Why and When are Preferences Convex? Threshold Effects and Uncertain Quality

Trenton G. Smith, Attila Tasnádi

Yixin LU

LARGE Meeting 2010
Department of Decision and Information Sciences
Rotterdam School of Management

February 3rd, 2010

Summary

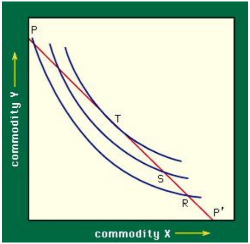
This paper consider circumstances under which *convex* preferences are optimal, with a specific setting:

- goods/products possess some hidden quality with *known distribution*
- consumer chooses a bundle of goods that maximizes the probability that he/she receives some threshold level of this quality

It is shown that

- if the threshold is small w.r.t. consumption level, convex preferences
- if the threshold is large w.r.t. consumption level, nonconvex preferences

Convexity of preferences



Convexity of preferences:

- one of the canonical assumptions in economic theory;
- combination of bundles are at least as good as the extreme bundles.

Convexity is appealing in part because it is conducive to marginal analysis and single-valued continuous demand function.

However, preferences are not always convex in practice

Advertising Effects Shifts in individual demand in response to product advertisements.

Milgrom & Roberts 1986: informative signaling effects
Smith & Tasnádi 2009: thresholds are sensitive to information

When convex preferences are beneficial?

Diversity in consumption

Example: human diet

How to measure the optimality?

Follow the Behavioral Ecology,

- natural selection favors agents who maximizes their expected payoff(utility) in a stochastic environment;
- preferences shall be considered optimal w.r.t underlying stochastic payoff structure.

Problem Description

A decision maker is face with a menu of two products: x and y must choose how much of each to consume, given

- fixed income m;
- prices p for x and 1 for y;
- a critical threshold k for a single unobservable characteristic (quality), i.e., the consumer seeks to maximize the probability that he acquires k units of this quality;
- the quality per unit of x and y are independent random variables, denoted by C_x and C_y with distribution functions F and G.

Mathematical Formulation

The decision problem can be stated as follows:

$$\begin{aligned} \max_{(x,y)} \quad & U(x,y) \\ \text{s.t.} \quad & px + y \leq m \\ & x, y \geq 0 \end{aligned}$$

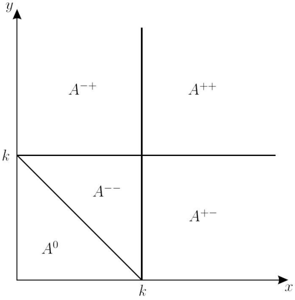
Assuming the random variables are continuous and

- have density functions f and g;
- the support of them is the unit interval.

Then we have

$$U(x,y) = P(C_x x + C_y y \geq k) = \int_k^\infty \int_{\max\{0,t-y\}}^{\min\{x,t\}} \frac{1}{xy} f\left(\frac{z}{x}\right) g\left(\frac{t-z}{y}\right) dz dt$$

Five regions in commodity space



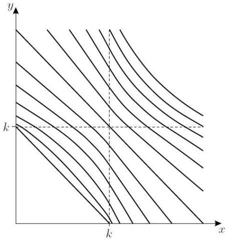
- "death zone" A^0;
- low consumption region A^{--};
- low consumption of x region A^{-+};
- low consumption of y region A^{+-};
- high consumption region A^{++}.

Case 1: uniform case

Assuming the random variables C_x and C_y follow uniform distribution:

$$F(x) = G(x) = \begin{cases} 0, & \text{if} x < 0; \\ x, & \text{if} x \in [0,1]; \\ 1, & \text{if} x > 1. \end{cases}$$

When $k = 1$, the corresponding indifference curves are:



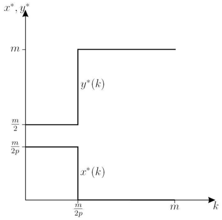
Optimal solutions for Case 1

Finding 1

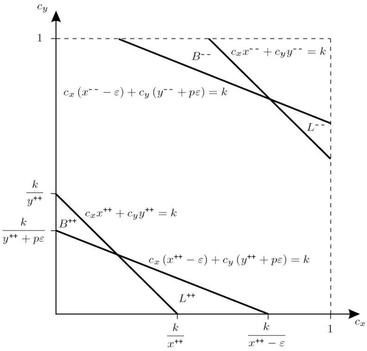
: In Case 1, we observe a discontinuous change in behavior at $k = m/(2p)$ if $p \geq 1$ and at $k = pm/2$ if $p < 1$.

Illustration

For $p > 1$, there is a discontinuous change in the demand corresponding as the threshold k increases.



Geometrical interpretation of threshold effects



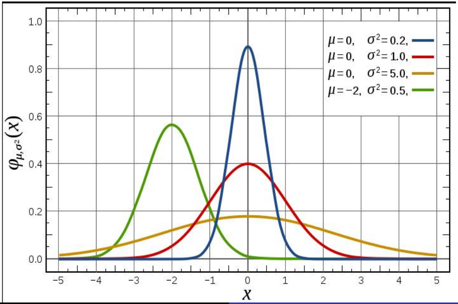
Implications

- when k is small, minimize probability associated with "very bad" outcomes;
- when k is large, maximize probability associated with "very good " outcomes;

Discontinuous threshold effect: I

Lemma

(Proschan 1965): Suppose the independent nonnegative random variables C_x and C_y and have *log-concave* density f, for any given $m > 0$, $Z_{\lambda,m} := \lambda C_x + (m - \lambda)C_y$ is strictly increasing in peakness in λ on $[0, m/2]$.



Discontinuous threshold effect: II

Main Theorem

Suppose the independent nonnegative random variables C_x and C_y are symmetric around their common means $\mu = EC_x = EC_y$, have *log-concave* density f and $supp(C_x) = supp(C_y) = [0, 2\mu]$.

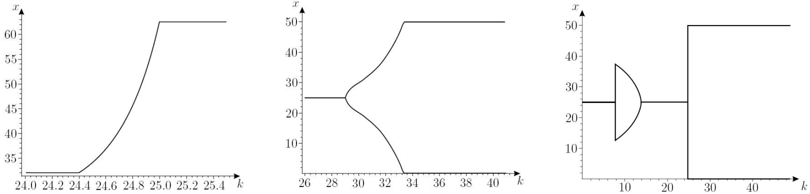
- k increases from 0 to $m\mu$ the optimal consumption bundle of the consumer remains $(m/2, m/2)$;
- k increases from $m\mu$ to $2m\mu$ the optimal consumption bundles are $(0, m)$ and $(m, 0)$.

In particular, if k increases from $m\mu - \epsilon$ to $m\mu + \epsilon$, then we observe a discontinuous shift in the consumer's behavior.

What happens if the assumptions are violated

Assumptions used by the main theorem

- budget lines with slope -1;
- symmetry of the random variables;
- log-concavity of the density function.



Economic behavior and threshold-induced nonconvexities

Example

Advertising effects in food industry: "medical miracle" might effectively shift demand b inducing a local convexity.

Example

Decision about family size: the returns to education (parental investment) much higher, hence parents decide to devote more resources to less children.

Example

Potential bankruptcy faced by modern firms: go for extreme and high-risk (non-convex) business strategies

Discussion

Strengths

- propose one reason why preferences might be convex
- develop a normative theory with thorough analysis to the stochastic decision problem
- results and findings are consistent with the content and form of many modern marketing messages

Weaknesses

- strong assumptions with questionable validity in practice
- lack of empirical evidence
- only considers single-attribute quality (utility)

Conclusion and Future Work

Conclusion

Preferences are not always convex in real world. The convexity (concavity) of preferences are associated with the pay-off structure and the level of threshold.

Future Work

- Check the applicability/validity of the underlying assumptions associated with the threshold theory;
- Generalize the threshold theory to more complicated cases, e.g., multi-product, multi-attribute.

Thanks for Your Attention!