Why and When are Preferences Convex?
Threshold Effects and Uncertain Quality

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This paper considers circumstances under which convex preferences are optimal, with a specific setting:

- goods/products possess some hidden quality with known distribution
- consumer chooses a bundle of goods that maximizes the probability that he/she receives some threshold level of this quality

It is shown that

- if the threshold is small w.r.t. consumption level, convex preferences
- if the threshold is large w.r.t. consumption level, nonconvex preferences
Convexity of preferences:

- one of the canonical assumptions in economic theory;
- combination of bundles are at least as good as the extreme bundles.

Convexity is appealing in part because it is conducive to marginal analysis and single-valued continuous demand function.
However, preferences are not always convex in practice.

**Advertising Effects** Shifts in individual demand in response to product advertisements.

Milgrom & Roberts 1986: informative signaling effects
Smith & Tasnádi 2009: thresholds are sensitive to information
When convex preferences are beneficial?

Diversity in consumption

Example: human diet

How to measure the optimality?

Follow the Behavioral Ecology,

- natural selection favors agents who maximizes their expected payoff (utility) in a stochastic environment;
- preferences shall be considered optimal w.r.t underlying stochastic payoff structure.
A decision maker is face with a menu of two products: $x$ and $y$ must choose how much of each to consume, given

- fixed income $m$;
- prices $p$ for $x$ and 1 for $y$;
- a critical threshold $k$ for a single unobservable characteristic (quality), i.e., the consumer seeks to maximize the probability that he acquires $k$ units of this quality;
- the quality per unit of $x$ and $y$ are independent random variables, denoted by $C_x$ and $C_y$ with distribution functions $F$ and $G$. 
Mathematical Formulation

The decision problem can be stated as follows:

$$\max_{(x,y)} \quad U(x, y)$$

s.t. \quad px + y \leq m

$$x, y \geq 0$$

Assuming the random variables are continuous and

- have density functions $f$ and $g$;
- the support of them is the unit interval.

Then we have

$$U(x, y) = P(C_x x + C_y y \geq k) = \int_{k}^{\infty} \int_{\max\{0, t-y\}}^{\min\{x, t\}} \frac{1}{xy} f\left(\frac{z}{x}\right) g\left(\frac{t-z}{y}\right) dz dt$$
Five regions in commodity space

- “death zone” $A^0$;
- low consumption region $A^{--}$;
- low consumption of $x$ region $A^{--}$;
- low consumption of $y$ region $A^{+-}$;
- high consumption region $A^{++}$. 
Case 1: uniform case

Assuming the random variables $C_x$ and $C_y$ follow uniform distribution:

$$F(x) = G(x) = \begin{cases} 
0, & \text{if } x < 0; \\
x, & \text{if } x \in [0, 1]; \\
1, & \text{if } x > 1.
\end{cases}$$

When $k = 1$, the corresponding indifference curves are:
Finding 1

In Case 1, we observe a discontinuous change in behavior at $k = m/(2p)$ if $p \geq 1$ and at $k = pm/2$ if $p < 1$.

Illustration

For $p > 1$, there is a discontinuous change in the demand corresponding as the threshold $k$ increases.
Geometrical interpretation of threshold effects

Implications

- when $k$ is small, minimize probability associated with “very bad” outcomes;
- when $k$ is large, maximize probability associated with “very good” outcomes;
 Lemma

(Proschan 1965): Suppose the independent nonnegative random variables $C_x$ and $C_y$ and have log-concave density $f$, for any given $m > 0$, $Z_{\lambda,m} := \lambda C_x + (m - \lambda)C_y$ is strictly increasing in peakness in $\lambda$ on $[0, m/2]$. 

![Graph showing log-concave distributions with different parameters]
Main Theorem

Suppose the independent nonnegative random variables $C_x$ and $C_y$ are symmetric around their common means $\mu = EC_x = EC_y$, have log-concave density $f$ and $\text{supp}(C_x) = \text{supp}(C_y) = [0, 2\mu]$.

- $k$ increases from 0 to $m\mu$ the optimal consumption bundle of the consumer remains $(m/2, m/2)$;
- $k$ increases from $m\mu$ to $2m\mu$ the optimal consumption bundles are $(0, m)$ and $(m, 0)$.

In particular, if $k$ increases from $m\mu - \epsilon$ to $m\mu + \epsilon$, then we observe a discontinuous shift in the consumer’s behavior.
What happens if the assumptions are violated

Assumptions used by the main theorem

- budget lines with slope $-1$;
- symmetry of the random variables;
- log-concavity of the density function.
Economic behavior and threshold-induced nonconvexities

Example
Advertising effects in food industry: “medical miracle” might effectively shift demand by inducing a local convexity.

Example
Decision about family size: the returns to education (parental investment) much higher, hence parents decide to devote more resources to fewer children.

Example
Potential bankruptcy faced by modern firms: go for extreme and high-risk (non-convex) business strategies.
Discussion

Strengths
- propose one reason why preferences might be convex
- develop a normative theory with thorough analysis to the stochastic decision problem
- results and findings are consistent with the content and form of many modern marketing messages

Weaknesses
- strong assumptions with questionable validity in practice
- lack of empirical evidence
- only considers single-attribute quality (utility)
Conclusion

Preferences are not always convex in real world. The convexity (concavity) of preferences are associated with the pay-off structure and the level of threshold.

Future Work

- Check the applicability/validity of the underlying assumptions associated with the threshold theory;
- Generalize the threshold theory to more complicated cases, e.g., multi-product, multi-attribute.
Thanks for Your Attention!