid Reserves, and Insolvency Risk*

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February 24, 2014

Abstract

We develop a dynamic model of banking to assess the effects of liquidity and capital requirements on banks' insolvency risk. The model features endogenous capital structure, liquid asset holdings, payout, and default decisions. In the model, banks face taxation, flotation costs of securities, and default costs and are financed with equity, insured deposits, and risky debt. Using the model, we demonstrate that *(i)* regulatory requirements affect bank behavior even when they are not binding; *(ii)* liquidity requirements constraining banks to hold a minimum amount of liquid reserves have no long-run effects on default risk but may increase default risk in the short run; *(iii)* capital requirements increase banks' franchise value ex post and reduce default risk; *(iv)* levered banks have no risk shifting incentives when outside financing is costly.

KEYWORDS: banks; liquidity buffers; capital structure; insolvency risk; regulation

JEL CLASSIFICATION NUMBERS: G21, G28, G32, G33.

*We thank Lucy White for encouraging us to develop this model and Darrell Duffie and Jean-Charles Rochet for stimulating discussions on the topic. We also thank Pierre Collin-Dufresne, Ruediger Fahlenbrach, Damir Filipovic, Thierry Foucault, Nataliya Klimenko, Lorian Mancini, Semyon Malamud, Norman Schuerhoff, Anders Trolle, Francesca Zucchi, and seminar participants at Banque de France, EPFL, and the University of Lausanne for helpful comments. Financial support from the Swiss Finance Institute and from the Fondation Banque de France is gratefully acknowledged. The usual disclaimer applies.

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Banks can impose major risks on the economy. Avoidance of these risks and the associated social costs is the overwhelming concern of prudential regulation. Given the experience in the financial crisis of 2007-2008, in which insufficient liquidity buffers and excessive leverage made the financial system unable to withstand large negative shocks, the debate on banking regulation has recently evolved around two main ideas, that have been reflected in proposals for regulatory reform. First, equity requirements should be significantly increased, so that if the value of the banks' assets were to decline this would not automatically lead to distress and the resulting losses would be borne by the bank owners (see e.g. Admati and Hellwig (2013)).¹ Second, because most banks assets are illiquid and raising fresh equity at short notice is costly, banks should hold a buffer of liquid reserves in order to be able to cope with short-term losses (see e.g. Gorton (2012)).

While many qualitative discussions of capital and liquidity requirements are available in the literature, financial theory has made little headway in developing models that provide quantitative guidance for the use of different instruments of prudential regulation. Moreover, as useful as they are, many of the recent discussions on banking regulation ignore important incentive effects that regulatory requirements may have on bank behavior. Our objective in this paper is therefore twofold. First, we seek to develop a dynamic tractable model of banks' choices of liquid asset holdings, financing, payout, and default policies in the presence of realistic market frictions. Second, we want to use this model to characterize the endogenous response of banks to the imposition of liquidity and capital requirements and to measure the effects of such requirements on banks' insolvency risk.

We begin our analysis by formulating a dynamic structural model in which banks face taxation, issuance costs of securities, and default costs and may be constrained by a regulator to hold a minimum amount of liquid reserves and of equity capital. In the model, banks are financed with equity, insured deposits, and risky debt. They hold risky, illiquid assets (e.g.

¹For example, U.S. banking regulators have proposed in July 2013 a new restriction on borrowing for U.S. banks. This new restriction would require a ratio of regulatory (Tier 1) capital to total assets of 6 percent for 8 Systemically Important Financial Institutions and 5 percent for their bank holding companies, independently of the riskiness of these assets. Similarly, the Vickers Commission in the U.K. has recommended a 4 percent ratio rather than the 3 percent global standard set by the Basel III regime.

loan portfolios) that yield high returns but are subject to both small and frequent shocks (i.e. Brownian shocks) as well as large negative shocks capturing tail risk (i.e. negative jumps). Banks also hold risk-free, liquid assets (i.e. cash reserves or safe government bonds) that can be used to absorb these shocks and save on recapitalization costs. In the model, banks maximize the present value of future dividends to incumbent shareholders by choosing their buffers of liquid assets as well as their payout, financing, and default policies.

We start our analysis by solving the policy choices that maximize shareholder value in an hypothetical environment in which banks are unregulated. We show that when outside financing is costly, inside and outside equity are not perfect substitutes and banks find it optimal to hold buffers of liquid assets. Indeed, while cash is an asset with a low return, it is also the most liquid store of value, the one that can be used to absorb potential losses without issuing costly securities. To reduce the risk of default and save on recapitalization costs, banks manage their liquidity buffers dynamically by adjusting their dividend payments to shareholders. Banks facing higher recapitalization costs are less levered, pay less dividends, hold more liquid reserves, and default earlier. Moreover, banks behave in a risk-averse fashion. That is, we find that levered banks have no incentives to increase risk, even though shareholders have limited liability, consistent with the evidence presented in Stein (2014).

We also show that banks absorb small and intermediate losses using their buffers of liquid assets or by raising outside funds at a cost when these buffers are insufficient. That is, in our model, small and intermediate losses do not lead to insolvency. We show, however, that large losses due to a realization of tail risk may lead banks to default. Notably, we find that banks' insolvency risk increases with tail risk, financial leverage, and financing costs and decreases with franchise value, i.e. the expected cash flow from running the banks' assets. As will become clear, one key feature of our framework is that it highlights a trade-off between managing insolvency risk ex ante via capital structure policy versus ex post via liquidity buffers. That is, in our model, liquidity management, capital structure policies, and default decisions are jointly and endogenously determined.

After solving for the policy choices of unregulated banks, we examine the effects of prudential regulation on these policy choices and insolvency risk. We first consider liquidity requirements that mandate banks to hold a minimum amount of liquid reserves. We show that when facing such a requirement, banks voluntarily choose to hold liquid buffers in excess of the required minimum in order to reduce recapitalization costs associated with breaches of the requirement. Notably, we show that as liquidity requirements rise, banks raise their *target level* of liquid reserves by a like amount, thereby making their cushion of liquid reserves independent of the level chosen by the regulator. We then investigate the implication of this behavior for default risk. Because raising outside funds is costly, an increase in liquidity requirements will not generally lead to an immediate adjustment to the new target buffer. As a result, in the *short-run*, liquidity requirements will lead to a drop in franchise value and to an increase in insolvency risk. We also demonstrate that once there has been an opportunity to build up liquid reserves towards the desired level, changes in liquidity requirements have no impact on insolvency risk. That is, contrary to received wisdom, we find that constraining liquid reserves has no *long-run* impact on the risk of failure of banks.

In addition to liquidity requirements, banks may be subject to capital (or equity) requirements. Such requirements do not impose any restriction on what assets banks should hold but impose constraints on the way they fund their operations. Indeed, as argued by Admati and Hellwig (2013), an increase of equity capital transfers a large fraction of the bank's risks to the bank's owners, which otherwise might be passed on to creditors or taxpayers. Capital requirements are generally expressed in terms of leverage ratios, indicating how much equity capital banks should have relative to their total assets. For example, banks are expected to maintain a Tier 1 capital to asset ratio of 3% under Basel III. In July 2013, the US Federal Reserve Bank announced that the minimum Basel III leverage ratio would be 6% for 8 Systemically Important Financial Institutions. Consistent with these recommendations, we consider requirements that fix *ex-ante* a minimum amount of equity capital for banks. Using a calibrated version of the model, we show that such requirements have a significant effect on insolvency risk. For example, increasing equity capital from 6% (as recommended by U.S. regulators) to 20% (as recommended by Admati and Hellwig) decreases the probability of

default over a one year horizon from 0.1518% to less than 0.0001%. We also show that such requirements have a significant effect on bank value. In our base environment for example, we find that increasing equity capital from the equity value-maximizing level to 20% reduces total bank value by 6.25%.

The literature examining default risk in banks has started with the early contributions of Merton (1977, 1978), in which the objective is to determine the cost of deposit insurance and loan guarantees. Although an important milestone, the analysis of Merton has four limitations. First, it assumes that banks default whenever the assets-to-deposits ratio falls below some *exogenous* barrier. This provision approximates the case in which banks face a positive net worth covenant, but it is by no means the only or typical situation. Second, the capital structure of banks is set exogenously and does not depend on frictions that banks face such as taxes, bankruptcy costs, or issuance costs of securities. Third, the dynamics of the banks assets are governed by an exogenous process, implying that there is no connection between the banks' asset and capital structures. Fourth, raising equity is costless, so that liquid reserves are irrelevant.

Most of the recent quantitative banking models examine variants to the first of these assumptions. By doing so, these studies are able to determine the effects of a given, exogenous capital structure on insolvency risk, while maintaining the Modigliani and Miller (1958) assumption that investment policy is independent of financing policy. In these contributions, insolvency is endogenous and triggered by shareholders decision to cease injecting funds in the bank (see e.g. Fries, Mella-Barral, and Perraudin (1997), Bhattacharya, Planck, Strobl, and Zechner (2002), or Décamps, Rochet, and Roger (2004)). While identifying some prime determinants of insolvency risk, these theories assume that asset and liability structures are exogenously given. As a result, they leave open the question of how financing structure and asset structure interact and jointly affect insolvency risk. In addition, these models also maintain the assumption that banks can raise outside funds at no cost, thereby leaving no role for liquidity. In that respect, they ignore a number of key determinants of insolvency risk, which are at the centre of most regulatory frameworks, for example those of the Basel Committee on Banking Supervision.

Our analysis inherits some of the assumptions of this literature. For example, bank shareholders are protected by limited liability and the bank's objective is shareholder value maximization. However, it differs from these contributions in three important respects. First, we consider that at least part of a bank's assets are illiquid and that it is costly to issue securities, thereby providing a role for liquidity buffers. Second, we incorporate some of the key market imperfections and regulatory requirements that banks face in practice and relate banks' payout, financing, and default policies to these various frictions. Third, in our model, financing structure and asset structure interact and jointly affect insolvency risk. We show that these unique features have important implications. Notably, in standard models of bank moral hazard limited liability leads banks to increase risk as much as possible. In addition, shareholder value is always increased by making dividend payments to shareholders. This is not the case in our model with frictions, in which equity value is concave and shareholders have incentives to protect their franchise value by maintaining adequate liquid reserves.

There is a large literature on bank capital that analyzes its role in regulation (see e.g. Hellman, Murdoch, and Stiglitz (2001), Décamps, Rochet and Roger (2004), Morrison and White (2005), or Repullo and Suarez (2013)). But it is only recently that the question of bank optimal capital structure has begun to be addressed. The papers that are most closely related to ours in this literature are Froot and Stein (1988) and Subramanian and Yang (2013). Subramanian and Yang use a dynamic model to examine banks' insolvency risk. In their paper, there is no role for liquid reserves as outside financing is costless. In addition, the authors mostly focus on regulators' incentives to step in as banks become distressed. In that respect, the two papers are complementary.

Froot and Stein (1998) build a two-period model in which capital is initially costless but may become costly in the future. In their model, decisions are made by bank shareholders in the first period, in which the future increase in financing costs leads the bank to increase equity capital and to engage in risk management activities to hedge future potential losses. Our model differs from Froot and Stein by considering that banks always face the same set of frictions and choose their payout, financing, and default decisions in response to these frictions as well as regulatory requirements. Another difference is that we consider a dynamic

model. Indeed, as suggested by Hellwig (1998), a static framework fails to capture important intertemporal effects. For example, in a static model, a regulatory requirement can only affect a bank's behavior if it is binding. In practice however, regulatory requirements are binding for a small minority of banks and yet seem to influence the behavior of other banks.

From a methodological perspective, our paper relates to the inventory models of Milne and Whalley (2001), Milne (2004), Peura and Keppo (2006), or Décamps, Mariotti, Rochet, and Villeneuve (2011), in which financing costs lead banks or corporations to have liquidity buffers. In these models, uncertainty is solely driven by Brownian shocks and there is no risk of default if firms can raise outside equity. Our paper advances this literature in two important dimensions. First, we incorporate jumps in our analysis to account for tail risk and show that banks may find it optimal to default, even if they have access to outside funds. Second, we endogenize not only banks' payout decisions but also their financing and default policies. This allows us to examine the effects of regulation on banks' insolvency risk. As shown in the paper, this problem is difficult to solve because we now have two free boundaries (the default and the payout thresholds) instead of one (the payout threshold). Another contribution of this paper is therefore to develop a new method based on fixed-point arguments to derive the solution to shareholders' optimization problem in this context.

The paper is organized as follows. Section I presents the model. Section II derives the policies choices of unregulated banks. Section III examines the effects of liquidity and capital requirements on these policy choices and insolvency risk. Section IV discusses the model's implications. Section 5 concludes. The proofs are gathered in a separate Appendix.

I. Model

We develop a dynamic banking model in which financing structure and asset structure interact and jointly affect insolvency risk. Our focus is on the liquidity management, payout, financing, and default policies that maximize the bank's equity value. We also analyze the effects of capital and liquidity requirements on these policy choices and banks' insolvency risk. Throughout the paper, agents are risk neutral and discount cash flows at a constant

rate ρ . Time is continuous and uncertainty is modeled by a probability space $(\Omega, \mathcal{F}, P; \mathbb{F})$, with the filtration $\mathbb{F} = \{\mathcal{F}_t : t \geq 0\}$ satisfying the usual conditions.

The subject of study is a bank held by shareholders that have limited liability. This bank is subject to taxation at the rate $\theta \in (0, 1)$. It owns a portfolio of risky assets (e.g. a portfolio of risky loans) as well as liquid reserves (i.e. cash reserves or safe government bonds) and is financed by equity, insured deposits, and risky debt. Risky assets generate after-tax cumulative cash flows A_t that evolve according to:

$$dA_t = (1 - \theta)\mu dt + \sigma dB_t - Y_{N_t} dN_t$$

where B_t is a Brownian motion, N_t is a Poisson process, (μ, σ) are constants, and $(Y_n)_{n=1}^{\infty}$ is a sequence of independently and identically distributed random variables that are drawn from \mathbb{R}_+ according to an exponential distribution with cumulative density function $F(y) = 1 - e^{-\beta y}$ with $\beta > 0$. In this equation, the increments of the Brownian motion represent small and frequent shocks to the bank cash flows. The jumps of the Poisson process represent large losses that may be due, for example, to defaults across the loan portfolio of the bank. We denote the intensity of the Poisson process by λ so that over an infinitesimal time interval there is a probability λdt that the bank makes a large loss. Because the distribution of the jump magnitudes has unbounded support, the bank will be unable to withstand this loss with some positive probability, leading to insolvency. That is, the bank is subject to tail risk (see for example Acharya, Cooley, Richardson, and Walter (2009) for an analysis of the role of tail risk in the 2007-2009 financial crisis).

The relative illiquidity of bank assets generally constitutes a main source of banking fragility.² As discussed in Froot and Stein (1998), “one of the fundamental roles of banks and financial intermediaries is to invest in assets which, because of their information-sensitive nature, cannot be frictionlessly traded in the capital markets.” The standard example of such

²This illiquidity problem can be due for example to the special expertise of the bank in evaluating its long-term investments. If the bank tries to sell them before they pay out, other investors might infer that the bank got bad news about the future returns from these investments leading to a decrease of the price at which the bank could sell these assets. That is, liquidity problems may be fundamentally informational. But informational problems are also why we need a banking system in the first place.

an illiquid asset is a loan to a small or medium-sized company. To capture this important feature, we consider that *risky* assets are illiquid and assume for simplicity that they have zero liquidation value, as in Biais, Mariotti, Rochet, and Villeneuve (2010), DeMarzo, Duffie, and Varas (2013), or Rochet and Zargari (2013).

In addition to its risky assets, the bank can (or may be constrained by the regulator to) hold liquid, risk-free reserves. We denote by S_t the liquid reserves of the bank at any time $t \geq 0$. Holding liquid reserves generally involves deadweight costs. We capture these costs by assuming that the rate of return on liquid reserves is zero.³ When optimizing its liquid asset holdings, the bank trades-off the lower returns of these assets with the benefits of liquidity.

The bank’s capital structure comprises equity, a fixed volume of insured deposits D , and risky debt. Deposits are insured against bank failure by a deposit insurance fund and require the bank to make a payment $c_D \geq 0$ per unit of time, which includes the interest paid to depositors and a deposit insurance premium.⁴ Risky debt requires the bank to make a payment $c_L \geq 0$ per unit of time and has a face value L that is endogenized in section III.B. The bank’s debt payments are thus $c = c_D + c_L$ and its cumulative earnings satisfy

$$C_t = A_t - (1 - \theta)ct = (1 - \theta)(\mu - c)t + \sigma B_t - \sum_{n=1}^{N_t} Y_n. \quad (1)$$

Given that sudden losses can eliminate an arbitrarily large fraction of the bank’s assets, the bank is subject to default risk. In the following, we use a stock based definition of default whereby the bank services debt as long as equity value is positive (as in e.g. Leland (1994)). That is, default is the result of the optimizing behavior of shareholders. We will show that in our model limited liability only has value if the bank cash flows are subject to tail risk, i.e. only if $\lambda > 0$, for otherwise the bank would not be subject to default risk.

³A necessary condition for a well-defined payout policy is that there exists a cost of holding liquidity in that the rate of return on liquid reserves is strictly less than the risk-free rate of return ρ .

⁴As in Hanson, Shleifer, Stein and Vishny (2014), “we depart from much of the rest of the literature by downplaying the vulnerability of bank deposits to runs. Indeed, we emphasize precisely the opposite aspect of deposit finance: Relative to other forms of private-money creation that occur in the shadow-banking sector—notably, short-term collateralized claims such as broker-dealer repurchase agreements (repos)—bank deposits are noteworthy because, in the modern institutional environment, they are highly sticky and not prone to run at the first sign of trouble.”

In the model, equity capital and liquid reserves serve as buffers against default risk. The bank can increase its liquid reserves either by retaining earnings or by issuing new equity. We consider that when raising outside funds, the bank has to pay a lump-sum cost ϕ . Because of this fixed cost, the bank will raise new funds through lumpy and infrequent issues. In addition, in order to reduce the costs associated with having to issue fresh equity, the bank will choose to retain earnings and hold liquidity buffers.

A payout and financing strategy for the bank is a pair $\pi = (P^\pi, R^\pi)$ of adapted and non-decreasing processes with initial value zero, where P_t^π and R_t^π respectively represent the cumulative payouts to shareholders and the cumulative net financing raised from investors up until time $t \geq 0$. The liquid reserves process associated with a strategy π is defined by:

$$S_t^\pi = s + C_t - P_t^\pi + R_t^\pi,$$

where C_t is defined in equation (1) and s is the initial level of liquid reserves. This equation is a general accounting identity, in which P_t^π and R_t^π are endogenously determined by the bank. It shows that liquid reserves increase with the bank's earnings and with the funds raised from outside investors and decrease with payouts to shareholders. The liquidation time associated with the payout and financing strategy π is then defined by:

$$\tau_\pi = \inf\{t \geq 0 : S_{t+}^\pi = \lim_{u \downarrow t} S_u^\pi \leq 0\}.$$

In the model, bank shareholders make their financing, payout, and default decisions after observing the increment of the cash flow process. As a result, we use left-continuous processes in the definition of strategies and right-hand limits in the definition of liquidation times. Notably, the occurrence of a cash flow jump

$$\Delta C_t = C_t - \lim_{s \uparrow t} C_s \leq -S_{t-}^\pi$$

that depletes the liquid reserves of the bank results in default if shareholders do not provide sufficient funds for the right hand limit

$$S_{t+}^{\pi} = S_{t-}^{\pi} + \Delta C_t + R_{t+}^{\pi} - R_t^{\pi} - P_{t+}^{\pi} + P_t^{\pi}$$

to be strictly positive. In the following, we will consider strategies such that $P_{t+}^{\pi} - P_t^{\pi} \leq S_t^{\pi} + R_{t+}^{\pi} - R_t^{\pi}$, implying that the bank cannot pay out amounts that it does not hold.

Management chooses the payout, financing, and default policies of the bank to maximize the present value of future dividends to incumbent shareholders, net of the claim of new investors on future cash flows. That is, management solves:

$$v(s) = \sup_{\pi \in \Pi(s)} \mathbb{E}_s \left[\int_0^{\tau_{\pi}} e^{-\rho t} (dP_t^{\pi} - d\Phi_t(R^{\pi})) \right], \quad (2)$$

where $\Pi(s)$ denotes the set of admissible strategies and dP_t^{π} and $d\Phi_t(R^{\pi})$ are respectively the bank's incremental payout and the contribution of shareholders to the bank (including the cumulative cost of external financing) over the time interval $(t, t + dt)$.⁵ Because risky assets have no liquidation value, there is no cash flow to shareholders in default. That is, we have $(S_{\tau_{\pi}}^{\pi} - D - L)^+ = 0$, where $x^+ \equiv \max(0, x)$.

To gain some intuition on the solution to problem (2), note that because of the fixed costs of financing, the bank raises external funds only when cash flow shocks deplete its liquid reserves, i.e. when $s \leq 0$. In such instances, the bank has to either immediately raise new funds to finance the shortfall and continue operating, or default. The bank may choose to default when the shortfall or the cost of refinancing are too large. When this is not the case, the bank pays the financing cost ϕ and raises an amount that maximizes

⁵Formally, the cumulative net financing raised from outside investors R_t^{π} and the total contribution of shareholders to the bank $\Phi_t(R^{\pi})$ up until time t are defined by

$$R_t^{\pi} = \sum_{n=1}^{\infty} 1_{\{t > \xi_n\}} r_n \quad \text{and} \quad \Phi_t(R^{\pi}) = R_t^{\pi} + \sum_{n=1}^{\infty} 1_{\{t > \xi_n\}} \phi,$$

for some increasing sequence of stopping times $(\xi_n)_{n=1}^{\infty}$ that represent the dates at which the bank raises funds from outside investors and some sequence of nonnegative random variables $(r_n)_{n=1}^{\infty}$ that represent the net financing amounts. See the Technical Appendix to the paper for more details.

equity value. Taking into account these two possibilities, we thus have that the equity value function satisfies for all $s \leq 0$:

$$v(s) = \sup_{b \geq 0} (v(b) - b + s - \phi)^+ = (\alpha_0^* + s)^+, \quad (3)$$

where b is the level of the bank's liquid reserves after raising external funds and

$$\alpha_0^* = v(0) = \sup_{b \geq 0} (v(b) - (b + \phi))^+$$

is the maximal shortfall that bank shareholders can accept to refinance. Anticipating, equation (3) shows that two key determinants of default risk in our model are financing costs ϕ and tail risk, i.e. the possibility for s to become (very) negative.

Because the likelihood of costly refinancing decreases with s , we expect the marginal value of liquid reserves to be decreasing and, therefore, the equity value function to be concave. If this conjecture is verified, then there should exist some barrier level b_0^* such that $v'(s) \geq 1$ if and only if $s \leq b_0^*$ and the optimal payout policy should consist in distributing dividends to maintain liquid reserves at or below the target level b_0^* .

The main difficulty in solving problem (2) is that one needs to simultaneously determine the values of α_0^* and b_0^* . To circumvent this difficulty, we proceed in two steps. In the first step, we fix the value of the constant α and solve for the payout policy in an auxiliary problem where the bank cannot raise funds but produces the payoff $(\alpha + s)^+$ to shareholders when it runs out of liquid reserves and defaults. In the second step, we show that the payoff to shareholders in default can be chosen in such a way that the value of this auxiliary problem coincides with the solution to problem (2) and derive the equity value-maximizing payout, financing, and default policies.

Before moving to these two steps, we first determine the equity value and the bank's policy choices when there are no refinancing costs. This allows us to derive a first best *franchise value* of equity that will be of repeated use when solving shareholders' optimization problem.

II. Value of an unregulated bank

A. First best franchise value

When there are no costs of raising funds, in that $\phi = 0$, there is no need for the bank to hold liquid reserves because any loss can be covered by issuing equity at no cost, if optimal to do so. As a result, it is optimal to distribute all earnings and the optimization problem reduces to choosing the default policy that maximizes equity value. That is, management solves

$$v^* = \sup_{\tau \in \mathcal{S}} \mathbb{E} \left[e^{-\rho\tau} |\Delta C_\tau| + \int_0^\tau e^{-\rho t} dC_t \right],$$

where cumulative earnings evolve according to equation (1), \mathcal{S} denotes the set of all stopping times, and $|\Delta C_\tau|$ represents the size of the loss leading to default. Solving this problem leads to the following result.

PROPOSITION 1 (First best franchise value): *When issuing equity is costless, it is optimal to distribute all earnings and the optimal default time is given by*

$$\tau^* = \inf\{t \geq 0 : v^* + \Delta C_t \leq 0\}$$

where the first best franchise value of equity $v^* > 0$ is the unique solution to

$$\rho v^* = (1 - \theta)(\mu - c) - \lambda \mathbb{E}[\min(v^*, Y_1)]. \quad (4)$$

when $c < \mu$ and $v^* = 0$ otherwise. In particular, for $c < \mu$, the first best franchise value of equity is increasing in the drift rate μ , and decreasing in the coupon rate c , the jump intensity λ , and the mean jump size $1/\beta$.

Proposition 1 shows that for any $c < \mu$, it is optimal for shareholders to default the first time that the absolute value of a jump of the cash flow process exceeds equity value. In effect, the quantity $|\Delta C_t| = Y_{N_t}$ plays the role of a cost of investment that shareholders have to pay to keep the bank alive following the occurrence of a large loss. To better understand

this feature, we can rewrite the Bellman equation (4) for the first best franchise value as

$$v^* = p(1 - \theta) \left(\frac{\mu - c}{\rho} \right) + (1 - p) \mathbb{E} [(v^* - Y_1)^+].$$

with the constant $p = \frac{\rho}{\rho + \lambda} \in (0, 1]$. This equation shows that when issuing equity is costless, the banks' problem can be interpreted as a discrete-time, infinite horizon problem in which shareholders earn $(1 - \theta) \frac{\mu - c}{\rho}$ each period with probability p and otherwise face a random liquidity shock that they can decide to pay, in which case the bank continues for an additional period, or not, in which case the bank is liquidated.

To determine the effect of limited liability on bank policy choices in this first best environment, let

$$v_0 = \mathbb{E} \left[\int_0^\infty e^{-\rho t} dC_t \right] = \frac{1}{\rho} \left((1 - \theta)(\mu - c) - \frac{\lambda}{\beta} \right)$$

stand for the equity value under the assumption that shareholders never default, and denote by $\Omega = v^* - v_0$ the excess value generated by the option of declaring default. Using the Bellman equation (4), it is easily seen that this quantity satisfies:⁶

$$\Omega = \left(\frac{\lambda}{\rho} \right) \mathbb{E} [(Y_1 - v_0 - \Omega)^+].$$

This equation shows that limited liability only has value if the bank cash flows are subject to jumps, i.e. only if $\lambda > 0$, for otherwise the bank would not be subject to default risk. Importantly, the fact that the first best franchise value of equity v^* is strictly positive for all $c < \mu$, irrespective of the frequency and magnitude of the jumps, implies that it is optimal for the bank to operate even if the present value of cash flows v_0 is negative. That is, in the presence of tail risk, limited liability leads shareholders to overinvest in risky assets.⁷

⁶When the jumps Y_n of the cumulative cash flow process are exponentially distributed, the solution to this equation is given in closed form by $\Omega = (1/\beta) \mathcal{W}((\lambda/\rho)e^{-v_0})$ where $\mathcal{W}(x)$ is the Lambert W function (see for example Corless, Gonnet, Hare, Jeffrey, and Knuth (1996)). While this closed form solution is useful in numerical computation, it is not necessary for our analysis of the Bellman equation.

⁷This behavior is reminiscent of the collapse of AIG who was providing insurance by issuing credit default swaps (CDSs). AIG was collecting premia on these CDSs and was expected to make payments only if certain

B. Equity value and bank policies with no refinancing

Having determined the first best franchise value of equity, we now turn to the solution of the auxiliary problem in which the bank has no access to outside funds. In this case, shareholders cannot refinance, have to default if the bank runs out of liquid reserves, and can only optimize equity value over the bank's payout policy.

Let $\alpha \geq 0$ be a constant and consider a bank whose assets produce after-tax net cash flows to shareholders given by dC_t as long as it is in operation, and a cash flow $(\alpha + s)^+$ to shareholders if the bank is liquidated at a point where $S_t = s$. The optimization problem of shareholders in such a bank can be written as

$$w(s; \alpha) = \sup_{\pi \in \Pi_0(s)} \mathbb{E}_s \left[\int_0^{\tau_\pi} e^{-\rho t} dP_t^\pi + e^{-\rho \tau_\pi} (\alpha + S_{\tau_\pi}^\pi)^+ \right], \quad (5)$$

where $\Pi_0(s)$ denotes the subset of admissible strategies such that $R^\pi = 0$ (i.e. no refinancing) and τ_π denotes the first time that liquid reserves are negative. The first term on the right-hand side of (5) captures the present value of all dividend payments until the time of default. The second term gives the present value of the cash flow to shareholders in default.

If the value of α is sufficiently large, it should be optimal for shareholders to liquidate the bank assets and distribute all the available cash. The following result confirms this intuition and shows that the threshold above which immediate liquidation is optimal coincides with the first best franchise value of equity determined in the previous section.

LEMMA 2: *If $\alpha \geq v^*$, it is optimal for shareholders to immediately liquidate.*

Given the above result, we now assume that $\alpha < v^*$ and solve problem (5) by deriving the payout policy that maximizes shareholder value. Following the literature on optimal dividend policies (see for example Albrecher and Thonhauser (2009) for a recent survey), it is natural to conjecture that the optimal payout strategy for shareholders should be of barrier type. Specifically, we conjecture that for any $\alpha < v^*$ there exists a constant barrier

bonds defaulted, which unexpectedly happened in the wake of the subprime crisis. Once the bonds started defaulting, the CDSs had to pay out and AIG was on the hook for billions, which eventually led them to default.

$b^*(\alpha)$ such that the optimal policy for problem (5) consists in paying dividends to maintain liquid reserves at or below the level $b^*(\alpha)$. To verify this conjecture, we start by calculating the value of the bank's equity under an arbitrary barrier strategy.

Fix a constant $b > 0$ and consider the strategy $\pi_b = (P^b, 0)$ that consists in paying dividends to maintain the liquid reserves of the bank at or below the level b . The cumulative payout associated with this barrier strategy is

$$P_t^b = \max_{0 \leq u < t} 1_{\{t > 0\}} (X_u - b)^+$$

where $X_t = s + C_t$ denotes the uncontrolled liquid reserves process (i.e. assuming that there are no dividend payments) and the corresponding value is defined by

$$w(s; \alpha, b) = \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho \tau_{\pi_b}} (\alpha + S_{\tau_{\pi_b}}^{\pi_b})^+ \right].$$

Letting ζ_0 denote the first time that the uncontrolled liquid reserves of the bank X_t become negative and using the dividend–penalty identity (see Gerber, Lin, and Yang (2006) and Gerber and Yang (2010)), we have that this value is given by:

$$w(s; \alpha, b) = \begin{cases} (\alpha + s)^+, & \text{for } s \leq 0, \\ \psi(s; \alpha) + \frac{W(s)}{W'(b)} (1 - \psi'(b; \alpha)), & \text{for } 0 < s \leq b, \\ s - b + w(b; \alpha, b), & \text{for } s > b, \end{cases} \quad (6)$$

where the function

$$\psi(s; \alpha) = \mathbb{E}_s [e^{-\rho \zeta_0} (\alpha + X_{\zeta_0})^+]$$

gives the present value of the payment that shareholders receive in liquidation if no dividends are distributed prior to bankruptcy, and $W(s)$ is the ρ –scale function of the uncontrolled liquid reserves process. Closed-form expressions for both of these functions as linear combinations of exponentials are provided in the Appendix. In the payout region (i.e. for $s > b$),

the bank pays dividends to maintain its liquid reserves at or below b , so that equity value grows linearly with s . In the earnings retention region (i.e. for $s \leq b$), the term

$$\frac{W(s)}{W'(b)} = w(s, 0, b) = \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b \right]$$

gives the present value of the dividend payments that shareholders receive until default. The term

$$\psi(s; \alpha) - \psi'(b; \alpha) \frac{W(s)}{W'(b)} = \mathbb{E}_s \left[e^{-\rho \tau_{\pi_b}} (\alpha + S_{\tau_{\pi_b}}^{\pi_b})^+ \right]$$

gives the present value of the payoff that the bank shareholders receive in liquidation (the second term on the left hand side reflecting the effects of payouts on this present value).

Equation (6) shows that in the earnings retention region, the value of a barrier strategy depends on the barrier level only through the function

$$H(b; \alpha) = \frac{1 - \psi'(b; \alpha)}{W'(b)}.$$

In the Appendix, we show that there exists a unique payout barrier $b^*(\alpha)$ that maximizes this function over \mathbb{R}_+ and, relying on a verification theorem for the Bellman equation associated with our optimization problem, we prove that the corresponding strategy is optimal among all strategies. This leads to the following result.

PROPOSITION 3 (Auxiliary value function): *Consider a bank with no access to outside funds that produces a cash flow $(\alpha + s)^+$ to shareholders in default. The equity value of such a bank is concave and twice continuously differentiable over $(0, \infty)$ and given by*

$$w(s; \alpha) = w(s; \alpha, b^*(\alpha)),$$

where $b^*(\alpha)$ is the unique solution to $H'(b^*(\alpha); \alpha) = 0$. Furthermore, the optimal policy for bank shareholders is to distribute dividends to maintain liquid reserves at or below $b^*(\alpha)$.

As can be seen from equation (6), the value function of problem (5) is linear with slope equal to one in the payout region where $s > b^*(\alpha)$. Combining this property with the smoothness established in Proposition 3 shows that this value function satisfies:

$$0 = w'(b^*(\alpha); \alpha) - 1 = w''(b^*(\alpha); \alpha).$$

The first of these two conditions is known as the smooth-pasting condition and indicates that liquid reserves are reflected down through dividend payments at the constant barrier level $b^*(\alpha)$. The second is usually referred as the high-contact condition and guarantees the optimality of the dividend barrier; see Dumas (1991).

Importantly, the fact that the value function of problem (5) is concave in liquid reserves shows that shareholders optimally behave in a risk averse way. In particular, it is never optimal for shareholders to engage in risk shifting in the sense that, given the opportunity, shareholders would never decide to increase the level of risk of the bank's assets. This is true even though the bank has risky debt outstanding and the price of deposit insurance is fixed. As we will show in the next section, this important finding carries over to the solution of the bank's optimization problem (2).

C. *Equity value and optimal bank policies*

Having solved for the payout policy of a bank that defaults the first time it runs out of liquid reserves, we now show how to obtain the optimal policies for problem (2) by endogenizing the value of the constant α that determines the equity value of the bank at the point where it runs out of liquid reserves.

Let $b > 0$ be a constant and consider the strategy $\hat{\pi}_b$ that consists in paying dividends to maintain liquid reserves at or below b and in raising funds back to b whenever liquid reserves become negative if that is profitable. Denote by

$$v(s; b) = \mathbb{E}_s \left[\int_0^{\tau_{\hat{\pi}_b}} e^{-\rho t} (dP_t^{\hat{\pi}_b} - d\Phi_t(R^{\hat{\pi}_b})) \right]$$

the corresponding equity value. By definition, this function satisfies

$$v(s; b) = (v(b; b) - b + s - \phi)^+, \quad \forall s \leq 0.$$

Since the bank does not raise funds before the first time τ_{π_b} that liquid reserves are negative, we have that $R_t^{\hat{\pi}_b} = P_t^{\hat{\pi}_b} - P_t^b = 0$ for all $0 \leq t \leq \tau_{\pi_b}$. Therefore, using the above equations together with the law of iterated expectations, we get that

$$\begin{aligned} v(s; b) &= \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho \tau_{\pi_b}} v(S_{\tau_{\pi_b}}^{\pi_b}; b) \right] \\ &= \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho \tau_{\pi_b}} (v(b; b) - b + S_{\tau_{\pi_b}}^{\pi_b} - \phi)^+ \right] \\ &= w(s; v(0; b), b), \end{aligned} \tag{7}$$

where $w(s; \alpha, b)$ is defined as in equation (6). Evaluating the bank's equity at the payout barrier b , subtracting $b + \phi$ from both sides of equation (7), and taking the positive part shows that the equity value associated with zero liquid reserves solves

$$\hat{\alpha}(b) = (v(b; b) - b - \phi)^+ = v(0; b) = (w(b; \hat{\alpha}(b), b) - b - \phi)^+.$$

Given equation (7), the result of the previous section suggest that to obtain the optimal strategy, we should look for a barrier level b such that $b = b^*(\hat{\alpha}(b))$ or, equivalently, for a fixed point of the function defined by

$$g(\alpha) = (w(b^*(\alpha), \alpha, b^*(\alpha)) - b^*(\alpha) - \phi)^+.$$

We show in the Appendix that this function admits a unique fixed point α_0^* in the interval $[0, v^*)$. If the cost of financing is such that

$$\phi \geq \phi^* \equiv w(b^*(0); 0, b^*(0)) - b^*(0),$$

then this fixed point is given by $\alpha_0^* = 0$ and it is not profitable for shareholders to raise funds when the bank runs out of liquid reserves. If the cost of refinancing is such that $\phi < \phi^*$, then we have that $\alpha_0^* > 0$ and it is profitable for shareholders to refinance when the bank runs out of liquid reserves. In this case, we have $\alpha_0^* = w(b_0^*; \alpha_0^*, b_0^*) - b_0^* - \phi$ with $b_0^* = b^*(\alpha_0^*)$ and it follows that α_0^* gives both the bank's equity value at the point where it runs out of liquid reserves and the size of the maximum loss that shareholders are willing to absorb.

The identity (7) and the definition of the constants α_0^* and b_0^* imply that the equity value associated with the strategy $\hat{\pi}_{b_0^*}$ is then given by

$$v(s; b_0^*) = w(s; \alpha_0^*, b_0^*) = w(s; \alpha_0^*).$$

Using the properties of the auxiliary value function derived in the previous section, we show in the Appendix that this function satisfies

$$v(s; b_0^*) = (\alpha_0^* + s)^+ = \max_{b \geq 0} (w(b; \alpha_0^*) - b + s - \phi)^+, \quad s \leq 0,$$

and, relying on a verification theorem for the Bellman equation associated with the bank's optimization problem, we prove that the barrier strategy $\hat{\pi}_{b_0^*}$ is optimal, not only in the class of barrier strategies, but among all strategies. This leads to the following result.

PROPOSITION 4 (Equity value function): *The equity value function is concave and twice continuously differentiable in liquid reserves over $(0, \infty)$ and given by*

$$v(s) = v(s; b_0^*) = w(s; \alpha_0^*, b_0^*),$$

where α_0^* denotes the unique fixed point of $g(\alpha)$ in the interval $[0, v^*)$. The optimal payout strategy consists in paying dividends to maintain liquid reserves at or below $b_0^* = b^*(\alpha_0^*)$. When $\phi < \phi^*$, the bank raises funds to move to b_0^* whenever liquid reserves become negative with a shortfall smaller than $\alpha_0^* > 0$, and liquidate otherwise. When $\phi \geq \phi^*$, the bank never raises equity and defaults the first time that liquid reserves reach $\alpha_0^* = 0$.

Using the concavity of the equity value function, we immediately get the following result:

COROLLARY 5 (Risk shifting): *Under the optimal payout, financing, and default policies, levered banks have no risk shifting incentives.*

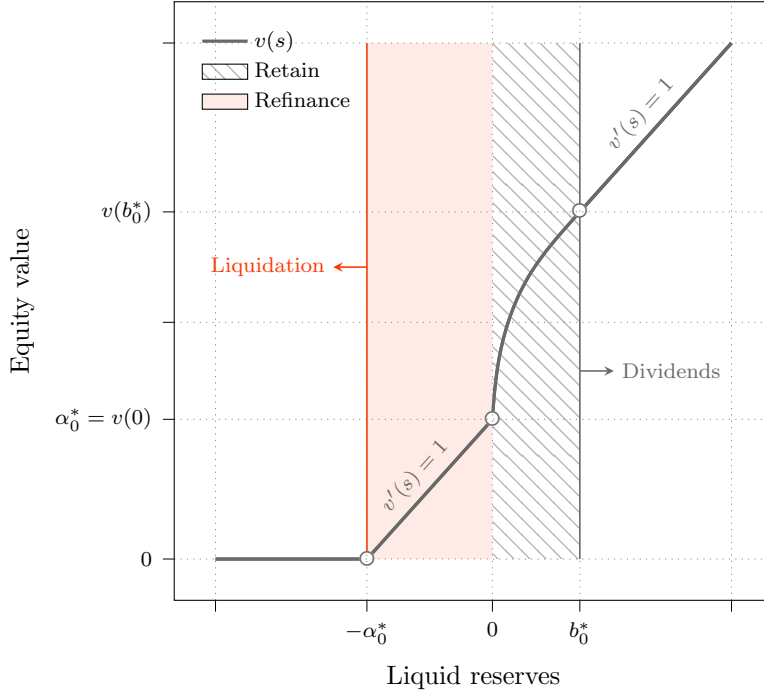
Proposition 4 and Corollary 5 show that when outside funds are costly, the value of equity is a concave function of the bank's liquid reserves over $(0, \infty)$, so that there are *no* incentives for shareholders to engage in *risk-shifting* even if the bank is levered. Specifically, if shareholders could choose the volatility of cash flows from an interval $[\underline{\sigma}, \bar{\sigma}]$, they would pick $\sigma = \underline{\sigma}$. Indeed, if the bank suffers from a series of shocks that deplete its liquid reserves, it incurs some cost to raising external funds. In an effort to avoid these costs and preserve equity value, the bank behaves in a risk-averse fashion. Our results are therefore consistent with the evidence presented in Stein (2014), who shows that banks with the highest levels of capital—i.e. those that are less likely to impose losses on the Deposit Insurance Fund—invest in the same mix of loans and securities and have the same risk exposure as less well capitalized banks. In our model, the bank's risk aversion is endogenous and depends on the level of its liquid reserves. In particular, for liquid reserves above the target level b_0^* , the equity value function satisfies

$$v''(s) = w''(s; \alpha_0^*) = 0,$$

so that the bank becomes risk-neutral. Our results are therefore in sharp contrast with those of previous quantitative models, in which banks can raise equity at no cost and equity value is convex; see for example Merton (1977, 1978), Sharpe (1978), or Subramanian and Yang (2013) among others.

For negative values of liquid reserves, the equity value function is linear as long as shareholders find it optimal to refinance and equal to zero in the default region where $s + \alpha_0^* \leq 0$. The value of equity is therefore convex for negative values of liquid reserves. However, since these values can only be reached following a large loss, the bank does not spend any time in this region. The equity value function is represented in Figure 1.

Figure 1: Equity value for an unregulated bank



Notes. This figure illustrates the shape of the equity value function and the optimal strategy for an unregulated bank. The regions \square on each side of the graph correspond respectively to liquidation (left) and dividend payments (right) while the two intermediate regions \square and \square correspond respectively to earnings retention and refinancing.

Under the optimal payout policy, the bank pays dividends in a minimal way to maintain its liquid reserves at or below b_0^* . This is illustrated by the smooth-pasting and high-contact conditions:

$$0 = v'(b_0^*) - 1 = v''(b_0^*),$$

which show that liquid reserves are optimally reflected down at b_0^* . When liquid reserves exceed b_0^* , the bank is fully capitalized and places no premium on internal funds so that it is optimal to make a lump sum payment $s - b_0^*$ to shareholders. The desired level of reserves,

b_0^* , results from the trade-off between the cost of raising funds and the cost of holding liquid reserves. The next section investigates how this tradeoff is affected by regulatory constraints.

Lastly, note that in our model, banks are subject to insolvency risk even though they can raise outside funds. Notably, Proposition 4 shows that it is optimal for shareholders to default following a shock that brings liquid reserves below $-\alpha_0^* = -v(0)$. Our results are thus in sharp contrast with those of Brownian-driven inventory models (see e.g. Milne and Whalley (2001), Peura and Keppo (2006), or Décamps, Mariotti, Rochet, and Villeneuve (2011)) in which there is no default if banks can raise outside equity. Also, while in those models firms always raise the same amount of capital, there exists some time series variation in the amount of funds raised from investors in our model.

III. The effects of liquidity and capital requirements

This section examines the effects of prudential regulation on the policy choices that maximize equity value and on the risk of insolvency. We introduce prudential regulation in our setup by considering that the regulator can set a lower bound on equity capital when making leverage choices and can force the bank to maintain its liquid reserves in excess of a given regulatory level. That is, our focus is on the instruments of regulation that are being considered in the Third Basel Accord, namely capital and liquidity requirements.

A. *Liquidity requirements*

We start by examining the effects of liquidity requirements on the bank's policy choices and insolvency risk by considering a regulatory constraint that mandates the bank to have a minimum level of liquid reserves $T > 0$ at all times. When $s \leq T$, the bank has the choice of recapitalizing by issuing new equity at a fixed cost ϕ or liquidating. By introducing this liquidity requirement in the model, we are interested in answering the following questions. Can a liquidity requirement affect bank behavior even if it is not binding? If liquidity requirements were to rise, would banks raise their target level of liquid reserves? What are the implications of these behaviors on default risk and on incentives for risk-taking?

The value of equity in a bank subject to a minimum regulatory level of liquid reserves is defined by the optimization problem

$$v(s; T) = \sup_{\pi \in \Pi(s, T)} \mathbb{E}_s \left[\int_0^{\tau_{\pi, T}} e^{-\rho t} (dP_t^\pi - d\Phi_t(R^\pi)) + e^{-\rho \tau_{\pi, T}} (S_{\tau_{\pi, T}}^\pi - D - L)^+ \right], \quad (8)$$

where $\tau_{\pi, T} = \inf\{t \geq 0 : S_{t+}^\pi \leq T\}$ denotes the default time associated with the strategy π in the presence of a liquidity requirement at the level T , the constant L is the face value of risky debt, and $\Pi(s, T)$ denotes the set of admissible payout and financing strategies such that $P_{t+}^\pi - P_t^\pi \leq S_t^\pi - T + R_{t+}^\pi - R_t^\pi$. This constraint implies that the regulatory amount of liquid reserves T cannot be distributed as dividends and therefore guarantees that creditors will receive at least this amount in liquidation.

Let $\ell(s) = (s - D - L)^+$ denote the payment that shareholders receive if liquidation occurs at point where the bank's liquid reserves are equal to s and let $a \wedge b = \min(a, b)$. Solving shareholders' optimization problem leads to the following result.

PROPOSITION 6 (Liquidity requirements): *Equity value in a bank subject to a minimum regulatory level of liquid reserves $T \geq 0$ is given by:*

$$v(s; T) = \begin{cases} v(s - T; 0), & \text{if } \ell(T) \leq \alpha_0^*, \\ w(s - T; \ell(T)), & \text{otherwise.} \end{cases} \quad (9)$$

Liquidity requirements increase the target level of liquid reserves b_T^ in that*

$$b_T^* = \begin{cases} T + b_0^*, & \text{if } \ell(T) \leq \alpha_0^*, \\ T + b^*(\ell(T) \wedge v^*), & \text{otherwise.} \end{cases} \quad (10)$$

When $\ell(T) \leq \alpha_0^$, the bank pays dividends to maintain liquid reserves at or below $T + b_0^*$, raises funds to move to $T + b_0^*$ whenever liquid reserves fall below T with a shortfall smaller than α_0^* , and liquidates otherwise. When $\ell(T) > \alpha_0^*$, the bank never raises equity, defaults as soon as $s \leq T$, and pays dividends to maintain liquid reserves at or below $T + b^*(\ell(T) \wedge v^*)$.*

Proposition 6 shows that the effect of minimum liquidity requirements on the bank's policy choices depends on their stringency. When the liquidity requirement is such that $\ell(T) > \alpha_0^* = v(0)$, it is never optimal for shareholders to refinance at the minimum regulatory level of liquid reserves T since the cash flow to shareholders in liquidation exceeds the continuation value of equity. In this case, the bank liquidates as soon as liquid reserves fall short of T . By contrast, when $\ell(T) \leq \alpha_0^*$, it is optimal for shareholders to refinance at T . In this case, liquidity requirements lead to a drop in equity value as

$$v(s; T) = v(s - T; 0) = v(s - T) \leq v(s), \quad \forall s \geq T,$$

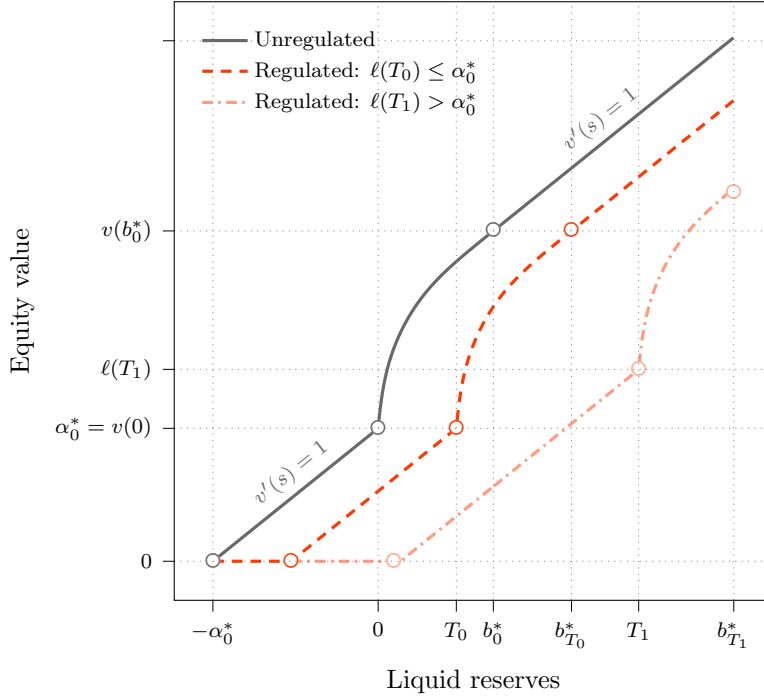
and make it optimal for shareholders to default when liquid reserves fall below $T - \alpha_0^*$. Figure 2 illustrates these effects by plotting the value equity for different liquidity requirements.

Given that liquidity requirements reduce equity value, it is immediate to see that in the short-run, or when the bank has not had the opportunity to move to its new target buffer of liquid reserves, such requirements increase insolvency risk (see also Figure 5 below). That is, liquidity requirements have a negative transitory effect on insolvency risk. We therefore focus below on the long-run effects of liquidity requirements, i.e. on their effects once the bank has had the opportunity to build up liquid reserves. In addition, given that setting $T > \alpha_0^* + D + L$ leads to liquidation whenever liquid reserves fall below the regulatory trigger, we consider that the regulator sets a minimum liquidity requirement satisfying $\ell(T) \leq \alpha_0^*$, consistent with the instruments of regulation of the Third Basel Accord.

Consider first the long-run effects of liquidity requirements on insolvency risk. When $\ell(T) \leq \alpha_0^*$, it is optimal for shareholders to refinance when liquid reserves reach the regulatory level T , at which point the bank raises $b_T^* - T = b_0^*$ to go to the target level of liquid reserves. As a result, equity value at the regulatory trigger T is given by:

$$v(T; T) = (v(b_T^*; T) - (b_T^* - T) - \phi)^+.$$

Figure 2: Equity value for a regulated bank



Notes. This figure illustrates the change in the equity value when going from an unregulated bank (solid line) to either a regulated bank with T_0 such that $\ell(T_0) \leq \alpha_0^*$ (dashed line) or to a regulated bank with T_1 such that $\ell(T_1) > \alpha_0^*$ (dashed-dotted line).

This value corresponds by definition to the maximum amount that shareholders are willing to contribute to keep the bank alive following a large loss. Using equations (9) and (10), we then have

$$v(T; T) = (v(b_T^* - T) - (b_T^* - T) - \phi)^+ = (v(b_0^*) - b_0^* - \phi)^+ = \alpha_0^*,$$

so that shareholders default when liquid reserves fall below T with a shortfall greater than α_0^* . This shows that the presence of liquidity requirement increases the default threshold from the unregulated value $-\alpha_0^*$ to the regulated value $T - \alpha_0^*$. Since the target level of liquid reserves satisfies $b_T^* = b_0^* + T$, we immediately get the following result:

COROLLARY 7 (Liquidity requirements and insolvency risk): *Liquidity requirements have no effect on insolvency risk when banks are optimally capitalized.*

Given that $b_T^* = b_0^* + T$, the cushion of liquid reserves that the bank holds above the required level does not depend on the level chosen by the regulator. In effect, liquidity requirements simply shifts the support of the distribution of liquid reserves from $(-\alpha_0^*, b_0^*]$ to $(-\alpha_0^* + T, b_0^* + T]$. This is due to the fact that raising the level of liquid reserves does not change the trade-off between the cost of carrying liquidity and the cost of outside capital. Lastly, since the bank's attitude towards risks depends on the amount of liquid reserves held in excess of the refinancing threshold, this result in turn implies that, once there has been an opportunity to build up liquid reserves towards the desired level, liquidity requirements do not affect the attitude of the bank towards risks. Therefore, our analysis implies that liquidity requirements have no long-run impact on either risk taking or insolvency risk.

B. Endogenous leverage and capital requirements

So far, our analysis has considered that the debt level of the bank was exogenously given. In this section, we examine the privately optimal mix between equity and debt on the balance sheet of the bank and the effects of regulatory constraints on this mix.

To do so, consider a bank that raises an amount D of deposits and issues at par a perpetual debt contract, with face value L and coupon rate c_L . Assume that the bank is subject to a fixed setup cost $\Psi \geq \phi$ and let us start by determining the set of face values that lead to nonnegative payoffs for both shareholders and debtholders. If creditors agree to the proposed face value L , then the present value to shareholders of setting up the bank is

$$\eta_s(c_L, L, T) = \sup_{e \geq (T + \Psi - D - L)^+} (-e + v(D + L + e - \Psi, T | c_L, L)),$$

where e represents the amount of equity to be injected by shareholders and $v(s, T | c_L, L)$, defined as in (8), gives the equity value of a bank that holds s in liquid reserves, is subject to a minimum liquidity requirement $T \geq 0$, and has issued a perpetual debt contract with

coupon rate c_L and face value L . In order to derive conditions under which this net present value is nonnegative, consider the function

$$N(c_L, T) = T + b_0^*(c_L) + \Psi - D - v(b_0^*(c_L)|c_L),$$

where $b_0^*(c_L)$ and $v(s|c_L)$ respectively give the optimal payout barrier and the equity value of an unregulated bank with coupon rate c_L . Note that, since shareholders in such a bank do not receive any payment in liquidation, these quantities depend neither on the face value of the debt contract L nor on the amount of deposits D that the bank initially raises. In the Appendix, we establish the following result:

PROPOSITION 8 (Net present value): *The net present value of setting up the bank to shareholders is positive if and only if $L \geq N(c_L, T)$, in which case it satisfies:*

$$\eta_s(c_L, L, T)^+ = (L - N(c_L, T))^+,$$

and shareholders' strategy $\Theta^(c_L) = (T, \alpha_0^*(c_L), b_0^*(c_L))$ consists in paying dividends to maintain liquid reserves at or below $T + b_0^*(c_L)$, raising funds to move to $T + b_0^*(c_L)$ whenever liquid reserves fall below T with a shortfall smaller than $\alpha_0^*(c_L)$, and liquidating otherwise.*

The first part of Proposition 8 shows that shareholders break even only if they can obtain at least $N(c_L, T)^+$ from creditors in exchange for the debt contract. If creditors agree to such a price, then the second part of the proposition shows that shareholders will first adjust liquid reserves to the optimal level

$$b_T^*(c_L) = T + b_0^*(c_L)$$

and will then run the bank according to the barrier strategy $\Theta^*(c_L)$. Accordingly, the present value to creditors will be

$$\eta_c(c_L, L, T) = d(T + b_0^*(c_L), \Theta^*(c_L)|c_L, L) - L, \tag{11}$$

where the function

$$d(s, \Theta | c_L, L) = \mathbb{E} \left[\int_0^{\tau(s, \Theta)} e^{-\rho t} c_L dt + e^{-\rho \tau(s, \Theta)} \min (S_{\tau(s, \Theta)}(s, \Theta) - D, L)^+ \right] \quad (12)$$

gives the market value of the bank's debt for all s under the assumption that it follows a barrier strategy $\Theta = (T, a, b)$. The first term on the right-hand side of (12) captures the present value of all coupon payments to debtholders until the occurrence of the default time $\tau(s, \Theta)$ associated with the use of the strategy Θ starting from the initial level s . The second term is the present value of the cash flow to debtholders in default, where we have assumed that deposits were senior to risky debt.⁸

In the Appendix, we derive a closed-form expression for the value of the bank's debt and show that, for any coupon rate $c_L > 0$, the creditors' present value is strictly decreasing in L and admits a unique root $L^*(c_L, T)$ that lies in $(0, c_L/\rho]$. This result implies that the present value of creditors in equation (11) is positive if and only if the proposed face value lies below $L^*(c_L, T)$. Combining this with Proposition 8 then shows that the set of individually rational face values associated to a coupon rate level is given by the interval

$$\{L > 0 : N(c_L, T)^+ \leq L \leq L^*(c_L, T)\}.$$

If the chosen coupon rate is such that this set is empty, then there is no face value of debt that generates nonnegative payoffs for both parties. If the coupon rate is such that $0 < N(c_L, T) \leq L^*(c_L, T)$, then any face value between these points generates a nonnegative present value for both parties, and it remains to determine how the surplus is shared between shareholders and creditors. To do so, we assume that creditors are competitive and do not have any bargaining power against the bank. In this case, the usual undercutting argument implies that shareholders will be able to sell debt at the highest possible price $L^*(c_L, T)$.

We can now examine the debt-equity mix that maximizes initial shareholder wealth as well as the effects of prudential regulation on this mix. In our model, issuing debt is beneficial

⁸Helberg and Lindset (2013) examine the effects of alternative priority rules on bank leverage and credit spreads in a Leland (1994) style model.

for the bank as it reduces its taxes. At the same time, for any given level of liquid reserves, debt financing increases expected bankruptcy costs and the frequency at which the bank raises costly outside capital. When choosing the coupon payment c_L on risky debt, the objective of shareholders is to maximize the value of equity after debt has been issued plus the proceeds from the debt issue net of the cost to shareholders of acquiring the bank assets through the provision of required capital. That is, shareholders solve:

$$\sup_{c_L \in \mathcal{C}(T)} \eta_s(c_L, L^*(c_L, T), T) = \sup_{c_L \in \mathcal{C}(T)} (v(b_0^*(c_L)) + D + L^*(c_L, T) - T - b_0^*(c_L) - \Psi), \quad (13)$$

where the feasible set is given by

$$\mathcal{C}(T) = \{c_L \geq 0 : N(c_L, T) \leq L^*(c_L, T)\}.$$

One key difference between the optimization problem (13) and leverage choices in dynamic corporate finance models such as Fischer, Heinkel, and Zechner (1989) or Leland (1994) is that the initial contribution of shareholders to the bank's capital is endogenous and depends on the liquidity constraints faced by the bank. Another important difference is that, in our model, the bank determines its optimal capital structure by balancing tax benefits against both issuance costs and bankruptcy costs.

While problem (13) is not amenable to an explicit solution, we can show that as long as the minimum liquidity requirement does not exceed the amount of deposits—which is the relevant case in practice—the presence of a liquidity requirement will either deter shareholders from setting up the bank or will have no effect on the optimal coupon rate. In particular, we establish the following result in the Appendix.

PROPOSITION 9 (Optimal leverage and liquidity requirements): *Assume that the optimal coupon rate $c_L^*(0) \in \mathcal{C}(0)$ is well-defined. If the liquidity requirement is such that $T \leq D$ then either $\mathcal{C}(T)$ is empty or $c_L^*(T) = c_L^*(0)$.*

In addition to liquidity requirements, the bank may also face capital requirements. That is, in an attempt to reduce insolvency risk, the regulator may constrain the bank to have a

minimum level of equity capital, i.e. to choose a debt level $c_L \in [0, \hat{c})$, where $\hat{c} \geq 0$ is a cap on the coupon payment on risky debt. We investigate in the next section the effects of such requirements on the bank's policy choices and the risk of insolvency.

IV. Model analysis

A. Parameter values and implied variables

This section provides additional results and illustrates the effects of frictions and regulatory constraints on bank's policy choices using numerical examples. The values of the model parameters are set as follows: the risk-free rate of return $\rho = 5\%$, the after-tax mean return on risky assets $(1 - \theta)\mu - \frac{\lambda}{\beta} = 13.5\%$, the diffusion coefficient $\sigma = 8\%$, the frequency of large losses $\frac{1}{\lambda} = 2$, the mean size of large losses $\frac{1}{\beta} = 9\%$, the corporate tax rate $\theta = 35\%$, the fixed volume of demand deposits $D = 4.5$ and the corresponding cash outflow $c_D = 4.5\%$, and the cost of outside funds $\phi = 0.75\%$. Table 1 summarizes these parameter values.

These parameter values imply that large losses represent on average 50% of yearly income and occur every other year in expectation and that the cost of refinancing represents between $\frac{\phi}{b_T^* + \alpha_T^*} = 1.27\%$ (at the default threshold) and $\frac{\phi}{b_T^*} = 6.36\%$ (at the regulatory threshold) of the capital raised. For these parameter values, the optimal coupon on risky debt is $c_L = c_L^*(0) = 11.49\%$ and the face value of risky debt is $L = 2.25$. Because for any coupon level, liquid reserves fluctuate between their minimum regulatory level and the target buffer set by bank shareholders, the leverage ratio of the bank effectively remains between two bands as in the dynamic capital structure models of Fischer, Heinkel, and Zechner (1989), Hackbarth, Miao, and Morellec (2006), Strebulaev (2007), and Morellec, Nikolov, and Schürhoff (2012). Notably, the leverage ratio of the bank – defined as debt value plus deposits over total bank value – fluctuates between 91.89% and 93.49% at the optimal coupon level and deposits represent between 61.25% and 62.31% of total bank value, consistent with the figures reported in Gropp and Heider (2010).⁹ Lastly, the 1-year default probability for the bank at optimal

⁹Interestingly, changing the pricing of deposit insurance has little effect on the leverage ratio selected by shareholders. For example, the overall leverage of the bank fluctuates between 91.45% and 93.26% if we

Table I: Parameter values and implied variables

A. Parameter values		
Parameter	Symbol	Value
Drift rate net of taxes	$(1 - \theta)\mu$	0.18
Volatility	σ	0.08
Arrival intensity	λ	0.50
Mean jump size	$1/\beta$	0.09
Tax rate	θ	0.35
Face value of debt	L	2.25
Face value of deposits	D	4.50
Coupon rate	c_L	0.1149
Cost of deposits	c_D	0.045
Discount rate	ρ	0.05
Financing cost	ϕ	0.0075
Liquidity requirement	T	0.00
B. Implied variables		
Variable	Symbol	Value
First best value	v^*	0.6220
Equity value when $s = T = 0$	α_0^*	0.4699
Target level of liquid reserves	b_0^*	0.1179
Financing cost (%)		[0.0127, 0.0636]
Leverage		[0.9189, 0.9349]
Leverage from deposits		[0.6125, 0.6231]

leverage is 0.1057%, consistent with those reported in Crossen and Zhang (2012) or Hamilton, Munves, and Smith (2010) for 1-year default probabilities in the financial industry.

B. Default and payout decisions

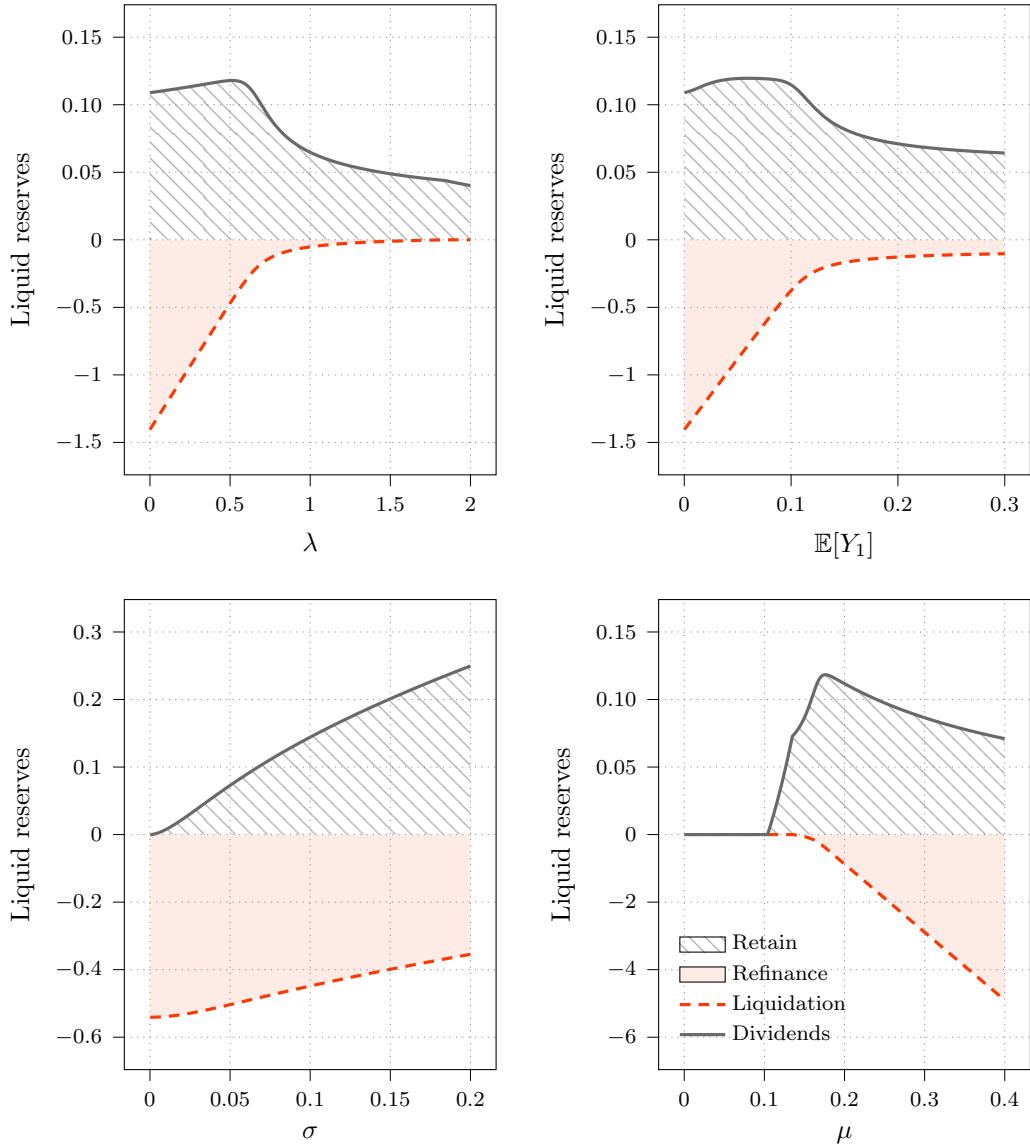
We start our analysis by examining the determinants of the target level of liquid reserves. To do so, we plot in Figure 3 the unconstrained target buffer b_0^* as a function of the intensity of large losses λ , the size of large losses $\frac{1}{\beta}$, the volatility coefficient σ , and the the rate of return μ . In the model, the target buffer of liquid reserves maximizes the value of equity and results from a trade-off between the carry cost of liquidity ρ and the cost of raising increase c_D from 4.5% to 9% and between 91.46% and 93.56% if we increase c_D to 13.5%. This is due to the fact that an increase in c_D leads not only to a decrease in $c_L^*(T)$ but also to a decrease in equity value.

new capital. Because an increase in volatility increases the likelihood of a costly equity issuance, the target buffer b_0^* increases with σ . An increase in the frequency of large losses λ or in their size $\frac{1}{\beta}$ has two opposite effects on b_0^* . First, it increases the likelihood of a costly security issuance and, hence, shareholders' incentives to build up liquid reserves. Second, it reduces the expected cash flow from operating the bank's assets (and thus the bank's franchise value) and therefore shareholders' incentives to contribute capital. The first effect dominates for low values of λ and $\frac{1}{\beta}$; the second effect dominates for larger expected losses. The effect of the rate of return μ on the target level of liquid reserves also results from two opposite effects. On the one hand, a rate of return increases the bank's franchise value and shareholders' incentives to contribute capital. On the other hand a higher rate of return increases revenues and reduces the role of liquid reserves as a buffer to absorb losses.

Consider next the determinants of the bank's default threshold. As shown in Proposition 4, when there is no liquidity requirement in that $T = 0$, default occurs when the bank is hit with a negative shock that takes its liquid reserves below $-\alpha_0^*$. This threshold is given by the value of equity at the target, i.e. $v(b_0^*)$, net of the new provision of capital b_0^* and of the refinancing cost ϕ . By decreasing $v(b_0^*)$, an increase in the intensity of large losses λ , in their size $\frac{1}{\beta}$, in the cost of financing ϕ , or in the diffusion coefficient σ lead to an increase in the default threshold. By contrast, an increase in the rate of return μ (or a decrease in the coupon rate c) leads to an increase in the franchise value of the bank and, therefore, to a decrease in the default threshold. These effects are illustrated in Figure 3.

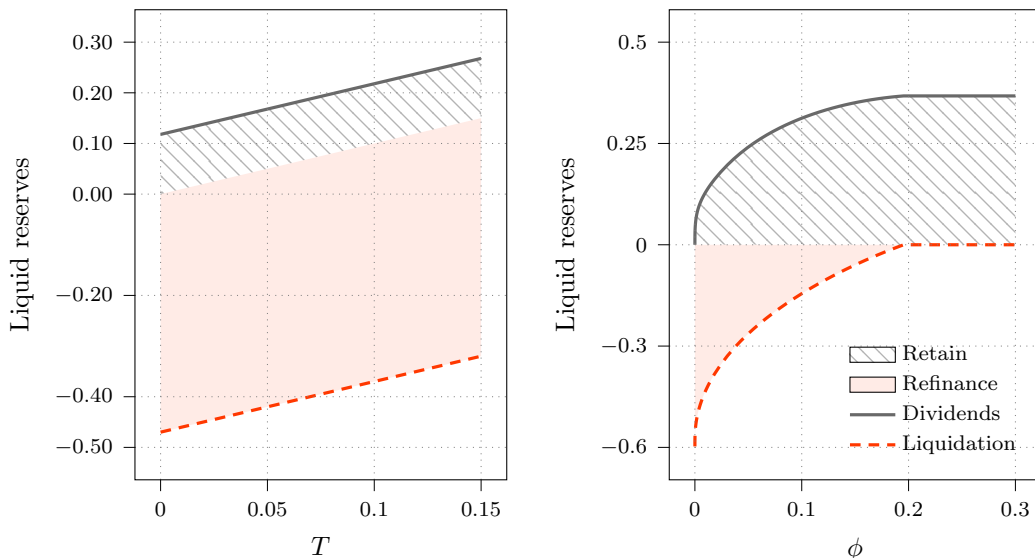
To illustrate the effects of liquidity requirements on bank's policy choices, Figure 4 plots the target level of liquid reserves as well as the default threshold as functions of the minimum requirement T . Consistent with Proposition 6, the figure shows that both thresholds grow linearly with the minimum liquidity requirement T . That is, regulatory requirements affect bank behavior even when they are not binding. The figure also shows that banks will generally hold a significant buffer of liquid reserves in excess of the liquidity requirements in order to reduce refinancing costs. Therefore, our theory predicts that liquidity requirements should be non-binding for most banks. Lastly, Figure 4 shows that both thresholds increase with the cost of external funds ϕ . Indeed, an increase in the cost of raising funds decreases

Figure 3: Effect of the cash-flow parameters



Notes. This figure plots the target level of liquid reserves (solid line) and the liquidation threshold (dashed line) as functions of the jump arrival intensity, the mean jump size, the cash flow volatility and the cash flow drift. In each panel the upper and lower regions \square correspond respectively to dividend payments and liquidation while the two intermediate regions \square and \square correspond respectively to earnings retention and refinancing.

Figure 4: Effect of liquidity requirements and financing costs



Notes. This figure plots the target level of liquid reserves (solid line) and the liquidation threshold (dashed line) as functions of the required level of liquid reserves and the fixed cost of equity financing. In each panel the upper and lower regions \square correspond respectively to dividend payments and liquidation while the two intermediate regions \square and \square correspond respectively to earnings retention and refinancing.

the benefits of refinancing, leading to an increase in the selected default threshold. In addition, an increase in the cost of raising funds raises the value of inside equity and therefore shareholders' incentives to build up liquid reserves.

C. Regulation and insolvency risk

An important question that our model allows to answer is whether regulatory requirements affects insolvency risk. Proposition 6 and Corollary 7 show that altering the liquid reserves of banks has no long-run impact for the risk of insolvency (this is also illustrated by Figure 4 in which the spread between the target level of liquid reserves b_T^* and the default threshold $T - \alpha_0^*$ is independent of T). To examine the short-run effects of liquidity requirements on insolvency risk, Figure 5 plots the relative change in the probability of default of the bank

that results from imposing a minimum liquidity requirement . When the bank follows equity value-maximizing policies and is optimally financed, this relative change is given by:

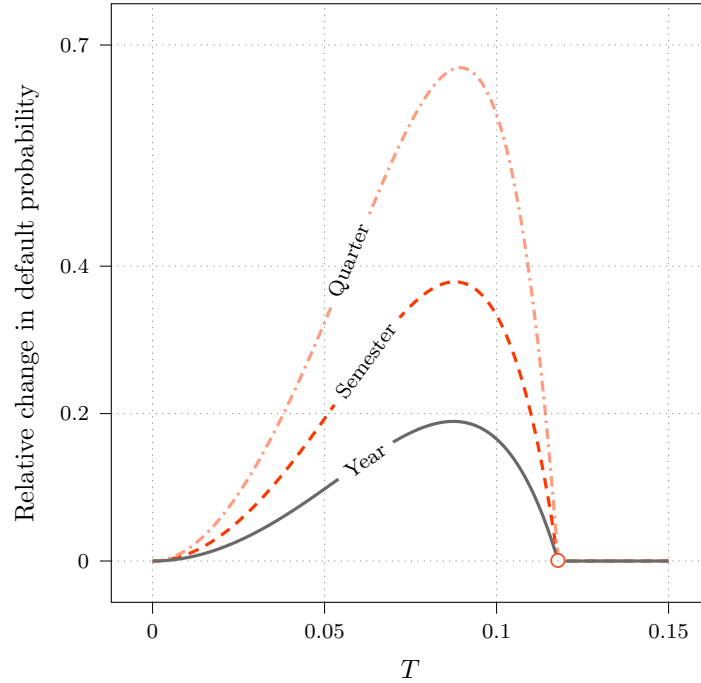
$$\frac{P(\tau(b_0^*; T, \alpha_0^*, b_0^*) < t)}{P(\tau(b_0^*; 0, \alpha_0^*, b_0^*) < t)} - 1,$$

where $\tau(b_0^*; T, \alpha_0^*, b_0^*)$ is the time of default for a bank with liquid reserves b_0^* , default trigger at $-\alpha_0^*$, payout trigger at b_0^* , and facing a liquidity requirement T . Figure 5 considers three different horizons to compute the change in the probability of default: A three-month horizon ($t = 0.25$), a six-month horizon ($t = 0.5$), and a one-year horizon ($t = 1$). Consistent with the discussion in section III.A, Figure 5 shows that imposing a minimum liquidity requirement always increases insolvency risk in the short-run. In addition, the figure shows that the increase in the probability of default is larger for shorter horizons, since it is less likely that the bank has had the opportunity to build up liquid reserves. The figure also shows that for large liquidity requirements (i.e. when T is large), it is optimal for the bank to recapitalize immediately. In such cases, liquidity requirements do not change the probability of default.

One way to constrain the bank's leverage is to constrain its liquid asset holdings by imposing liquidity requirements. However, we have just shown that imposing such constraints leads to a short-run increase in insolvency risk for any given coupon c_L . In addition, as shown by Proposition 9, liquidity requirements have no effects on banks' *ex ante* choice of debt level, i.e. on the liability side of the balance sheet, in that $c_L^*(T) = c_L^*(0)$ for all $T \leq D$. As a result, a more direct mechanism, constraining directly banks' leverage ratios, may be necessary. One measure advocated by practitioners and academics to limit default risk in the banking sector is to impose capital requirements (see e.g. Admati and Hellwig (2013)), i.e. a minimum level of equity capital (or a maximum coupon level) for banks.

To illustrate the effects of such capital requirements on insolvency risk, Figure 6 plots the probability of default of the bank over a one-year horizon as a function of its coupon payment c_L , assuming that the bank holds the target amount of liquid reserves $b_T^*(c_L)$. (The Appendix shows how to compute this default probability.) The three curves represent this default probability for different values of the cost of financing ϕ . On each curve the

Figure 5: Insolvency risk and liquidity requirements

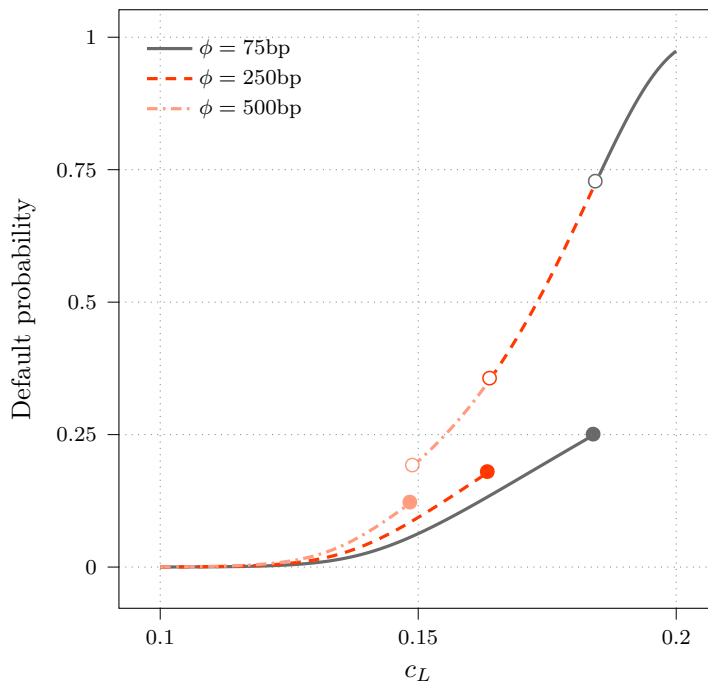


Notes. The figure plots the relative change in default probability at a 3 month (dotted line), 6 month (dashed line) and 1 year horizon (solid line) induced by a tightening of the liquidity requirement for a bank with liquid reserves equal to the unregulated target b_0^* . The dot indicates the point where the liquidity requirement level reaches the unregulated target.

discontinuity occurs at the point where the bank no longer uses outside equity financing, i.e. at the point where $\ell(T)$ becomes larger than α_0^* .

As shown by the figure, capital requirements have a very strong quantitative effect on insolvency risk. In our base case environment for example, reducing the coupon rate of the bank from $c_L = 12.8\%$ – corresponding to a equity capital ratio of 6% advocated by U.S. regulators – to $c_L = 5.34\%$ – corresponding to a the equity capital ratio of 20% recommended by Admati and Hellwig (2013) – reduces the probability of default over a one-year horizon from 0.1518% to less than 0.0001%. In effect, imposing capital requirements leads to an

Figure 6: Insolvency risk and coupon level



Notes. This figure plots the default probability at an horizon of one year for an optimally capitalized, unregulated bank as a function of the coupon rate c_L on long term debt for different level of the fixed cost of financing. On each curve the discontinuity occurs at the point where the bank no longer uses outside equity financing.

increase in the *ex post* franchise value of the bank (i.e. in the cash flow rate of the bank net of the payments to debtholders) and, hence, in shareholders' willingness to absorb losses.¹⁰

D. Regulation and value

Our analysis so far has shown that while liquidity requirements increase banks' insolvency risk in the short-run, capital requirements reduce this risk. Another important aspect of

¹⁰In our model, the bank only has access to one class of risky assets. As a result, the ex-ante constraint on the coupon level that we consider in the paper is equivalent to a risk-weighted capital ratio. When banks have access to multiple risky assets, risk-based capital charges constrain the bank's asset choices to reflect the trade-off between risk and return. See e.g. Rochet (1992) for an analysis of the benefits and costs of risk-weights.

regulation that has been widely discussed in the financial press relates to its effects on valuations. To measure the value effects of liquidity and capital requirements, Figure 7 plots the net present value to shareholders (top left panel) as well as the bank's leverage (top right panel) as functions of the selected coupon payment.

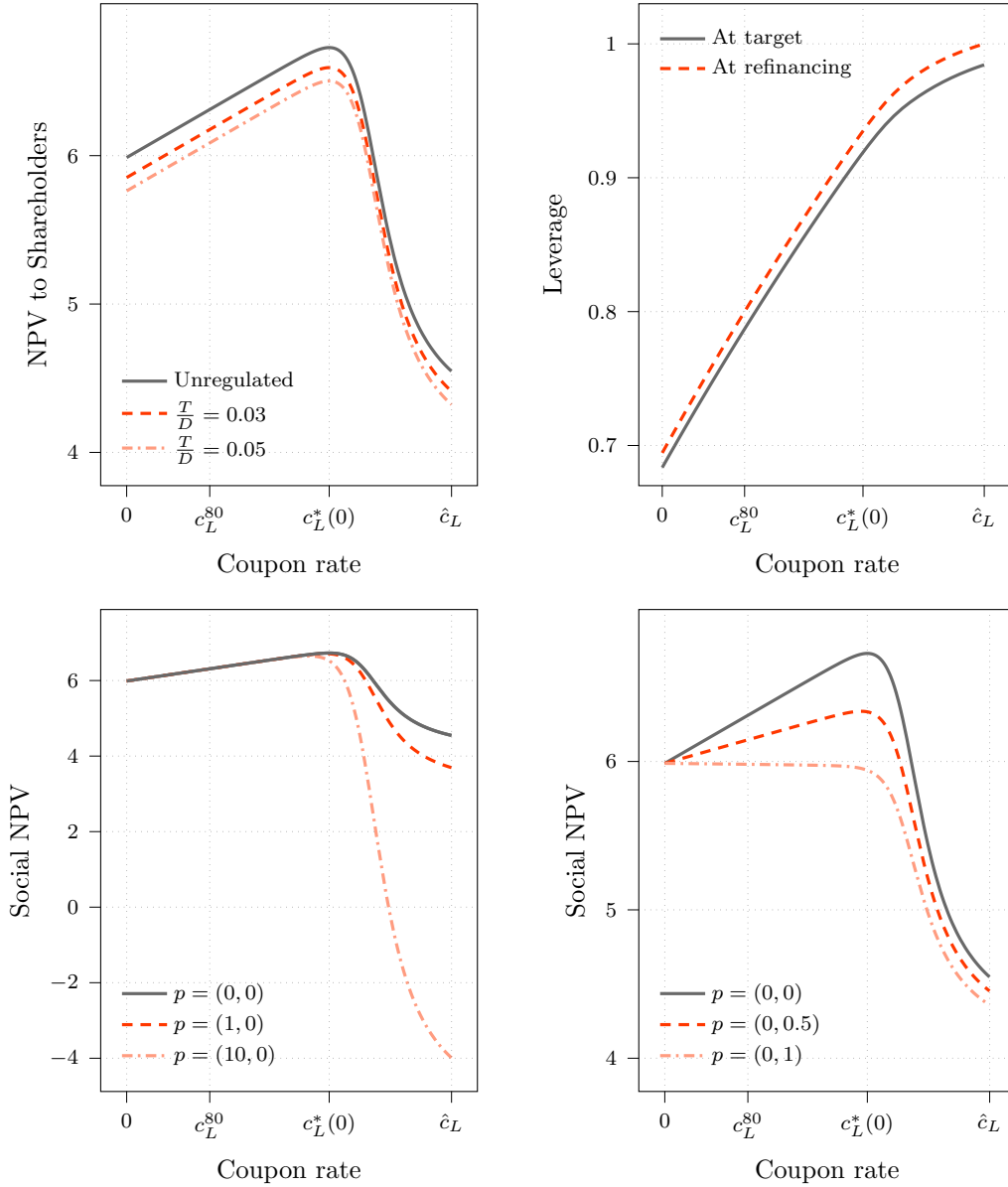
As shown by Figure 7, both liquidity and capital requirements have significant value effects. For example, the total value of the bank, given by $v(b_0^*(c_L)) + D + L^*(c_L, T)$, is reduced by 6.25% when moving from the equity value-maximizing leverage ratio to a leverage ratio with 20% equity capital. In our model, this decrease in value is due to an increase in the bank's cost of capital. Similarly, the figure shows that imposing a liquidity requirement leads not only to a short-run increase in default risk but also in a large reduction in the value of the bank to shareholders. In our base case environment for example, requiring a minimum liquid reserves representing 3% of the value of deposits (i.e. $\frac{T}{D} = 0.03$) reduces the total value of the bank by 1.97%. Two additional features are worth being mentioned. First, because of the presence of deposits, leverage fluctuates between 61.25% and 62.31% of total bank value when $c = 0$ on the figure. Second, Figure 7 shows that we have $c_L^*(T) = c_L^*(0)$ for all T , consistent with Proposition 9.

Next, to measure the social benefits of regulation, we compute the social net present value of the bank defined as the net present value to shareholders net of the cost of debt to the regulator. We consider that this cost has two components. First, it includes a flow cost that (partially) balances the tax benefits of debt to the bank shareholders. Second, it includes an additional cost of default, related for example to systemic effects. Specifically, we define the net present value to the regulator $\eta_r(p, c_L)$ or social net present value as:

$$\eta_r(p, c_L) \equiv \eta_s(c_L) - \mathbb{E}_{b_T^*(c_L)} \left[p_0 e^{-\rho \tau_T^*(c_L)} + p_1 \int_0^{\tau_T^*(c_L)} e^{-\rho t} \theta c_L dt \right],$$

where $\eta_s(c_L)$ is the value of the bank to shareholders (plotted on the top left panel of Figure 7), p_0 represents the systemic cost borne by the regulator in default, and p_1 represents the flow cost of debt to the regulator.

Figure 7: Regulation, leverage, and value



Notes. This figure plots the net present value to shareholders as a function of the coupon level for different values of the ratio $\frac{T}{D}$ (upper left panel), the leverage at the target level of liquid reserves $b_T^*(c_L)$ and at the refinancing point T as functions of the coupon level (upper right panel), and the net present value to the regulator as a function of the coupon level for different values of the cost parameters $p = (p_0, p_1)$ (lower panels). On the horizontal axis $c_L^{80} = 0.0471$ indicates the coupon level where the leverage at refinancing equals 80%, $c_L^*(0) = 0.1149$ indicates the coupon level that is optimal for the shareholders of the bank given any liquidity requirement $T \leq D$, and $\hat{c}_L = 0.1843$ indicates the coupon level where the bank switches to a no-refinancing strategy.

The bottom panels of Figure 7 plot the social net present value for different systemic costs of default (bottom left panel) as well as different flow costs of debt (bottom right panel) as functions of the selected coupon payment. As shown by the figure, systemic default costs have little effect on the coupon payment that maximizes the social net present value. For example, increasing systemic default costs from $p_0 = 0$ (no systemic costs) to $p_0 = 10$ (systemic costs corresponding to 150% of the total bank value) only changes the range of leverage ratios from [91.89%; 93.49%] to [89.78%; 91.32%]. This result is due to the fact that the default probability is very low at the equity value-maximizing leverage ratio so that increasing systemic default costs only has a marginal effect on the coupon payment maximizing the social value of the bank.

Similarly, reducing the tax benefit of debt to the regulator only has a large effect if the reduction is drastic, i.e. if p_1 is very large. For example, increasing the flow cost of debt to the regulator from $p_1 = 0$ to $p_1 = 0.5$ (i.e. reducing the tax benefit of debt to the regulator by 50%) only changes the range of leverage ratios from [91.89%; 93.49%] to [91.26%; 92.84%]. Overall, our analysis therefore suggests that while constraining capital ratios clearly reduces insolvency risk, it seems difficult to obtain optimal leverage ratios in the range of 80%, unless one completely removes the tax deductibility of interest payments.

V. Conclusion

We develop a dynamic model of banking in which banks face taxation, flotation costs of securities, and default costs and are financed with equity, insured deposits, and risky debt. In this model, liquidity management, capital structure policies, and default decisions are jointly and endogenously determined. Shareholders have limited liability and banks' policies maximize shareholder value.

Using this model, we show that when raising outside funds is costly, inside and outside equity are not perfect substitutes and banks find it optimal to hold buffers of liquid assets. To reduce the risk of default and save on recapitalization costs, banks manage their liquidity buffers dynamically by adjusting their dividend payments to shareholders. Banks facing

higher cost of outside financing are less levered, pay less dividends, hold more liquid reserves, and default earlier. In addition, we show that equity value is concave, implying that shareholders in levered banks have no incentives to increase risk.

Turning to regulation, we show that liquidity requirements constraining banks to hold a minimum amount of liquid reserves have no long-run effects on default risk or bank risk-taking but may increase default risk in the short run. We also show that capital requirements, indicating how much equity capital banks should have relative to their total assets, increase banks' *ex post* franchise value and reduce default risk. *Ex ante*, however, such requirements may reduce significantly total bank value by increasing the bank's cost of capital. In our base environment for example, we find that increasing capital ratios from the equity value-maximizing level to 20%, as suggested by Admati and Hellwig (2013), reduces total bank value by 6.25%.

While the framework developed in this paper is based on a number of simplifying assumptions, we believe that it is a natural starting point to think about bank equity capital and liquid reserves and to examine the effects of prudential regulation on bank behavior and insolvency risk. In future research, we plan to extend this framework to examine additional issues related to bank optimal capital structures and asset choices.

References

- Acharya, V., T. Cooley, M. Richardson, and I. Walter, 2009, "Manufacturing Tail Risk: A Perspective on the Financial Crisis of 2007-2009," *Foundations Trend in Finance* 4, 247-325.
- Admati, A., P. DeMarzo, M. Hellwig, and P. Pfleiderer, 2011, "Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is *Not* Expensive," Working Paper, Stanford University.
- Admati, A., and M. Hellwig, 2013, *The Bankers' New Clothes: What's Wrong with Banking and What to Do about It*, Princeton: Princeton University Press.
- Albrecher, H., and S. Thonhauser, 2009, "Optimality Results for Dividend Problems in Insurance," *Revista de la Real Academia de Ciencias Exactas, Fisicas y Naturales. Serie A. Mathematicas* 103, 295-320.
- Bhattacharya, S., M. Plank, G. Strobl, and J. Zechner, 2002, "Bank Capital Regulation with Random Audits," *Journal of Economic Dynamics and Control* 26, 1301-1321.
- Biais, B., T. Mariotti, J.C. Rochet, and S. Villeneuve, 2010, "Large Risks, Limited Liability, and Dynamic Moral Hazard," *Econometrica* 78, 73-118.
- Black, F., and M. Scholes, 1973, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economy* 81, 637-659.
- Corless, R., Gonnet, G., Hare, D., Jeffrey, D. and Knuth, D., 1996, "On the LambertW function," *Advances in Computational Mathematics* 5, 329-359.
- Crossen, C., and X. Zhang, 2012, "Validating the Public EDF Model for Global Financial Firms," Moody's Analytics.
- Décamps, J.P., T. Mariotti, J.C. Rochet, and S. Villeneuve, 2011, "Free Cash Flow, Issuance Costs, and Stock Prices," *Journal of Finance*, 66, 1501-1544.
- Décamps, J.P., J.C. Rochet, and B. Roger, 2004, "The Three Pillars of Basel II: Optimizing the Mix," *Journal of Financial Intermediation* 13, 132-155.

- DeMarzo, P., D. Duffie, and F. Varas, 2012, "Contingent Capital Requirements," Working paper, Stanford University.
- Dumas, B., 1991, "Super Contact and Related Optimality Conditions," *Journal of Economic Dynamics and Control* 15, 675-685.
- Fischer, E., R. Heinkel, and J. Zechner, 1989, "Dynamic Capital Structure Choice: Theory and Tests," *Journal of Finance* 43, 19-40.
- Froot, K. and J. Stein, 1998, "Risk Management, Capital Budgeting, and Capital Structure Policy for Financial Institutions: An Integrated Approach," *Journal of Financial Economics* 47, 55-82.
- Giammarino, R., T. Lewis, and D. Sappington, 1993, "An Incentive Approach to Banking Regulation," *Journal of Finance* 48, 1523-1542.
- Gerber, H., and H. Yang, 2010, "Obtaining the Dividend-Penalty Identities by Interpretation," *Insurance: Mathematics and Economics* 47, 206-207.
- Gerber, H., S. Lin, and H. Yang, 2006, "A Note on the Dividend-Penalty Identity and the Optimal Dividend Barrier," *ASTIN Bulletin* 36, 489-503.
- Gorton, G., 2012, *Misunderstanding Financial Crises: Why we Don't See Them Coming*, Oxford University Press.
- Gropp, R., and F. Heider, 2010, "The Determinants of Bank Capital Structure," *Review of Finance* 14, 587-622.
- Hackbarth, D., J. Miao, and E. Morellec, 2006, "Capital Structure, Credit Risk and Macroeconomic Conditions," *Journal of Financial Economics* 82, 519-550.
- Hamilton, D., D. Munves, and A. Smith, 2010, "Banks and their EDF Measures Now and Through the Credit Crisis," Moody's Analytics.
- Hanson S., A. Shleifer, J. Sein, and R. Vishny, 2014, "Banks as Patient Investors," Working paper, Harvard University.

- Helberg, S., and S. Lindset, 2013, "Debt Bank Regulations: Implications for Bank Capital and Bond Risk," Working paper, Norwegian University of Science and Technology.
- Hellmann, T., K. Murdock, and J. Stiglitz, 2000, "Liberalization, Moral Hazard in Banking, and Prudential Regulation: Are Capital Requirements Enough?" *American Economic Review* 90, 147-165.
- Hellwig, M., 1998, "Banks, Markets, and the Allocation of Risk in an Economy," *Journal of Institutional and Theoretical Economics* 154, 328-345.
- Merton, R., 1977, "An Analytic Derivation of the Cost of Deposit Insurance and Loan Guarantees," *Journal of Banking and Finance* 1, 512-520.
- Merton, R., 1978, "On the Cost of Deposit Insurance When There are Surveillance Costs," *Journal of Business* 51, 439-452.
- Milne, A., and E. Whalley, 2001, "Bank Capital Regulation and Incentives for Risk-Taking," Working paper, Cass Business School.
- Milne, A., 2004, "The Inventory Perspective on Bank Capital," Working paper, Cass Business School.
- Modigliani, F., and M. Miller, 1958, "The Cost of Capital, Corporation Finance, and the Theory of Investment," *American Economic Review* 48, 261-297.
- Morrison, A., and L. White, 2005, "Crises and Capital Requirements in Banking," *American Economic Review* 95, 1548-1572.
- Morellec, E., B. Nikolov, and N. Schürhoff, 2012, "Corporate Governance and Capital Structure Dynamics," *Journal of Finance* 67, 801-847.
- Peura, S., and J. Keppo, 2006, "Optimal Bank Capital with Costly Recapitalization," *Journal of Business* 79, 2163-2201.
- Repullo, R., and J. Suarez, 2013, "The Procyclical Effects of Bank Capital Regulation," *Review of Financial Studies* 26, 452-490.

- Rochet, J.-C., 1992, "Capital Requirements and the Behaviour of Commercial Banks," *European Economic Review* 36, 1137-1170.
- Rochet, J.-C., and Y. Zargari, 2013, "Bail-Ins: A Possible Solution to Deal with Systemically Important Financial Institutions," Working paper, University of Zurich.
- Sharpe, W., 1978, "Bank Capital Adequacy, Deposit Insurance, and Security Values," *Journal of Financial and Quantitative Analysis* 13, 701-718.
- Stein, J., 2014, "Banks as Patient Investors," Speech, Board of Governors of the Federal Reserve System.
- Strebulaev, I., 2007, "Do Tests of Capital Structure Mean What They Say?" *Journal of Finance* 62, 1747-1787.
- Subramanian, A. and B. Yang, 2012, "Dynamic Prudential Regulation," Working paper, Georgia State University.