Health and Mortality Delta: Assessing the Welfare Cost of Household Insurance Choice*

Ralph S. J. Koijen† Stijn Van Nieuwerburgh‡ Motohiro Yogo§

May 20, 2011

Abstract

We develop a pair of risk measures for the universe of health and longevity products that includes life insurance, annuities, and supplementary health insurance. Health delta measures the differential payoff that a policy delivers in poor health, while mortality delta measures the differential payoff that a policy delivers at death. Optimal portfolio choice simplifies to the problem of choosing a combination of health and longevity products that replicates the optimal exposure to health and mortality delta. For each household in the Health and Retirement Study, we calculate the health and mortality delta implied by its ownership of life insurance, annuities including defined-benefit plans, supplementary health insurance, and long-term care insurance. For the median household aged 51 to 58, the lifetime welfare cost of market incompleteness and suboptimal portfolio choice is 27 percent of total wealth.

JEL classification: D14, D91, G11, G22, I10

Keywords: Annuity, Health insurance, Life insurance, Medical expenditure

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*For comments and discussions, we thank Ben Heijdra, Theo Nijman, Sam Schulhofer-Wohl, and seminar participants at Columbia University, Federal Reserve Bank of Minneapolis, University of Minnesota, University of Tokyo, and Vanderbilt University. This paper is based upon work supported under a Netspar research grant. The Health and Retirement Study is sponsored by the National Institute of Aging (grant U01-AG009740) and is conducted by the University of Michigan. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis, the Federal Reserve System, or the National Bureau of Economic Research.

†University of Chicago, NBER, and Netspar-Tilburg University (e-mail: ralph.koijen@chicagobooth.edu)

‡New York University, NBER, and CEPR (e-mail: svnieuwe@stern.nyu.edu)

§Federal Reserve Bank of Minneapolis and NBER (e-mail: myogo@nber.org)

1. Introduction

Retail financial advisors and insurance companies offer a large variety of health and longevity products including life insurance, annuities, supplementary health insurance, and long-term care insurance. Each of these products comes in a potentially confusing variety of maturities and payout structures. Consider, for example, a simplified menu of life insurance and annuity products offered by TIAA-CREF in Table 1. This variety of products begs for a risk measure that allows households to assess to what extent these products are substitutes and to ultimately choose an optimal combination of products. Such risk measures already exist in other parts of the retail financial industry. For example, beta measures the exposure of an equity product to aggregate market risk, and duration measures the exposure of a fixed-income product to interest-rate risk. The existence of such risk measures, based on sound economic theory, has proven to be tremendously valuable in quantifying and managing financial risk for both households and institutions alike.

This paper develops a pair of risk measures for health and longevity products, which we refer to as health and mortality delta. Health delta measures the differential payoff that a policy delivers in poor health, while mortality delta measures the differential payoff that a policy delivers at death. Each household has an optimal exposure to health and mortality delta that depends on preferences (i.e., risk aversion and bequest motive) and characteristics (i.e., cohort, age, health, and wealth). Optimal portfolio choice simplifies to the problem of choosing a combination of health and longevity products, not necessarily unique, that replicates the optimal health and mortality delta.

Using our theory of optimal portfolio choice, we assess the welfare cost of the observed choices of health and longevity products. Figure 1 reports the ownership rates for term- and whole-life insurance, annuities including defined-benefit plans, supplementary health insurance, and long-term care insurance for households in the Health and Retirement Study. The ownership rate for term-life insurance exceeds 60 percent for households aged 51 to 58, while the ownership rate for annuities including defined-benefit plans exceeds 60 percent for households aged 67 to 74. In comparison, the ownership rates for supplementary health insurance and long-term care insurance are much lower. For example, the ownership rate for long-term care insurance is only slightly above 10 percent for households aged 67 to 74. How close are these observed choices to achieving the optimal private demand for health and longevity products, given the public provision of insurance through Social Security and Medicare?

To answer this question, we calculate the health and mortality delta for each household, implied by its ownership of health and longevity products. We then calculate the welfare cost for each household as a function of deviations of the observed health and mortality delta from the optimal health and mortality delta. For the median household aged 51 to 58, the lifetime welfare cost is 27 percent of total wealth, which includes the present value of future income in excess of out-of-pocket health expenses. We interpret this welfare cost as the joint cost of market incompleteness and suboptimal portfolio choice. For households younger than 91, most of the welfare cost is explained by deviations of the observed mortality delta from the optimal mortality delta, rather than deviations of the observed health delta from the optimal health delta. In other words, choices over life insurance and annuities have a much larger impact on the welfare cost than do choices over supplementary health insurance and long-term care insurance.

The remainder of the paper is organized as follows. In Section 2, we develop a life-cycle model in which a household faces health and mortality risk and invests in life insurance, annuities, and supplementary health insurance. In Section 3, we derive the optimal health and mortality delta under complete markets as well as a key formula for measuring the welfare cost of deviations from optimal health and mortality delta. In Section 4, we calibrate the life-cycle model using the Health and Retirement Study. In Section 5, we measure the welfare cost of market incompleteness and suboptimal portfolio choice. In Section 6, we provide two illustrations of how a household can replicate the optimal health and mortality using existing health and longevity products. Section 7 concludes. The appendices contain proofs and details about the data that are omitted in the main text.

2. A Life-Cycle Model with Health and Mortality Risk

In this section, we develop a life-cycle model in which the household faces health and mortality risk that affects life expectancy, health expenses, and the marginal utility of consumption or wealth. The household can invest in life insurance, annuities, supplementary health insurance, and a bond. The household may face borrowing or portfolio constraints, which may prevent it from achieving full insurance of health and mortality risk.

2.1 Health and Mortality Risk

In our model, health refers to any information that is verifiable through medical underwriting that involves a health examination and a review of medical history. For tractability, we do not model residual private information such as self assessments of health that might affect the demand for health and longevity products. However, we will examine private information as a potential explanation for the heterogeneity in demand for health and longevity products in our empirical work.

2.1.1 Health Transition Probabilities

The household lives for at most T periods and dies with certainty in period $T+1$. In each period $t \in [1, T]$, the household's health is in one of three states, indexed as $h_t \in \{1, 2, 3\}$.[1] The health states are ordered so that $h_t = 1$ corresponds to death, $h_t = 2$ corresponds to poor health, and $h_t = 3$ corresponds to good health. The three-state model can be interpreted as a discrete-time analog of a continuous-time model in which a continuous process drives health risk and a jump process drives mortality risk (Milevsky and Promislow, 2001).

[1] While three states is appropriate for our empirical application, it is straightforward to extend the theoretical framework to more than three states.

The household's health evolves from period t to $t+1$ according to a Markov chain with a 3×3 transition matrix π_t. We denote the (i,j)th element of the transition matrix as

$$\pi_t(i,j) = \Pr(h_{t+1} = j | h_t = i). \tag{1}$$

Conditional on being in health state i in period t, $\pi_t(i,j)$ is the probability of being in health state j in period $t+1$. Death is an absorbing state so that $\pi_t(1,1) = 1$. Let $\mathbf{e}_i$ denote a 3×1 vector whose ith element is one and whose other elements are zero. We define an n-period transition probability as

$$\pi_t^n(i,j) = \mathbf{e}_i' \prod_{s=0}^{n-1} \pi_{t+s} \mathbf{e}_j. \tag{2}$$

Conditional on being in health state i in period t, $\pi_t^n(i,j)$ is the probability of being in health state j in period $t+n$.

We define an n-period mortality rate as

$$p_t(n|i) = \begin{cases} \mathbf{e}_i' \pi_t \mathbf{e}_1 & \text{if } n = 1 \\ \mathbf{e}_i' \prod_{s=0}^{n-2} \pi_{t+s} \begin{bmatrix} \mathbf{0} & \mathbf{e}_2 & \mathbf{e}_3 \end{bmatrix} \pi_{t+n-1} \mathbf{e}_1 & \text{if } n > 1 \end{cases}. \tag{3}$$

Conditional on being in health state i in period t, $p_t(n|i)$ is the probability of being alive in period $t+n-1$ but dead in period $t+n$. We also define an n-period survival probability as

$$q_t(n|i) = 1 - \pi_t^n(i,1). \tag{4}$$

Conditional on being in health state i in period t, $q_t(n|i)$ is the probability of being alive in period $t+n$.

2.1.2 Out-of-Pocket Health Expenses

Although most households are covered by employer-provided health insurance or Medicare, they still face the risk of significant out-of-pocket health expenses, especially in old age. Many health plans only cover basic or in-network care, have capped benefits, or do not cover entire categories of health expenses. For example, Medicare does not cover nursing home care, and Medicaid only covers a limited and capped amount of nursing home care for those that qualify. Moreover, a household can lose health insurance through a layoff or a divorce. Health insurance must specify coverage for each type of future health contingency and treatment, some of which are not known to exist in advance. The fact that health insurance coverage can be short term or incomplete is perhaps a natural consequence of the complexity of these policies.

We model the consequences of imperfect health insurance as follows. In each period, the household faces an exogenous out-of-pocket health expense whose distribution depends on age and health. We denote the out-of-pocket health expense in period t as M_t, or as $M_t(h_t)$ to denote its realization for a particular health state. Naturally, worse health states are associated with higher out-of-pocket health expenses. There is no health expense at death so that $M_t(1) = 0$.

2.2 Health and Longevity Products

In each period t, the household can invest in life insurance, annuities, and supplementary health insurance of maturities one through $T - t$. In addition, the household can save in a one-period bond, which earns a gross interest rate R.

2.2.1 Term-Life Insurance

Let $1_{\{h_{t+s}=j\}}$ denote an indicator function that is equal to one if the policyholder is in health state j in period $t + s$. Life insurance of term n issued in period t pays out a death benefit of

$$D_{L,t+s}(n-s|h_{t+s}) = 1_{\{h_{t+s}=1\}}, \tag{5}$$

upon death of the policyholder in any period $t+s \in [t+1, t+n]$. In each period t, $T-t$ is the maximum available term since the policyholder dies with certainty in period $T+1$. For the purposes of this paper, we treat whole-life insurance as a special case of term-life insurance with maximum term $T-t$.

The pricing of life insurance depends on the policyholder's age and health at issuance of the policy. Naturally, younger and healthier policyholders with longer life expectancy pay a lower premium.[2] Conditional on being in health state h_t in period t, the price of n-period life insurance per unit of death benefit is

$$P_{L,t}(n|h_t) = \sum_{s=1}^{n} \frac{p_t(s|h_t)}{R_L^s}, \tag{6}$$

where $R_L \leq R$ is the discount rate. The pricing of life insurance is actuarially fair when $R_L = R$, while $R_L < R$ implies that life insurance sells at a premium.

2.2.2 Annuities

Let $1_{\{h_{t+1}\neq 1\}}$ denote an indicator function that is equal to one if the policyholder is alive in period $t+1$. An annuity of term n issued in period t pays out a constant stream of income

$$D_{A,t+s}(n-s|h_{t+1}) = 1_{\{h_{t+s}\neq 1\}}, \tag{7}$$

in each period $t+s \in [t+1, t+n]$ while the policyholder is alive. In each period t, $T-t$ is the maximum available term since the policyholder dies with certainty in period $T+1$.

[2] The insurer could charge a premium that does not depend on health in a pooling equilibrium (e.g., group life insurance). In that case, we would have to solve for the pooling price that allows the insurer to break even, given the aggregate demand for a given policy. While a conceptually straightforward extension of our framework, such an exercise would be computationally challenging.

The pricing of annuities depends on the policyholder's age and health at issuance of the policy. Naturally, younger and healthier policyholders with longer life expectancy pay a higher premium. Conditional on being in health state h_t in period t, the price of an n-period annuity per unit of income is

$$P_{A,t}(n|h_t) = \sum_{s=1}^{n} \frac{q_t(s|h_t)}{R_A^s}, \tag{8}$$

where $R_A \leq R$ is the discount rate.

The annuities that we have introduced here are building blocks for so-called deferred annuities. In period t, suppose the policyholder goes long an annuity of term $T-t$ and short an annuity of term $n < T-t$. This long-short portfolio of annuities is effectively an n-period deferred annuity whose income payments start in period $t+n+1$.

2.2.3 Supplementary Health Insurance

Supplementary health insurance of term n issued in period t pays out a benefit of

$$D_{H,t+s}(n-s|h_{t+s}) = 1_{\{h_{t+s}=2\}}(M_{t+s}(2) - M_{t+s}(3)), \tag{9}$$

in each period $t+s \in [t+1, t+n]$ while the policyholder is alive. Insofar as out-of-pocket health expenses include nursing home or home health care expenses, we can also think about this policy as long-term care insurance. A unit of this policy represents full insurance, equating out-of-pocket health expenses across all health states in which the policyholder is alive. In each period t, $T-t$ is the maximum available term since the policyholder dies with certainty in period $T+1$.

The pricing of supplementary health insurance depends on the policyholder's age and health at issuance of the policy. Naturally, younger and healthier policyholders with lower expected health expenses pay a lower premium. Conditional on being in health state h_t in period t, the price of n-period health insurance per unit of benefit is

$$P_{H,t}(n|h_t) = \sum_{s=1}^{n} \frac{\pi_t^s(h_t, 2)(M_{t+s}(2) - M_{t+s}(3))}{R_H^s}, \tag{10}$$

where $R_H \leq R$ is the discount rate.

2.3 Health and Mortality Delta for Health and Longevity Products

For each policy $i = \{L, A, H\}$ of term n, we define its health delta in period t as

$$\Delta_{i,t}(n) = P_{i,t+1}(n-1|2) + D_{i,t+1}(n-1|2) - (P_{i,t+1}(n-1|3) + D_{i,t+1}(n-1|3)). \tag{11}$$

Health delta measures the differential payoff that a policy delivers in poor health relative to good health in period $t+1$. Similarly, we define its mortality delta in period t as

$$\delta_{i,t}(n) = D_{i,t+1}(n-1|1) - (P_{i,t+1}(n-1|3) + D_{i,t+1}(n-1|3)). \tag{12}$$

Mortality delta measures the differential payoff that a policy delivers at death relative to good health in period $t+1$.

Figure 2 explains the relation between the payoffs of a policy and its health and mortality delta. In this illustration, short-term policies have maturity of two years (i.e., the frequency of interviews in the Health and Retirement Study), while long-term policies mature at death. We normalize the death benefit of life insurance and the income payments of annuities to be \$1k. Section 4 contains details about how we calibrate the prices of long-term policies, which are not important for the purposes of this illustration. The solid line represents the payoffs of a policy in the three health states. Health delta is the payoff of a policy in poor health relative to good health, which is minus the slope of the dashed line if the horizontal distance between good and poor health is one. Mortality delta is the payoff of the policy at death relative to good health, which is minus two times the slope of the dotted line if the horizontal distance between good health and death is two.

Short-term life insurance pays out \$1k only if the policyholder dies. Therefore, short-term life insurance has zero health delta and a mortality delta of \$1k. Even if the policyholder remains alive, long-term life insurance is worth the present value of \$1k in the event of future death, which is higher in poor health when he has impaired mortality. Therefore, long-term life insurance has both positive health delta and positive mortality delta.

The short-term annuity pays out \$1k only if the policyholder remains alive. Therefore, the short-term annuity has zero health delta and a mortality delta of $-\$1$k. In addition to the income if the policyholder remains alive, the long-term annuity is worth the present value of \$1k in each future period that he remains alive, which is higher in good health when he has longer life expectancy. Therefore, the long-term annuity has both negative health delta and negative mortality delta.

Short-term health insurance pays out a benefit only in poor health when the policyholder has high out-of-pocket health expenses. Therefore, short-term health insurance has positive health delta and zero mortality delta. In addition to the benefit in poor health, long-term health insurance is worth the present value of benefits in the event of future poor health, which is higher in poor health when the policyholder has higher expected health expenses. Therefore, long-term health insurance has positive health delta and negative mortality delta.

Figure 3 reports the health and mortality delta per dollar investment for these health and longevity products over the life cycle. In comparison to long-term life insurance, short-term life insurance generates high mortality delta per dollar investment. Therefore, short-term life insurance is a relatively inexpensive way to deliver wealth to death, especially for younger policyholders. Short- and long-term annuities deliver similar mortality delta per dollar investment, implying that they are close substitutes. In comparison to long-term health insurance, short-term health insurance generates high health delta per dollar investment. Therefore, short-term health insurance is a relatively inexpensive way to deliver wealth to poor health, especially for younger policyholders.

2.4 Budget Constraint

In each period t that the household is alive, it receives labor or retirement income Y_t and pays out-of-pocket health expenses M_t. The realization of both income and health expenses can depend on age and health. Let W_t denote the household's cash-on-hand in period t, which is its wealth after receiving income and paying health expenses. The household consumes from cash-on-hand and saves the remaining wealth in life insurance, annuities, supplementary health insurance, and the bond. Let B_t denote the total face value of bonds, and let $B_{i,t}(n) \geq 0$ denote the total face value of policy i of term n. The household's savings in period t is

$$W_t - C_t = \frac{B_t}{R} + \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} P_{i,t}(n|h_t) B_{i,t}(n). \tag{13}$$

Let

$$A_{t+1}(j) = B_t + \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} (P_{i,t+1}(n-1|j) + D_{i,t+1}(n-1|j)) B_{i,t}(n) \tag{14}$$

denote the household's wealth, prior to receiving income and paying health expenses, if health state j is realized in period $t+1$. In particular, $A_{t+1}(1) = B_t + \sum_{n=1}^{T-t} B_{L,t}(n)$ is the household's wealth left as a bequest if it dies in period $t+1$. The household must die with non-negative net worth, that is $A_{t+1}(1) \geq 0$. The household's intertemporal budget constraint is

$$W_{t+1} = A_{t+1} + Y_{t+1} - M_{t+1}. \tag{15}$$

2.5 Loan from Health and Longevity Products

The household can borrow from its holdings of health and longevity products, which we model as a negative position in the bond. For our purposes, a loan from health and longevity products is a simple way to model actual features of these policies. For example, long-term health insurance and life insurance may have periodic payment of premiums during the term of the policy, which can be interpreted as a "mortgage" on the policy. In addition, a household can take out a loan from the cash surrender value of whole-life insurance or a loan from annuities in a defined contribution plan.

For each policy i of term n, the policyholder can borrow up to $\alpha_i(n)P_{i,t}(n|h_t)$ per unit of benefit, where $\alpha_i(n) \in [0,1]$. The loan accrues interest at the gross interest rate R. The policyholder can partially repay the loan including accrued interest at any time during the term of the policy. The policyholder must fully repay the loan at maturity of the policy or at death, whichever happens sooner. Hence, the household faces the borrowing constraint

$$\frac{B_t}{R} \geq -\sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} \alpha_i(n)P_{i,t}(n|h_t)B_{i,t}(n). \tag{16}$$

In addition, the household may face additional portfolio constraints, which we leave as unspecified in this general description of the life-cycle problem.

2.6 Objective Function

For each health state $h_t \in \{2,3\}$ in period t, we define the household's objective function recursively as

$$U_t(h_t) = \left\{\omega(h_t)^\gamma C_t^{1-\gamma} + \beta\left[\pi_t(h_t,1)\omega(1)^\gamma A_{t+1}(1)^{1-\gamma} + \sum_{j=2}^{3}\pi_t(h_t,j)U_{t+1}(j)^{1-\gamma}\right]\right\}^{1/(1-\gamma)} \tag{17}$$

with the terminal value

$$U_T(h_T) = \omega(h_T)^{\gamma/(1-\gamma)} C_T. \tag{18}$$

The parameter β is the subjective discount factor, and γ is relative risk aversion. The health state-dependent utility parameter $\omega(h_t)$ allows the marginal utility of consumption or wealth to vary across health states. The presence of a bequest motive is parameterized as $\omega(1) > 0$, in contrast to its absence $\omega(1) = 0$. The parameterization $\omega(2) < \omega(3)$ means that consumption and health are complements in the sense that the marginal utility of consumption is lower in poor health.

3. Solution to the Life-Cycle Problem under Complete Markets

In this section, we derive the solution to the life-cycle problem under complete markets. While markets may not be complete in practice, the closed-form solution that this assumption yields is a useful theoretical benchmark for thinking about the optimal management of health and mortality risk. We also derive a key formula for measuring the welfare cost of deviations from optimal insurance of health and mortality risk.

3.1 Optimal Health and Mortality Delta

When markets are complete, there are potentially many portfolio policies that achieve the same consumption and wealth allocations. Therefore, it is impractical to characterize the optimal portfolio policy as a combination of health and longevity products. Instead, we characterize the solution to the life-cycle problem as an optimal consumption policy and a set of health state-contingent wealth policies.

To simplify notation, we define disposable income as income in excess of out-of-pocket health expenses. We then define total wealth as cash-on-hand plus the present value of future disposable income:

$$\widehat{W}_t = W_t + \sum_{s=1}^{T-t} \frac{\mathbf{E}_t[Y_{t+s} - M_{t+s}|h_t]}{R^s}. \tag{19}$$

We define health delta in period t as the difference in wealth between poor health and good health in period $t+1$:

$$\Delta_t = A_{t+1}(2) - A_{t+1}(3). \tag{20}$$

Similarly, we define mortality delta in period t as the difference in wealth between death and good health in period $t+1$:

$$\delta_t = A_{t+1}(1) - A_{t+1}(3). \tag{21}$$

Proposition 1. *When markets are complete, the solution to the life-cycle problem is*

$$C_t^* = c_t(h_t)\widehat{W}_t, \tag{22}$$

$$\begin{aligned} \Delta_t^* =& \frac{(\beta R)^{1/\gamma} C_t^*}{\omega(h_t)} \left(\frac{\omega(2)}{c_{t+1}(2)} - \frac{\omega(3)}{c_{t+1}(3)} \right) \\ &- \left(\sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|2]}{R^{s-1}} - \sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|3]}{R^{s-1}} \right), \end{aligned} \tag{23}$$

$$\delta_t^* = \frac{(\beta R)^{1/\gamma} C_t^*}{\omega(h_t)} \left(\omega(1) - \frac{\omega(3)}{c_{t+1}(3)} \right) + \sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s}|3]}{R^{s-1}}. \tag{24}$$

The average propensity to consume in health state $h_t \in \{2, 3\}$ is

$$c_t(h_t) = \left[1 + \frac{\pi_t(h_t, 1)(\beta R)^{1/\gamma}\omega(1)}{R\omega(h_t)} + \sum_{j=2}^{3} \frac{\pi_t(h_t, j)(\beta R)^{1/\gamma}\omega(j)}{R\omega(h_t)c_{t+1}(j)} \right]^{-1} \tag{25}$$

with the terminal value $c_T(h_T) = 1$.

As shown in Appendix A, the optimal policy equates the marginal utility of consumption or wealth across all health states in period $t+1$. The expression for the optimal health delta Δ_t^* shows that there are three forces that drive the household's desire to insure poor health relative to good health. First, the household would like to deliver relatively more wealth to the health state in which the marginal utility of consumption is high, determined by the relative magnitudes of $\omega(2)$ and $\omega(3)$. Second, the household would like to deliver relatively more wealth to the health state in which the average propensity to consume is low, determined by the relative magnitudes of $c_{t+1}(2)$ and $c_{t+1}(3)$. Naturally, the household consumes more slowly out of wealth in better health states associated with longer life expectancy. Finally, the household would like to deliver relatively more wealth to the health state in which lifetime disposable income is low. Naturally, the household has lower lifetime disposable income in poor health associated with shorter life expectancy, higher health expenses, and potentially lower income.

The same three forces also explain the expression for the optimal mortality delta δ_t^*. First, the household would like to deliver relatively more wealth to death if the marginal utility of the bequest (i.e., $\omega(1)$) is high. Second, the household would like to deliver relatively more wealth to death if the average propensity to consume in good health (i.e., $c_{t+1}(3)$) is high. Finally, the household would like to deliver relatively more wealth to death if lifetime disposable income is high in good health.

3.2 Replicating the Optimal Health and Mortality Delta through Health and Longevity Products

Proposition 2. *Given an optimal consumption policy, a feasible portfolio policy that satisfies the budget constraint (13), the borrowing constraint (16), and additional portfolio constraints* *(if any) is optimal if it satisfies the equations*

$$\Delta_t^* = \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} \Delta_{i,t}(n) B_{i,t}(n), \tag{26}$$

$$\delta_t^* = \sum_{i=\{L,A,H\}} \sum_{n=1}^{T-t} \delta_{i,t}(n) B_{i,t}(n). \tag{27}$$

Proposition 2 emphasizes the fact that health and mortality delta are sufficient for constructing an optimal portfolio of health and longevity products. Health delta $\Delta_{i,t}(n)$ measures the incremental contribution that policy i of term n has to the household's exposure to health delta. Mortality delta $\delta_{i,t}(n)$ measures the incremental contribution that policy i of term n has to the household's exposure to mortality delta. A combination of health and longevity products, not necessarily unique, that satisfies equation (26) delivers the optimal amount of wealth to poor health in period $t+1$. Similarly, a combination of health and longevity products, not necessarily unique, that satisfies equation (27) delivers the optimal amount of wealth to death in period $t+1$.

3.3 Welfare Cost of Deviations from Optimal Health and Mortality Delta

Suppose the household's health and mortality delta were to deviate from the optimal health and mortality delta given in Proposition 1. As shown in Appendix A, we estimate the welfare cost of such deviations from optimal health and mortality delta through a second-order Taylor approximation around the known value function under complete markets. By the envelope theorem, the welfare cost is second order for sufficiently small deviations from optimal health and mortality delta (Cochrane, 1989).

Proposition 3. *Let V_t^* denote the value function associated with the sequence $\{\Delta_{t+s-1}^*(i), \delta_{t+s-1}^*(i)\}_{s=1}^n$ of optimal health and mortality delta under complete markets. Let V_t denote the value function associated with an alternative sequence $\{\Delta_{t+s-1}(i), \delta_{t+s-1}(i)\}_{s=1}^n$ of health and mortality* *delta that satisfies the budget constraint. The welfare cost of deviations from optimal health and mortality delta is*

$$\begin{aligned}
L_t(n) =& \frac{V_t}{V_t^*} - 1 \\
\approx& \frac{1}{2} \sum_{s=1}^{n} \sum_{i=2}^{3} \left[\frac{\partial^2 L_t(n)}{\Delta_{t+s-1}(i)^2} (\Delta_{t+s-1}(i) - \Delta^*_{t+s-1}(i))^2 \right. \\
&+ \frac{\partial^2 L_t(n)}{\delta_{t+s-1}(i)^2} (\delta_{t+s-1}(i) - \delta^*_{t+s-1}(i))^2 \\
&\left. +2 \frac{\partial^2 L_t(n)}{\partial \Delta_{t+s-1}(i) \partial \delta_{t+s-1}(i)} (\Delta_{t+s-1}(i) - \Delta^*_{t+s-1}(i))(\delta_{t+s-1}(i) - \delta^*_{t+s-1}(i)) \right],
\end{aligned} \tag{28}$$

where the expressions for the second partial derivatives are given in Appendix A.

A household may not achieve the optimal health and mortality delta under complete markets for two reasons. First, markets may be incomplete due to borrowing or portfolio constraints, or the menu of health and longevity products may be incomplete for certain demographic groups. Second, a nearly rational household may hold a suboptimal portfolio of health and longevity products even though markets are complete (Calvet, Campbell, and Sodini, 2007). This explanation is especially plausible for health and longevity products because there is no clear guidance on optimal portfolio choice, unlike for equity and fixed-income products. Because these two reasons are not mutually exclusive and difficult to distinguish based on the available data, we do not attempt to quantify the relative importance of these two hypotheses. Instead, we focus on estimating the joint cost of market incompleteness and suboptimal portfolio choice in this paper.

4. Calibrating the Life-Cycle Model

4.1 Health and Retirement Study

We use the Health and Retirement Study to calibrate the life-cycle model, which is a representative panel of older households in the United States since 1992. This household survey is uniquely suited for our study because it contains household-level data on health outcomes, health expenses, income, and wealth as well as ownership of life insurance, annuities, supplementary health insurance, and long-term care insurance. Some of these critical variables are missing in other household surveys such as the Panel Study of Income Dynamics or the Survey of Consumer Finances. We focus on households whose male respondent is aged 51 and older at the time of interview. We also require that households have both positive income and net worth to be included in our sample. Appendix B contains details on the construction of the relevant variables for our analysis.

Life insurance is written on the life of an individual, while resources like income and wealth are shared by the members of a household. Because the male respondent is typically married at the time of first interview, we must make some measurement assumptions when mapping the data to the model. We measure health outcomes and the ownership of life insurance, annuities, supplementary health insurance, and long-term care insurance for only the male respondent. We measure health expenses, income, and wealth at the household level. These measurement assumptions are consistent with our model insofar as the budget constraint holds for the household, and the male respondent buys life insurance to leave a bequest for surviving household members when he dies.

We calibrate the life-cycle model so that each period corresponds to two years, matching the frequency of interviews in the Health and Retirement Study. The model starts at age 51 to correspond to the youngest age at which respondents enter the survey. We assume that households die with certainty at age 111, so that there are a total of 30 periods (60 years) in the life-cycle model. We set the annualized riskless interest rate to 2 percent, which is roughly the average real return on the one-year Treasury note.

4.2 Definition of the Health States

In this section, we categorize health into three states including death, which is the minimum number of states that is necessary to model both health and mortality risk. For our purposes, the relevant criteria for poor health are that both the mortality rate and health expenses are high. This is precisely the state in which life insurance and supplementary health insurance are valuable to the household.

In Table 2, we use a probit model to predict future mortality based on observed health problems. The explanatory variables include dummy variables for doctor-diagnosed health problems, age, the interaction of the health problems with age, and cohort dummies. The marginal effect of high blood pressure on the mortality rate is 1.66 with a t-statistic of 3.52. This means that males with high blood pressure are 1.66 percentage points more likely to die within two years, holding everything else constant. Males with cancer are 13.62 percentage points more likely to die, while those with lung disease are 8.21 percentage points more likely to die. Past age 51, each additional ten years is associated with an increase of 3.26 percentage points in the mortality rate.

Using the estimated probit model, we calculate the predicted mortality rate for each household at each interview. We also calculate the ratio of out-of-pocket health expenses to income at each interview. We then define the following three health states.

1. Death.

2. Poor health: The predicted mortality rate is higher than the median conditional on cohort and age. In addition, the ratio of out-of-pocket health expenses to income is higher than the median conditional on cohort, age, and ownership of supplementary health insurance and long-term care insurance.

3. Good health: Alive and not in poor health.

To verify that our definition of the health states are reasonable, Panel A of Table 3 reports specific health problems that households face by age and health state. Within each age group, households in poor health have higher prevalence of doctor-diagnosed health problems. For example, among households aged 51 to 66, 28 percent of those in poor health have had heart problems, which is higher than 11 percent of those in good health. Older households, especially those in poor health, have higher prevalence of difficulty with activities of daily living. For example, among households aged 83 or older, 16 percent of those in poor health have some difficulty eating, which is higher than 7 percent of those in good health.

Panel B of Table 3 reports health care utilization by age and health state. Within each age group, households in poor health are more likely to have used health care in the two years prior to the interview. For example, among households aged 51 to 66, 79 percent of those in poor health use prescription drugs regularly, which is higher than 52 percent of those in good health. Among households aged 83 or older, 19 percent of those in poor health have stayed at a nursing home, which is higher than 8 percent of those in good health. These facts explain why households in poor health have higher out-of-pocket health expenses than those in good health.

Panel C of Table 3 reports health insurance coverage by age and health state. Among households aged 51 to 66, 22 percent of those in poor health are covered by Medicare, which is higher than 17 percent of those in good health. This difference is explained by the fact that some households in poor health are forced to take early retirement. Almost all households aged 67 or older are covered by Medicare. Among households aged 51 to 66, 58 percent of those in poor health are covered by an employer-provided health plan, which is lower than 63 percent of those in good health. Within each age group, the ownership rates of supplementary health and long-term care insurance are remarkably similar across health states.

Panel D of Table 3 reports the ownership rate of life insurance, the ownership rate of annuities including defined-benefit plans, and net worth by age and health state. Among households aged 51 to 66, 78 percent of those in poor health own some type of life insurance, which is comparable to 80 percent of those in good health. Although the ownership rate for life insurance declines in age, it remains remarkably high for older households. Among households aged 67 to 82, 65 percent of those in poor health receive annuity income that is not from Social Security, which is comparable to 61 percent of those in good health. Among households aged 67 to 82, the median net worth excluding life insurance and annuities is \$186k for those in poor health, which is comparable to \$187k for those in good health.

4.3 Health and Mortality Risk

4.3.1 Health Transition Probabilities

Once we have defined the three health states, we estimate the transition probabilities between the health states using an ordered probit model. The outcome variable is the health state at two years from the present interview. The explanatory variables include dummy variables for present health state and 65 or older, a quadratic polynomial in age, the interaction of the dummy variables with the quadratic polynomial in age, and cohort dummies. The dummy variable for 65 or older allows for potential changes in household behavior after retirement, when households qualify for Social Security and Medicare. Our estimated transition probabilities, which differ across cohorts, are the predicted probabilities from the ordered probit model.

To get a sense for these transition probabilities, Panel A of Table 4 reports the health distribution by age for a population of males born 1936 to 1940, who are in good health at age 51. By age 67, 30 percent of the population are dead, and 18 percent are in poor health. By age 83, 62 percent of the population are dead, and 14 percent are in poor health. Panel B reports the average life expectancy conditional on age and health. Households in poor health at age 51 are expected to live for 24 more years, which is shorter than 26 years for those in good health. The difference in life expectancy between poor health and good health remains relatively constant for older households. Households in poor health at age 83 are expected to live for 8 more years, which is shorter than 10 years for those in good health.

4.3.2 Out-of-Pocket Health Expenses

As explained in Appendix B, we use a panel regression model to estimate how out-of-pocket health expenses depend on cohort, age, and health. We use a comprehensive measure of out-of-pocket health expenses that includes payments of health insurance premiums and end-of-life health expenses. Panel C of Table 4 reports annual out-of-pocket health expenses by age and health for the cohort born 1936 to 1940. For comparison, Panel D reports average annual income by age, which includes Social Security but excludes annuities and defined-benefit plans.[3] Households in poor health at age 51 have annual out-of-pocket health expenses of \$2k, which is higher than \$0k for those in good health. Out-of-pocket health expenses rise rapidly in age, as emphasized by De Nardi, French, and Jones (2010). Households in poor health at age 83 have annual out-of-pocket health expenses of \$21k, which is higher than \$8k for those in good health. Since annual income at age 83 is \$18k, households in poor health must dissave in order to consume and pay health expenses.

Households in poor health not only face higher health expenses today, but they also face higher future health expenses. Panel E of Table 4 reports the present value of future disposable income (i.e., income in excess of out-of-pocket health expenses) by age and health. Households in poor health at age 59 have \$233k in lifetime disposable income that they can consume or bequeath, which is lower than \$271k for those in good health. A household in good health at age 51 is unlikely to be in poor health or die, at least in the near future. However, poor health or death can have a significant impact on lifetime resources. This leads to demand for health and longevity products that allow households to insure differences in lifetime resources across health states.

4.4 Health and Longevity Products

4.4.1 Pricing of Health and Longevity Products

In our benchmark calibration, we set the discount rate for health and longevity products to be the same as the riskless interest rate of 2 percent (i.e., $R_L = R_A = R_H = R$). In other words, we assume that the pricing of health and longevity products is actuarially

[3]For simplicity, our calibration assumes that income depends on cohort and age, but not on health. While there is some evidence that income varies with health, such variation is much smaller than the variation in out-of-pocket health expenses, which is the main focus of this paper.

fair conditional on age and health. This simplifying assumption is necessitated by the fact that we do not observe the premiums that households in the data pay for life insurance, supplementary health insurance, and long-term care insurance.

There are various reasons why the pricing of health and longevity products may not be actuarially fair in practice: rents arising from imperfect competition, discounts reflecting the poor credit quality of insurers, risk premia arising from aggregate health and mortality risk, and the presence of private information. The impact of private information on the pricing of insurance is ambiguous because adverse selection on health may be offset by advantageous selection on another dimension of private information such as risk aversion (de Meza and Webb, 2001). In life insurance markets, there is no evidence for private information about health (Cawley and Philipson, 1999). In long-term care insurance and Medigap insurance markets, private information about health appears to be offset by advantageous selection on risk aversion and cognitive ability (Finkelstein and McGarry, 2006; Fang, Keane, and Silverman, 2008). Given the ambiguous nature of both the theoretical predictions and the empirical findings, the absence of private information serves as a satisfactory starting point for our benchmark calibration. However, we will examine private information as a potential explanation for the heterogeneity in demand for health and longevity products in our empirical work.

4.4.2 Ownership of Health and Longevity Products

At each interview, the household reports its ownership of term- and whole-life insurance, annuities including defined-benefit plans, supplementary health insurance, and long-term care insurance. We do not have information on the maturity of term-life insurance or the exact coverage of supplementary health insurance or long-term care insurance. Therefore, we must make some measurement assumptions in order to map these health and longevity products to counterparts in the life-cycle model.

We assume that term-life insurance matures in two years and that whole-life insurance matures at death. We assume that annuity income starts at age 65, which is the typical retirement age, and terminates at death. We assume that the observed ownership of supplementary health insurance corresponds to owning half a unit of short-term health insurance in the life-cycle model. Similarly, the observed ownership of long-term care insurance corresponds to owning half a unit of short-term health insurance. Therefore, a household that has both supplementary health insurance and long-term care insurance has full insurance of out-of-pocket health expenses in the next period. This assumption is based on estimates that nursing home expenses account for about half of out-of-pocket health expenses for older households (Marshall, McGarry, and Skinner, 2010).

We model all health and longevity products as policies with real payments. We normalize the death benefit of life insurance and the income payments of annuities to be \$1k in 2005 dollars. Modeling nominal payments for health and longevity products would introduce inflation risk, which is beyond the scope of this paper. Moreover, a cost-of-living-adjustment rider that effectively eliminates inflation risk is available for life insurance, annuities, and long-term care insurance. In the data, we deflate the face value of life insurance and annuity income by the consumer price index to 2005 dollars.

As explained in Appendix B, we use a panel regression model to estimate how the face values of term- and whole-life insurance depend on cohort, age, and health. Panels A and B of Table 5 report the face values of term- and whole-life insurance, conditional on ownership, by age and health for the cohort born 1936 to 1940. We apply the same procedure to annuity income, which is reported in Panel C. Conditional on ownership, we use these estimated values to calculate the health and mortality delta for each household in Section 5. Our approach is more robust to missing observations and measurement error than the alternative of using the observed face values of term- and whole-life insurance and the observed annuity income.

5. Welfare Cost of the Observed Health and Mortality Delta

In this section, we use Proposition 3 to estimate the joint cost of market incompleteness and suboptimal portfolio choice, implied by the observed ownership of health and longevity products.

5.1 Health and Mortality Delta Implied by the Observed Ownership of Health and Longevity Products

For each household at each interview, we calculate the health and mortality delta implied by its ownership of term- and whole-life insurance, annuities including defined-benefit plans, supplementary health insurance, and long-term care insurance. A household's exposure to health delta is determined by positive health delta from whole-life insurance, supplementary health insurance, and long-term care insurance, which is offset by negative health delta from annuities. A household's exposure to mortality delta is determined by positive mortality delta from term- and whole-life insurance, which is offset by negative mortality delta from annuities.

Figure 4 reports the health and mortality delta for each household-interview observation, together with the median and mean at each age. For the median household, health delta is negative and has a slight U-shaped profile over the life cycle. This means that annuities have a predominant effect on the median household's exposure to health delta. For the median household, mortality delta is negative and has a pronounced U-shaped profile over the life cycle. This means that annuities have a predominant effect on the median household's exposure to mortality delta. There is more cross-sectional variation in mortality delta than in health delta at each age.

In Table 6, we examine whether household characteristics explain the variation in observed health and mortality delta. In the baseline specification in column (1), we regress health delta onto dummy variables for poor health and 65 or older, a quadratic polynomial in age, the interaction of the dummy variables with the quadratic polynomial in age, and cohort dummies. This baseline specification explains 17.69 percent of the variation in health delta. Column (2) shows that proxies for bequest motives and private information about health add virtually no explanatory power to the baseline specification, increasing the R^2 to only 17.85 percent. Health delta is \$0.32k lower for married households, and \$0.24k lower for households with living children. Net worth is an insignificant determinant of health delta. Health delta is \$0.29k higher for households in poor self-reported health, which is consistent with the presence of private information about health. Health delta is \$0.63k higher for households in excellent self-reported heath, which is consistent with advantageous selection.

We repeat the same exercise for mortality delta in columns (3) and (4). The baseline specification explains 48.74 percent of the variation in mortality delta. Column (2) shows that proxies for bequest motives and private information about health add virtually no explanatory power to the baseline specification, increasing the R^2 to only 48.98 percent. Mortality delta is \$1.23k higher for married households, and mortality delta is \$2.51k higher for households with living children. The sign of these coefficients are consistent with the presence of a bequest motive, although the magnitude of the coefficients are economically small. Net worth is an insignificant determinant of mortality delta. Mortality delta is \$2.39k higher for households in poor self-reported health, which is consistent with the presence of private information about health. Mortality delta is \$1.21k higher for households in excellent self-reported heath, which is consistent with advantageous selection.[4]

In summary, we find that household characteristics, such as marital status or the presence of children, and private information about health do not explain much of the observed heterogeneity in health and mortality delta. This finding suggests that preference heterogeneity along these observable dimensions would not explain the welfare cost of the observed health

[4]In addition to self-reported health status, we have ruled out significant explanatory power for other measures of private information about health including difficulty with activities of daily living, self-reported probability of living to age 75, and self-reported probability of moving to a nursing home.

and mortality delta that we document below.

5.2 Welfare Cost per Period

In this section, we estimate the welfare cost of deviating from the optimal health and mortality delta for one period, then following the optimal policy for the remaining lifetime. While the welfare cost per period is not our primary measure of interest, it allows us to estimate the unobserved preference parameters based on the observed ownership of health and longevity products alone, without an auxiliary model for how such ownership evolves over time.

For a given set of preference parameters, we can calculate the welfare cost per period by applying Proposition 3 for $n = 1$. We set the subjective discount factor to $\beta = 0.96$ annually, which is a common choice in the life-cycle literature. We set relative risk aversion to $\gamma = 4$, based on previous estimates in the Health and Retirement Study (Barsky et al., 1997). There is less guidance in the literature for the health state-dependent utility parameters. Therefore, we estimate $\omega(1)$ and $\omega(2)$ to minimize the implied welfare cost, under the normalization $\omega(3) = 1$. Our procedure leads to a lower-bound estimate of the welfare cost under the true preference parameters.

Let $L_h(\omega(1), \omega(2))$ denote the welfare cost per period for household-interview observation $h \in [1, H]$, given the preference parameters $\omega(1)$ and $\omega(2)$. We estimate the unknown preference parameters to minimize the sum of welfare cost across all observations:

$$\frac{1}{H} \sum_{h=1}^{H} L_h(\omega(1), \omega(2)). \tag{29}$$

We use continuous-updating generalized method of moments, based on the moment restriction

$$\mathbf{E} \begin{bmatrix} \frac{\partial L_h(\omega(1),\omega(2))}{\partial \omega(1)} \\ \frac{\partial L_h(\omega(1),\omega(2))}{\partial \omega(2)} \end{bmatrix} = \mathbf{0}. \tag{30}$$

As reported in Table 7, we obtain an estimate $\omega(1) = 3.76$ with a standard error of 0.09. In other words, households have a bequest motive that is equivalent to 3.76 periods (more than 7 years) of consumption. The strong bequest motive is consistent with previous estimates in the literature (Ameriks et al., 2011). We also obtain an estimate of $\omega(2) = 0.87$ with a standard error of 0.02. The fact that consumption and health are complements is consistent with previous estimates in the literature (Viscusi and Evans, 1990; Finkelstein, Luttmer, and Notowidigdo, 2010).

Panel A of Table 8 reports the median welfare cost over two years by age. The welfare cost for households aged 51 to 58 is 0.22 percent with a standard error of 0.02 percent. Using equation (28) for $n = 1$, we decompose this welfare cost into the sum of three parts. Deviations of the observed mortality delta from the optimal mortality delta explains virtually all of the welfare cost. Deviations of the observed health delta from the optimal health delta as well as the interaction between health and mortality delta, which is not reported in the table, explain a negligible share of the welfare cost. The welfare cost for households aged 83 to 90 is 0.13 percent with a standard error of 0.01 percent. Health delta explains 0.03 percent of the welfare cost, while mortality delta explains 0.14 percent of the welfare cost.

The top three panels of Figure 5 is a visual representation of Panel A of Table 8. This figure reports the welfare cost over two years for each household-interview observation, together with the median and mean at each age. The median welfare cost is lower than the mean, which is explained by a small number of households with very high welfare costs. Younger and older households have the highest welfare costs. For the younger households, mortality delta explains virtually all of the welfare cost. For the older households, health delta explains a more important share of the welfare cost.

Panel A of Table 9 reports the median welfare cost over two years by age and health. The welfare cost for households in poor health at age 51 to 58 is 0.68 percent with a standard error of 0.06 percent. This is higher than the welfare cost for households in good health, which is 0.11 percent with a standard error of 0.01 percent. The difference in welfare costs between poor health and good health persists for older households. The welfare cost for households in poor health at age 83 to 90 is 0.33 percent with a standard error of 0.04 percent. This is higher than the welfare cost for households in good health, which is 0.10 percent with a standard error of 0.01 percent.

5.3 Lifetime Welfare Cost

The welfare cost per period is based on the assumption that the household deviates from the optimal health and mortality delta for just one period, then follows the optimal policy for the remainder of its lifetime. In reality, a household that deviates from the optimal policy for one period will persist in the suboptimal policy for many periods. In this section, we measure the lifetime welfare cost by applying Proposition 3 for $n = T - t$.

In order to measure the lifetime cost, we must first model how the ownership of health and longevity products evolves over time, exploiting the panel dimension of the data. In Table 10, we use a probit model to predict ownership of a given type of policy at two years from the present interview. The key explanatory variable is whether the household is a present policy owner. A household aged 51 that is present owner of term-life insurance is 46 percent more likely to own it at the next interview. Similarly, a household aged 51 that is present owner of whole-life insurance is 67 percent more likely to own it at the next interview. A household aged 51 that is present owner of annuities including defined-benefit plans is 50 percent more likely to own them at the next interview. A household aged 51 that is present owner of supplementary health insurance is 33 percent more likely to own it at the next interview. Finally, a household aged 51 that is present owner of long-term care insurance is 24 percent more likely to own it at the next interview.

Based on the predicted probabilities from the probit model, we calculate the joint transition matrix for the health state and the ownership of health and longevity products. For each household, we then calculate the most likely sequence of future ownership of health and longevity products, conditional on the health state. Finally, we calculate the sequence of future health and mortality delta implied by the ownership of health and longevity products (i.e., $\{\Delta_{t+s-1}(i), \delta_{t+s-1}(i)\}_{s=2}^{T-t}$ in Proposition 3).

Panel B of Table 8 reports the median lifetime welfare cost by age. The lifetime welfare cost for households aged 51 to 58 is 26.61 percent with a standard error of 0.55 percent. This is a very large welfare cost that is equivalent to a 27 percent reduction in lifetime consumption, by the homogeneity of preferences. Using equation (28) for $n = T - t$, we decompose this lifetime welfare cost into the sum of three parts. Deviations of the observed health delta from the optimal health delta explain 0.57 percent, while deviations of the observed mortality delta from the optimal mortality delta explain 28.05 percent. The interaction between health and mortality delta, which is not reported in the table, explain the remainder of the lifetime welfare cost. The lifetime welfare cost for households aged 83 to 90 is 1.10 percent with a standard error of 0.07 percent. Health delta explains 0.13 percent of the lifetime welfare cost, while mortality delta explains 1.01 percent of the lifetime welfare cost.

The bottom three panels of Figure 5 is a visual representation of Panel B of Table 8. This figure reports the lifetime welfare cost for each household-interview observation, together with the median and mean at each age. The lifetime welfare cost is high for younger households, for whom the welfare cost per period accumulates over a longer expected lifetime. The lifetime welfare cost falls rapidly until age 83 and becomes insignificant for older households with shorter life expectancy. Mortality delta explains almost all of the lifetime welfare cost, which is explained by the fact that there is more cross-sectional variation in mortality delta than in health delta.

Panel B of Table 9 reports the median lifetime welfare cost by age and health. The lifetime welfare cost for households in poor health at age 51 to 58 is 22.03 percent with a standard error of 0.85 percent. This is lower than the lifetime welfare cost for households in good health, which is 29.53 percent with a standard error of 0.71 percent. As the lifetime welfare cost falls in age, the difference between poor health and good health disappears for older households. The lifetime welfare cost for households in poor health at age 83 to 90 is

1.12 percent with a standard error of 0.12 percent. This is similar to the lifetime welfare cost for households in good health, which is 1.10 percent with a standard error of 0.09 percent.

6. Optimal Portfolio of Health and Longevity Products

In this section, we provide two illustrations of how a household can replicate the optimal health and mortality delta through a portfolio of health and longevity products. In the first example, the household can buy short-term life insurance, a deferred annuity, short-term health insurance, and a bond. In the second example, the household can buy short- and long-term life insurance, short- and long-term annuities, and a bond. We do not impose any borrowing or portfolio constraints, so that the household achieves the optimal health and mortality delta under complete markets.

Our illustrations are for a male in good health at age 51, born 1936 to 1940. The household's initial wealth is $65k at age 51, which is chosen to match average net worth excluding life insurance and annuities for this cohort. The household's preference parameters are those given in Table 7.

6.1 Optimal Portfolio with Supplementary Health Insurance

Panel A of Table 11 reports the optimal health and mortality delta, which we calculate by applying Proposition 1. The optimal health delta is $4k at age 51, which implies that the household needs an additional $4k in poor health relative to good health at age 53. As equation (23) shows, there are three offsetting forces that determine the optimal health delta. First, the household has preference for consumption in good health over poor health (i.e., $\omega(2) < \omega(3)$), which pushes the optimal health delta to be more negative. Second, the household saves less in poor health because of shorter life expectancy (i.e., $c_{t+1}(2) > c_{t+1}(3)$), which pushes the optimal health delta to be more negative. Third, the household has lower lifetime disposable income in poor health, which pushes the optimal health delta to be more positive. The third force dominates the first two so that the optimal health delta is positive at age 51.

The optimal mortality delta is \$135k at age 51, which implies that the household needs an additional \$135k at death relative to good health at age 53. As equation (24) shows, there are three offsetting forces that determine the optimal mortality delta. First, the household has preference for bequest over consumption in good health (i.e., $\omega(1) > \omega(3)$), which pushes the optimal mortality delta to be more positive. Second, the household must save for future consumption in good health (i.e., $c_{t+1}(3) < 1$), which pushes the optimal mortality delta to be more negative. Third, the household has higher lifetime disposable income in good health, which pushes the optimal mortality delta to be more positive. The first and third forces dominate the second so that the optimal mortality delta is positive at age 51.

Panel B of Table 11 reports a portfolio of short-term life insurance, deferred annuities, and short-term health insurance that replicates the optimal health and mortality delta, which we calculate by applying Proposition 2. The optimal portfolio at age 51 consists of 135 units (i.e., death benefit of \$135k) of short-term life insurance, 0.98 units of short-term health insurance, and 63 units of the bond. Panel C reports the cost of the optimal portfolio, which is the sum of \$4k in short-term life insurance, \$1k in short-term health insurance, and \$61k in bonds. Figure 6 is a graphical illustration of how a portfolio of short-term life insurance and health insurance replicates the optimal health and mortality delta at age 51. A portfolio of only short-term policies leads to clean separation in the sense that health insurance replicates the optimal health delta, while life insurance replicates the optimal mortality delta.

The left panel of Figure 7 shows that the optimal health delta has a U-shaped profile over the life cycle. The positions in short-term health insurance that replicate the optimal health delta are 0.98 units at age 51, 0.06 units at age 67, and 0.77 units at age 83. Since one unit of short-term health insurance eliminates all uncertainty in out-of-pocket health expenses in the next period, these positions imply that the household demands only partial health insurance throughout the life cycle. The intuition for this result is that higher out-of-pocket health expenses in poor health are offset by shorter life expectancy, lowering the optimal health delta relative to full health insurance.

The right panel of Figure 7 shows that the optimal mortality delta declines over the life cycle. To replicate the optimal mortality delta, the household must hold short-term life insurance when young to generate positive mortality delta, then switch to deferred annuities when old to generate negative mortality delta. The optimal position in deferred annuities increases from 9 units (i.e., annuity income of $9k over two years) at age 59 to 45 units at age 83.

In this example, the household is exposed to reclassification risk because it can only invest in short-term life insurance and health insurance. In other words, a household in good health at age 51 has to pay a higher premium for life insurance and health insurance at age 53 if its health deteriorates. As emphasized by Cochrane (1995), the household can insure reclassification risk in a world with health state-contingent securities. Our example here shows that an optimal portfolio of short-term life insurance and health insurance essentially replicates health state-contingent securities, thereby insuring reclassification risk.

6.2 Optimal Portfolio with Life Insurance and Annuities Only

Panel A of Table 12 reports the optimal health and mortality delta, which are the same as in the previous example. Panel B reports a portfolio of short- and long-term life insurance, short- and long-term annuities, and a bond that replicates the optimal health and mortality delta. The optimal portfolio at age 51 consists of 93 units of short-term life insurance, 113 units of long-term life insurance, and -7 units of bonds. The optimal portfolio at age 67 switches to 3 units of short-term annuities, 15 units of long-term annuities, and 182 units of bonds. If the household survives until age 99, its optimal portfolio consists of 4,194 units of long-term life insurance, 629 units of short-term annuities, and $-4,040$ units of bonds.

In this example, long-term life insurance is the only policy that allows the household to generate positive health delta. Therefore, the household must hold a levered position in long-term life insurance to replicate the optimal health delta at ages 51 and 99. A loan of \$7k on long-term life insurance that is worth \$69k at age 51 seems achievable, given that whole-life insurance does not require an upfront payment of the entire premium in practice. However, a loan of \$3,883k on long-term life insurance that is worth \$3,851k at age 99 seems difficult to achieve in practice. This example shows that long-term life insurance can be a substitute for supplementary health insurance, but not a perfect one in the presence of borrowing constraints.

7. Conclusion

We have developed health and mortality delta as useful risk measures for thinking about health and longevity products such as life insurance, annuities, and supplementary health insurance. We believe that retail financial advisors and insurance companies should report the health and mortality delta of their health and longevity products, just as mutual fund companies report the market beta of their equity products and the duration of their fixed-income products. Financial advisors and insurance brokers should guide households on the optimal exposure to health and mortality delta over the life cycle, based on their preferences (i.e., risk aversion and bequest motive) and characteristics (i.e., cohort, age, health, and wealth). We hope that the introduction of these risk measures will facilitate standardization, identify overlap between existing products, identify risks that are not insured by existing products, and ultimately lead to new product development.

There are two potential interpretations for our empirical findings on the welfare cost of the observed choices of health and longevity products. Our preferred interpretation is that there are substantial welfare gains that can be achieved by completing missing insurance markets and by eliminating suboptimal portfolio choice. The existence of suboptimal portfolio choice is plausible for health and longevity products because there is no clear guidance on optimal portfolio choice, unlike for equity and fixed-income products. An alternative interpretation is unmodeled preference heterogeneity across households. We are skeptical of this possibility because our empirical findings suggest that such preference heterogeneity must be entirely disconnected from observable differences across households such as marital status, children, and measures of private information about health.

Appendix A. Proofs of Propositions

A.1 Proof of Proposition 1

We rewrite savings in period t as

$$W_t - C_t = \sum_{j=1}^{3} \frac{\pi_t(h_t, j)}{R} A_{t+1}(j). \tag{A1}$$

The household maximizes the objective function (17) subject to equation (A1) and the intertemporal budget constraint (15). In each period $t \in [1, T-1]$, the Bellman equation is

$$\begin{aligned} V_t(h_t, W_t) = & \max_{C_t, A_{t+1}(1), A_{t+1}(2), A_{t+1}(3)} \Big\{ \omega(h_t)^\gamma C_t^{1-\gamma} \\ & + \beta \left[\pi_t(h_t, 1)\omega(1)^\gamma A_{t+1}(1)^{1-\gamma} + \sum_{j=2}^{3} \pi_t(h_t, j) V_{t+1}(j, W_{t+1}(j))^{1-\gamma} \right] \Big\}^{1/1-\gamma}. \end{aligned} \tag{A2}$$

The proposition claims that the optimal health state-contingent wealth policies are given by

$$A^*_{t+1}(1) = \frac{(\beta R)^{1/\gamma} \omega(1) C^*_t}{\omega(h_t)}, \tag{A3}$$

$$A^*_{t+1}(j) = \frac{(\beta R)^{1/\gamma} \omega(j) C^*_t}{\omega(h_t) c_{t+1}(j)} - \sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s} | j]}{R^{s-1}} \; \forall j \in \{2, 3\}. \tag{A4}$$

The proof proceeds by backward induction.

Because the household dies with certainty in period $T+1$, optimal consumption in period T is $C^*_T = W_T$. Thus, the value function in period T is

$$V_T(h_T, W_T) = \omega(h_T)^{\gamma/(1-\gamma)} W_T. \tag{A5}$$

The first-order conditions in period $T-1$ are

$$
\begin{aligned}
\omega(h_{T-1})^{\gamma} C_{T-1}^{*-\gamma} =& \beta R \omega(1)^{\gamma} A_T^*(1)^{-\gamma} \\
=& \beta R \omega(h_T)^{\gamma} (A_T^*(j) + Y_T(j) - M_T(j))^{-\gamma} \; \forall j \in \{2,3\}.
\end{aligned} \tag{A6}
$$

These equations, together with equation (A1), imply the policy functions (22), (A3), and (A4) for period $T-1$. Substituting the policy functions into the Bellman equation, the value function in period $T-1$ is

$$
V_{T-1}(h_{T-1}, W_{T-1}) = \left(\frac{\omega(h_{T-1})}{c_{T-1}(h_{T-1})} \right)^{\gamma/(1-\gamma)} \widehat{W}_{T-1}. \tag{A7}
$$

Suppose that the value function in each period $t+1$ is

$$
V_{t+1}(h_{t+1}, W_{t+1}) = \left(\frac{\omega(h_{t+1})}{c_{t+1}(h_{t+1})} \right)^{\gamma/(1-\gamma)} \widehat{W}_{t+1}. \tag{A8}
$$

The first-order conditions in each period t are

$$
\begin{aligned}
\omega(h_t)^{\gamma} C_t^{*-\gamma} =& \beta R \omega(1)^{\gamma} A_{t+1}^*(1)^{-\gamma} \\
=& \frac{\beta R \omega(j)^{\gamma}}{c_{t+1}(j)^{\gamma}} \left(A_{t+1}^*(j) + \sum_{s=1}^{T-t} \frac{\mathbf{E}_{t+1}[Y_{t+s} - M_{t+s} | j]}{R^{s-1}} \right)^{-\gamma} \quad \forall j \in \{2,3\}.
\end{aligned} \tag{A9}
$$

These equations, together with equation (A1), imply the policy functions (22), (A3), and (A4) for each period t. Substituting the policy functions into the Bellman equation, the value function in each period t is

$$
V_t(h_t, W_t) = \left(\frac{\omega(h_t)}{c_t(h_t)} \right)^{\gamma/(1-\gamma)} \widehat{W}_t. \tag{A10}
$$

A.2 Proof of Proposition 3

To simplify notation, let $\pi_t^0(h_t, i) = 1_{\{h_t=i\}}$. Iterating forward on the budget constraint (A1),

$$\begin{aligned} W_t - C_t = &\sum_{s=1}^{n-1}\sum_{i=2}^{3} \frac{\pi_t^s(h_t,i)}{R^s}(C_{t+s}(i) - Y_{t+s}(i) + M_{t+s}(i)) \\ &+ \sum_{s=1}^{n}\sum_{i=2}^{3} \frac{\pi_t^{s-1}(h_t,i)\pi_{t+s-1}(i,1)}{R^s}(\delta_{t+s-1}(i) + A_{t+s}(i)) \\ &+ \sum_{i=2}^{3}\left[\frac{\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,2)}{R^n}(\Delta_{t+n-1}(i) + A_{t+n}(i)) \right. \\ &\left. + \frac{\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,3)}{R^n} A_{t+n}(i)\right]. \end{aligned} \tag{A11}$$

We consider perturbations of health and mortality delta that satisfy the budget constraint:

$$\partial\Delta_{t+n-1}(i) + \pi_{t+n-1}(i,2)\partial A_{t+n}(i) = 0, \tag{A12}$$

$$\partial\delta_{t+n-1}(i) + \pi_{t+n-1}(i,1)\partial A_{t+n}(i) = 0. \tag{A13}$$

We write the value function under complete markets as

$$\begin{aligned} &V_t(\Delta_{t+n-1}(i), \delta_{t+n-1}(i)) = \Bigg\{\omega(h_t)^\gamma C_t^{1-\gamma} + \sum_{s=1}^{n-1}\beta^s \sum_{i=2}^{3} \pi_t^s(h_t,i)\omega(i)^\gamma C_{t+s}(i)^{1-\gamma} \\ &+ \sum_{s=1}^{n}\beta^s \sum_{i=2}^{3} \pi_t^{s-1}(h_t,i)\pi_{t+s-1}(i,1)\omega(1)^\gamma(\delta_{t+s-1}(i) + A_{t+s}(i))^{1-\gamma} \\ &+ \beta^n \sum_{i=2}^{3}\big[\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,2)V_{t+n}(2, \Delta_{t+n-1}(i) + A_{t+n}(i) + Y_{t+n}(2) - M_{t+n}(2))^{1-\gamma} \\ &+ \pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,3)V_{t+n}(3, A_{t+n}(i) + Y_{t+n}(3) - M_{t+n}(3))^{1-\gamma}\big]\Bigg\}^{1/(1-\gamma)}. \end{aligned} \tag{A14}$$

Iterating forward on the first-order conditions (A9),

$$\begin{aligned}
&\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{\gamma/(1-\gamma)} V_t^{*-\gamma} = (\beta R)^n \omega(1)^\gamma (\delta^*_{t+n-1}(i) + A^*_{t+n}(i))^{-\gamma} \\
&= (\beta R)^n \left(\frac{\omega(2)}{c_{t+n}(2)}\right)^{\gamma/(1-\gamma)} V_{t+n}(2, \Delta^*_{t+n-1}(i) + A^*_{t+n}(i) + Y_{t+n}(2) - M_{t+n}(2))^{-\gamma} \\
&= (\beta R)^n \left(\frac{\omega(3)}{c_{t+n}(3)}\right)^{\gamma/(1-\gamma)} V_{t+n}(3, A^*_{t+n}(i) + Y_{t+n}(3) - M_{t+n}(3))^{-\gamma}. \qquad \text{(A15)}
\end{aligned}$$

Taking the partial derivative of equation (A14) with respect to $\Delta_{t+n-1}(i)$,

$$\begin{aligned}
&\frac{\partial V_t(\Delta_{t+n-1}(i), \delta_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)} = \beta^n \pi_t^{n-1}(h_t, i) \pi_{t+n-1}(i, 2) V_t^\gamma \\
&\times \Big[-\pi_{t+n-1}(i,1)\omega(1)^\gamma (\delta_{t+n-1}(i) + A_{t+n}(i))^{-\gamma} \\
&+ (1 - \pi_{t+n-1}(i,2)) \left(\frac{\omega(2)}{c_{t+n}(2)}\right)^{\gamma/(1-\gamma)} V_{t+n}(2, \Delta_{t+n-1}(i) + A_{t+n}(i) + Y_{t+n}(2) - M_{t+n}(2))^{-\gamma} \\
&-\pi_{t+n-1}(i,3) \left(\frac{\omega(3)}{c_{t+n}(3)}\right)^{\gamma/(1-\gamma)} V_{t+n}(3, A_{t+n}(i) + Y_{t+n}(3) - M_{t+n}(3))^{-\gamma}\Bigg]. \qquad \text{(A16)}
\end{aligned}$$

Evaluating at the optimal policy,

$$\frac{\partial V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)} = 0. \qquad \text{(A17)}$$

Similarly, the first partial derivative of the value function with respect to mortality delta, evaluated at the optimal policy, is

$$\frac{\partial V_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \delta_{t+n-1}(i)} = 0. \qquad \text{(A18)}$$

Taking the partial derivative of equation (A16) with respect to $\Delta_{t+n-1}(i)$ and evaluating at the optimal policy,

$$\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)^2} = -\gamma\beta^n\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,2)^2 V_t^{*\gamma}$$
$$\times\Big[\pi_{t+n-1}(i,1)\omega(1)^\gamma(\delta^*_{t+n-1}(i)+A^*_{t+n}(i))^{-1-\gamma}$$
$$+\frac{(1-\pi_{t+n-1}(i,2))^2}{\pi_{t+n-1}(i,2)}\left(\frac{\omega(2)}{c_{t+n}(2)}\right)^{2\gamma/(1-\gamma)} V_{t+n}(2,\Delta^*_{t+n-1}(i)+A^*_{t+n}(i)+Y_{t+n}(2)-M_{t+n}(2))^{-1-\gamma}$$
$$+\pi_{t+n-1}(i,3)\left(\frac{\omega(3)}{c_{t+n}(3)}\right)^{2\gamma/(1-\gamma)} V_{t+n}(3,A^*_{t+n}(i)+Y_{t+n}(3)-M_{t+n}(3))^{-1-\gamma}\Bigg]. \tag{A19}$$

Substituting the first-order conditions (A15),

$$\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)^2} = -\frac{\gamma\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,2)^2}{\beta^{n/\gamma}R^{n(1+1/\gamma)}V_t^*}\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{(1+\gamma)/(1-\gamma)}$$
$$\times\left[\frac{\pi_{t+n-1}(i,1)}{\omega(1)}+\frac{(1-\pi_{t+n-1}(i,2))^2c_{t+n}(2)}{\pi_{t+n-1}(i,2)\omega(2)}+\frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A20}$$

Similarly, the second partial derivative of the value function with respect to mortality delta, evaluated at the optimal policy, is

$$\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \delta_{t+n-1}(i)^2} = -\frac{\gamma\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,1)^2}{\beta^{n/\gamma}R^{n(1+1/\gamma)}V_t^*}\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{(1+\gamma)/(1-\gamma)}$$
$$\times\left[\frac{(1-\pi_{t+n-1}(i,1))^2}{\pi_{t+n-1}(i,1)\omega(1)}+\frac{\pi_{t+n-1}(i,2)c_{t+n}(2)}{\omega(2)}+\frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A21}$$

Finally, the cross-partial derivative of the value function with respect to health and mortality delta, evaluated at the optimal policy, is

$$\frac{\partial^2 V_t(\Delta^*_{t+n-1}(i),\delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)\partial\delta_{t+n-1}(i)} = -\frac{\gamma\pi_t^{n-1}(h_t,i)\pi_{t+n-1}(i,1)\pi_{t+n-1}(i,2)}{\beta^{n/\gamma}R^{n(1+1/\gamma)}V_t^*}\left(\frac{\omega(h_t)}{c_t(h_t)}\right)^{(1+\gamma)/(1-\gamma)}$$
$$\times\left[-\frac{1-\pi_{t+n-1}(i,1)}{\omega(1)}-\frac{(1-\pi_{t+n-1}(i,2))c_{t+n}(2)}{\omega(2)}+\frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A22}$$

Dividing by V_t^* and substituting the value function (A10),

$$\frac{\partial^2 L_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)^2} = -\frac{\gamma \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,2)^2 \omega(h_t)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} c_t(h_t) \widehat{W}_t^2}$$
$$\times \left[\frac{\pi_{t+n-1}(i,1)}{\omega(1)} + \frac{(1-\pi_{t+n-1}(i,2))^2 c_{t+n}(2)}{\pi_{t+n-1}(i,2)\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A23}$$

$$\frac{\partial^2 L_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \delta_{t+n-1}(i)^2} = -\frac{\gamma \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,1)^2 \omega(h_t)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} c_t(h_t) \widehat{W}_t^2}$$
$$\times \left[\frac{(1-\pi_{t+n-1}(i,1))^2}{\pi_{t+n-1}(i,1)\omega(1)} + \frac{\pi_{t+n-1}(i,2) c_{t+n}(2)}{\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A24}$$

$$\frac{\partial^2 L_t(\Delta^*_{t+n-1}(i), \delta^*_{t+n-1}(i))}{\partial \Delta_{t+n-1}(i)\partial \delta_{t+n-1}(i)} = -\frac{\gamma \pi_t^{n-1}(h_t, i)\pi_{t+n-1}(i,1)\pi_{t+n-1}(i,2) \omega(h_t)}{\beta^{n/\gamma} R^{n(1+1/\gamma)} c_t(h_t) \widehat{W}_t^2}$$
$$\times \left[-\frac{1-\pi_{t+n-1}(i,1)}{\omega(1)} - \frac{(1-\pi_{t+n-1}(i,2)) c_{t+n}(2)}{\omega(2)} + \frac{\pi_{t+n-1}(i,3)c_{t+n}(3)}{\omega(3)}\right]. \tag{A25}$$

Appendix B. Health and Retirement Study

The Health and Retirement Study is a panel survey designed to study the health and wealth dynamics of the elderly in the United States. The data consist of five cohorts: the Study of Assets and Health Dynamics among the Oldest Old (born before 1924), the Children of Depression (born 1924 to 1930), the initial HRS cohort (born 1931 to 1941), the War Baby (born 1942 to 1947), and the Early Baby Boomer (born 1948 to 1953). Many of the variables that we use are from the RAND HRS (Version I), which is produced by the RAND Center for the Study of Aging with funding from the National Institute on Aging and the Social Security Administration. Whenever necessary, we use variables from both the core and exit interviews to supplement the RAND HRS. The data consist of eight waves, covering every two years between 1992 and 2006.

The Health and Retirement Study continues to interview respondents that enter nursing homes. However, any respondent that enters a nursing home receives a zero sampling weight because these weights are based on the non-institutionalized population of the Current Population Survey. Therefore, the use of sampling weights would lead us to underestimate nursing home expenses, which account for a significant share of out-of-pocket health expenses for older households. Because nursing home expenses are important for this paper, we do not use sampling weights in any of our analysis.

Since wave 3, the survey asks bracketing questions to solicit a range of values for questions that initially receive a non-response. Based on the range of values implied by the bracketing questions, we use the following methodology to impute missing observations. For each missing observation, we calculate the minimum and maximum values that are implied by the responses to the bracketing questions. For each non-missing observation, we set the minimum and maximum values to be the valid response. We then estimate the mean and the standard deviation of the variable in question through interval regression, under the assumption of log-normality. Finally, we fill in each missing observation as the conditional mean of the distribution in the bracketed range.

B.1 Out-of-Pocket Health Expenses

Out-of-pocket health expenses from the RAND HRS are the total amount paid for hospitals, nursing homes, doctor visits, dentist visits, outpatient surgery, prescription drugs, home health care, and special facilities. Payments of health insurance premiums from the core interviews are the sum of premiums paid for Medicare/Medicaid HMO, private health insurance, long-term care insurance, and prescription drug coverage (i.e., Medicare Part D). We convert the premium reported at monthly, quarterly, semi-annual, or annual frequency to the total implied payment over two years.

Since wave 3, out-of-pocket health expenses at the end of life are available through the exit interviews. Without end-of-life expenses, we would underestimate the true cost of poor health in old age, especially in the upper tail of the distribution (Marshall, McGarry, and Skinner, 2010). Out-of-pocket health expenses from the exit interviews are the total amount paid for hospitals, nursing homes, doctor visits, prescription drugs, home health care, other health services, other medical expenses, and other non-medical expenses.

We measure out-of-pocket health expenses as the sum of out-of-pocket health expenses from the RAND HRS and payments of health insurance premiums from the core interviews. For the last core interview prior to death of the primary respondent, we also add out-of-pocket health expenses at the end of life from the exit interviews. We measure out-of-pocket health expenses at the household level as the sum of these expenses for both the male respondent and his wife, if married.

We estimate the life-cycle profile for out-of-pocket health expenses through a panel regression with household fixed effects. We model the logarithm of real out-of-pocket health expenses as a function of dummy variables for health and 65 or older, a quadratic polynomial in age, and the interaction of the dummy variables with the quadratic polynomial in age. The dummy variable for 65 or older allows for potential changes in household behavior after retirement, when households qualify for Social Security and Medicare. We use the estimated regression model, averaging the household fixed effects by cohort and ownership of supplementary health insurance and long-term care insurance, to predict out-of-pocket health expenses in the absence of these policies by cohort, age, and health.

B.2 Income

Income includes labor income, Social Security disability and supplemental security income, Social Security retirement income, and unemployment or workers compensation. Income excludes pension and annuity income and capital income. We calculate after-tax income by subtracting federal income tax liabilities, estimated through the NBER TAXSIM program (Version 9). We measure household income as the sum of income for both the male respondent and his wife, if married.

We estimate the life-cycle profile for income through a panel regression with household fixed effects. We model the logarithm of real after-tax income as a function of a dummy variable for 65 or older, a quadratic polynomial in age, and the interaction of the dummy variable with the quadratic polynomial in age. We use the estimated regression model, averaging the household fixed effects by cohort, to predict income by cohort and age.

B.3 Life Insurance

We measure the ownership and the face value of life insurance using the core interviews. Term-life insurance refers to individual and group policies that have only a death benefit. Whole-life insurance refers to policies that build cash value, from which the policyholder can borrow or receive cash upon surrender of the policy. In waves 1 through 3, we measure the total face value of all policies as the sum of the face value of term- and whole-life insurance. In wave 4, only the total face value of all policies, and not the breakdown between term- and whole-life insurance, is available. In waves 5 through 8, we measure the total face value of term-life insurance as the difference between the face value of all policies and whole-life insurance.

We estimate the life-cycle profile for the face value of life insurance through a panel regression with household fixed effects. We model the logarithm of the real face value of life insurance as a function of dummy variables for health and 65 or older, a quadratic polynomial in age, and the interaction of the dummy variables with the quadratic polynomial in age. We use the estimated regression model, averaging the household fixed effects by cohort, to predict the face value of life insurance by cohort, age, and health.

B.4 Annuities including Defined-Benefit Plans

We define ownership of annuities including defined-benefit plans as participation in a defined-benefit plan at the present employer or positive reported pension and annuity income. We measure household pension and annuity income as the sum of this income for both the male respondent and his wife, if married.

We estimate the life-cycle profile for pension and annuity income through a panel regression with household fixed effects. We model the logarithm of real pension and annuity income as a function of dummy variables for health and 65 or older, a quadratic polynomial in age, and the interaction of the dummy variables with the quadratic polynomial in age. We use the estimated regression model, averaging the household fixed effects by cohort, to predict pension and annuity income by cohort, age, and health.

B.5 Net Worth

Household assets include checking, savings, and money market accounts; CD, government savings bonds, and T-bills; bonds and bond funds; IRA and Keogh accounts; businesses; stocks, mutual funds, and investment trusts; and primary and secondary residence. Household liabilities include all mortgages for primary and secondary residence, other home loans for primary residence, and other debt. Net worth is the value of assets minus the value of liabilities.

We estimate the life-cycle profile for net worth through a panel regression with household fixed effects. We model the logarithm of real net worth as a function of dummy variables for health and 65 or older, a quadratic polynomial in age, and the interaction of the dummy variables with the quadratic polynomial in age. We use the estimated regression model, averaging the household fixed effects by cohort, to predict net worth by cohort, age, and health.

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Table 1: Life Insurance and Annuity Products Offered by TIAA-CREF

This table lists the life insurance and annuity products offered by TIAA-CREF, a financial services company based in New York, as of December 2010.

Name of product	Term	Income deferrable
Panel A: Life insurance products		
Annual Renewable Term	1 year	
Level Term	10, 15, 20, or 30 years	
Intelligent Life Universal	Life	
Panel B: Annuity products		
Single Premium Immediate	Life	
Investment Horizon	Life	14 months–90th birthday

Table 2: Predicting Future Mortality with Observed Health Problems

This table reports an estimate of a probit model for predicting death within two years from the present interview. The explanatory variables include dummy variables for doctor-diagnosed health problems, age, the interaction of the health problems with age, and cohort dummies. The omitted cohort is those born prior to 1911. The table reports the marginal effects on the mortality rate (in percentage points) with heteroskedasticity-robust t-statistics in parentheses. The sample consists of males aged 51 and older in the Health and Retirement Study for the period 1992 to 2006.

Explanatory variable	Marginal effect	t-statistic
Doctor-diagnosed health problems:		
High blood pressure	1.66	(3.52)
Diabetes	5.66	(6.41)
Cancer	13.62	(8.61)
Lung disease	8.21	(6.17)
Heart problems	2.60	(4.18)
Stroke	5.57	(4.40)
$(\text{Age} - 51)/10$	3.26	(12.71)
× High blood pressure	-0.44	(-2.31)
× Diabetes	-0.72	(-3.07)
× Cancer	-1.79	(-7.25)
× Lung disease	-0.28	(-1.05)
× Heart problems	0.04	(0.18)
× Stroke	-0.32	(-1.17)
Birth cohort:		
1911–1915	-1.69	(-5.23)
1916–1920	-2.39	(-8.16)
1921–1925	-3.32	(-12.17)
1926–1930	-3.58	(-11.69)
1931–1935	-3.74	(-8.79)
1936–1940	-4.08	(-8.37)
1941–1945	-3.46	(-8.33)
1946–1950	-3.51	(-9.47)
1951–1955	-3.03	(-4.97)
Correctly predicted (%):		
Both outcomes	92.87	
Dead only	55.13	
Alive only	93.01	
Observations	43,452	

Table 3: Health Problems, Health Care Utilization, and Health Insurance Coverage

Panel A reports the percentage of households who have ever had doctor-diagnosed health problems or have some difficulty with activities of daily living at the time of interview. Panel B reports the percentage of households who have used health care in the two years prior to the interview. Panel C reports the percentage of households who have health insurance at the time of interview. Panel D reports the percentage of households who own life insurance or annuities including defined-benefit plans at the time of interview. It also reports the median of total face value conditional on ownership, deflated by the consumer price index to 2005 dollars. Term-life insurance refers to individual and group policies that have only a death benefit. Whole-life insurance refers to policies that build cash value, from which the policyholder can borrow or receive cash upon surrender of the policy. Supplementary health insurance includes Medigap insurance and refers to any coverage that is not government, employer-provided, or long-term care insurance. The sample consists of males aged 51 and older in the Health and Retirement Study for the period 1992 to 2006.

Age	51–66		67–82		83–	
Health	Poor	Good	Poor	Good	Poor	Good
Panel A: Health problems (%)						
Doctor-diagnosed health problems:						
High blood pressure	59	32	65	46	57	44
Diabetes	22	9	34	14	20	14
Cancer	9	4	30	12	28	20
Lung disease	10	4	21	7	20	7
Heart problems	28	11	56	26	77	31
Stroke	7	3	21	6	33	11
Some difficulty with						
Bathing	5	1	10	4	27	15
Dressing	9	4	14	8	30	18
Eating	2	1	5	2	16	7
Panel B: Health care utilization (%)						
Monthly doctor visits	11	4	17	8	21	12
Hospital stay	27	14	43	26	54	37
Outpatient surgery	21	17	26	21	24	21
Nursing home stay	1	0	4	2	19	8
Home health care	4	2	12	6	24	13
Special facilities and services	7	4	10	6	15	11
Prescription drugs	79	52	94	76	97	80
Panel C: Health insurance (%)						
Medicare	22	17	98	97	99	98
Medicaid	3	2	3	5	5	7
Employer-provided health insurance	58	63	36	32	30	26
Supplementary health insurance	10	11	33	32	38	38
Long-term care insurance	7	7	12	13	10	9
Panel D: Life insurance, annuities including defined-benefit plans, and net worth (thousands of 2005 dollars)						
Ownership rate (%):						
All life insurance	78	80	71	71	59	57
Term-life insurance	62	65	50	51	38	38
Whole-life insurance	29	33	30	28	23	19
Annuities including defined-benefit plans	51	56	65	61	58	58
Median face value conditional on ownership:						
All life insurance	57	72	18	20	10	10
Term-life insurance	50	67	12	14	7	7
Whole-life insurance	35	40	20	20	11	11
Net worth excluding life insurance and annuities	120	161	186	187	170	153

Table 4: Health Distribution, Life Expectancy, and Out-of-Pocket Health Expenses
Panel A reports the health distribution at each age for a population of households who are in good health at age 51. Panel B reports the remaining life expectancy by age and health. Panel C reports annual out-of-pocket health expenses by age and health in thousands of 2005 dollars. Panel D reports annual income by age in thousands of 2005 dollars. Panel E reports the present value of future income in excess of out-of-pocket health expenses by age and health in thousands of 2005 dollars. The reported estimates are for males in good health at age 51, born 1936 to 1940 in the Health and Retirement Study.

Health	Age						
	51	59	67	75	83	91	99
Panel A: Health distribution (%)							
Dead	0	15	30	45	62	83	97
Poor	0	22	18	16	14	9	2
Good	100	63	52	39	23	8	1
Panel B: Life expectancy (years)							
Poor	24	20	15	11	8	5	4
Good	26	23	19	14	10	7	4
Mean	26	22	18	13	9	6	4
Panel C: Out-of-pocket health expenses (thousands of 2005 dollars per year)							
Poor	2	4	7	12	21	38	73
Good	0	1	3	5	8	11	14
Mean	0	2	4	7	12	25	56
Panel D: Income (thousands of 2005 dollars per year)							
Mean	51	38	26	21	18	16	14
Panel E: Present value of future disposable income (thousands of 2005 dollars)							
Poor	428	233	107	19	-43	-81	-104
Good	468	271	136	31	-51	-111	-147
Mean	468	261	129	27	-48	-95	-116

Table 5: Life Insurance, Annuity Income, and Net Worth

Panel A reports the total face value of term-life insurance, conditional on ownership, by age and health in thousands of 2005 dollars. Panel B reports the total face value of whole-life insurance, conditional on ownership, by age and health in thousands of 2005 dollars. Panel C reports annual annuity income including defined-benefit plans, conditional ownership, by age and health in thousands of 2005 dollars. Panel D reports net worth excluding life insurance and annuities by age and health in thousands of 2005 dollars. The reported estimates are for males born 1936 to 1940 in the Health and Retirement Study.

Health	Age						
	51	59	67	75	83	91	99
Panel A: Term-life insurance (thousands of 2005 dollars)							
Poor	56	50	30	22	21	26	43
Good	64	54	31	23	22	29	50
Mean	64	53	31	23	22	28	45
Panel B: Whole-life insurance (thousands of 2005 dollars)							
Poor	50	41	31	26	25	29	38
Good	50	41	31	25	22	22	26
Mean	50	41	31	25	23	26	34
Panel C: Annuity income including defined-benefit plans (thousands of 2005 dollars per year)							
Poor	11	12	14	13	12	11	9
Good	8	10	13	13	12	10	8
Mean	8	11	13	13	12	10	9
Panel D: Net worth excluding life insurance and annuities (thousands of 2005 dollars)							
Poor	65	112	145	155	148	127	97
Good	65	114	151	166	163	145	115
Mean	65	114	150	163	158	135	102

Table 6: Determinants of the Observed Health and Mortality Delta

The explanatory variables in the first specification include dummy variables for poor health and 65 or older, a quadratic polynomial in age, the interaction of the dummy variables with the quadratic polynomial in age, and cohort dummies. The omitted cohort is those born prior to 1911. Additional explanatory variables in the second specification include dummy variables for marital status and living children, logarithm of net worth excluding life insurance and annuities, and self-reported general health status. The table reports the regression coefficients with heteroskedasticity-robust t-statistics in parentheses. The sample consists of males aged 51 and older in the Health and Retirement Study for the period 1992 to 2006.

Explanatory variable	Health delta				Mortality delta			
	(1)		(2)		(3)		(4)	
Poor health	-0.14	(-1.18)	-0.18	(-1.49)	-15.20	(-12.53)	-15.30	(-12.52)
65 or older	-7.56	(-12.35)	-7.58	(-12.30)	35.40	(10.41)	35.51	(10.36)
(Age – 51)/10	1.23	(3.10)	1.24	(3.11)	167.76	(44.26)	167.73	(44.15)
× Poor health	-1.94	(-9.54)	-1.90	(-9.27)	0.59	(0.42)	1.16	(0.81)
× 65 or older	8.01	(12.51)	8.06	(12.51)	-100.00	(-23.44)	-100.00	(-23.34)
$(\text{Age} - 51)^2/100$	-1.32	(-4.36)	-1.32	(-4.33)	-63.04	(-24.74)	-62.90	(-24.61)
× Poor health	0.41	(7.18)	0.40	(6.98)	0.55	(1.62)	0.40	(1.17)
× 65 or older	-0.41	(-1.28)	-0.43	(-1.33)	56.49	(21.91)	56.42	(21.82)
Married			-0.32	(-3.92)			1.23	(2.31)
Has living children			-0.24	(-2.11)			2.51	(3.24)
Net worth			-0.01	(-0.79)			-0.06	(-0.50)
Self-reported health status:								
Poor			0.29	(3.07)			2.39	(3.72)
Fair			0.03	(0.39)			1.54	(3.02)
Very good			0.15	(1.62)			-0.50	(-0.88)
Excellent			0.63	(5.05)			1.21	(1.59)
Birth cohort:								
1911–1915	2.14	(10.15)	2.12	(10.03)	12.75	(18.92)	12.92	(18.90)
1916–1920	0.68	(2.94)	0.68	(2.94)	17.39	(22.55)	17.43	(22.16)
1921–1925	5.10	(20.42)	5.15	(20.51)	38.99	(45.08)	39.14	(44.31)
1926–1930	3.82	(14.06)	3.91	(14.32)	48.06	(47.43)	48.17	(46.67)
1931–1935	3.33	(11.75)	3.40	(11.93)	60.96	(53.60)	61.22	(52.89)
1936–1940	3.51	(12.03)	3.62	(12.30)	80.95	(65.13)	81.37	(64.31)
1941–1945	2.52	(8.34)	2.59	(8.51)	103.64	(75.70)	103.82	(74.52)
1946–1950	2.44	(8.02)	2.50	(8.16)	179.17	(111.80)	179.65	(110.29)
1951–1955	0.96	(3.08)	1.04	(3.31)	244.57	(118.38)	245.76	(117.71)
R^2 (%)	17.69		17.85		48.74		48.98	
Observations	32,778		32,405		32,778		32,405	

Table 7: Preference Parameters

The subjective discount factor is reported in annualized units. The value for relative risk aversion is based on a previous estimate in the Health and Retirement Study (Barsky et al., 1997). The utility weights for death and poor health are estimated by continuous-updating generalized method of moments with heteroskedasticity-robust standard errors in parentheses. The sample consists of males aged 51 and older in the Health and Retirement Study for the period 1992 to 2006.

Parameter	Symbol	Value
Subjective discount factor	β	0.96
Relative risk aversion	γ	4
Utility weight for death	$\omega(1)$	3.76
		(0.09)
Utility weight for poor health	$\omega(2)$	0.87
		(0.02)
Utility weight for good health	$\omega(3)$	1.00

Table 8: Welfare Cost of the Observed Health and Mortality Delta by Age

This table reports the median welfare cost by age, expressed as a percentage of total wealth. The welfare cost for each household is measured by the difference of the observed health and mortality delta from the optimal health and mortality delta. The lifetime cost is based on the probability of future ownership of health and longevity products, implied by the probit model in Table 10. The sample consists of males aged 51 and older in the Health and Retirement Study for the period 1992 to 2006.

	Age					
	51–58	59–66	67–74	75–82	83–90	91–
Panel A: Welfare cost per period (% of total wealth)						
Total cost	0.22	0.07	0.08	0.09	0.13	0.36
	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.07)
Cost due to health delta	0.00	0.00	0.01	0.01	0.03	0.09
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.06)
Cost due to mortality delta	0.22	0.07	0.08	0.10	0.14	0.24
	(0.02)	(0.01)	(0.01)	(0.01)	(0.02)	(0.03)
Panel B: Lifetime welfare cost (% of total wealth)						
Total cost	26.61	21.97	10.63	2.15	1.10	1.10
	(0.55)	(0.32)	(0.24)	(0.12)	(0.07)	(0.12)
Cost due to health delta	0.57	0.57	0.17	0.08	0.13	0.45
	(0.17)	(0.01)	(0.00)	(0.00)	(0.00)	(0.09)
Cost due to mortality delta	28.05	23.14	11.31	2.42	1.01	0.73
	(0.54)	(0.32)	(0.25)	(0.12)	(0.08)	(0.12)

Table 9: Welfare Cost of the Observed Health and Mortality Delta by Health

This table reports the median welfare cost by age and health, expressed as a percentage of total wealth. The welfare cost for each household is measured by the difference of the observed health and mortality delta from the optimal health and mortality delta. The lifetime cost is based on the probability of future ownership of health and longevity products, implied by the probit model in Table 10. The sample consists of males aged 51 and older in the Health and Retirement Study for the period 1992 to 2006.

Health	Age					
	51–58	59–66	67–74	75–82	83–90	91–
Panel A: Welfare cost per period (% of total wealth)						
Poor	0.68	0.38	0.56	0.52	0.33	0.25
	(0.06)	(0.03)	(0.02)	(0.03)	(0.04)	(0.18)
Good	0.11	0.04	0.05	0.05	0.10	0.40
	(0.01)	(0.00)	(0.00)	(0.00)	(0.01)	(0.06)
Panel B: Lifetime welfare cost (% of total wealth)						
Poor	22.03	20.23	9.55	2.12	1.12	0.78
	(0.85)	(0.54)	(0.42)	(0.19)	(0.12)	(0.25)
Good	29.53	22.02	11.30	2.41	1.10	1.23
	(0.71)	(0.39)	(0.30)	(0.14)	(0.09)	(0.14)

Table 10: Predicting the Future Ownership of Health and Longevity Products

This table reports an estimate of a probit model for predicting ownership of a given type of policy at two years from the present interview. The explanatory variables include dummy variables for present policy owner, poor health, and 65 or older; a quadratic polynomial in age; the interaction of the dummy variables with the quadratic polynomial in age; and cohort dummies. The omitted cohort is those born prior to 1911. The table reports the marginal effects on the probability of ownership (in percentage points) with heteroskedasticity-robust t-statistics in parentheses. The sample consists of males aged 51 and older in the Health and Retirement Study for the period 1992 to 2006.

Explanatory variable	Term-life insurance		Whole-life insurance		Annuities including defined-benefit plans		Supplementary health insurance		Long-term care insurance	
Present owner	46.36	(24.39)	66.80	(38.66)	49.98	(39.96)	33.46	(15.28)	23.84	(9.13)
Poor health	-2.39	(-1.05)	-1.23	(-0.62)	-4.81	(-3.06)	3.38	(2.67)	-1.42	(-1.97)
65 or older	-11.83	(-1.42)	-26.47	(-3.44)	-4.04	(-0.60)	19.82	(4.98)	1.80	(0.58)
$(\text{Age} - 51)/10$	17.02	(2.43)	-10.46	(-1.73)	-25.96	(-5.65)	-21.56	(-6.33)	-1.57	(-0.75)
$\times$ Present owner	2.97	(1.03)	-2.01	(-0.74)	11.37	(5.48)	6.38	(3.99)	9.90	(7.45)
$\times$ Poor health	-0.04	(-0.01)	0.03	(0.01)	3.68	(1.63)	-0.29	(-0.19)	0.82	(0.76)
$\times$ 65 or older	-4.35	(-0.45)	29.90	(3.41)	19.50	(2.67)	3.84	(0.83)	1.28	(0.37)
$(\text{Age} - 51)^2/100$	-12.95	(-2.77)	3.51	(0.85)	12.47	(4.16)	19.02	(8.60)	2.26	(1.61)
$\times$ Present owner	-0.48	(-0.60)	-0.16	(-0.21)	-0.79	(-1.33)	-1.17	(-2.86)	23.84	(9.13)
$\times$ Poor health	0.25	(0.29)	0.27	(0.34)	-0.60	(-0.89)	0.26	(0.62)	0.08	(0.25)
$\times$ 65 or older	11.03	(2.29)	-9.00	(-2.09)	-12.96	(-4.02)	-14.81	(-6.43)	-2.46	(-1.61)
Birth cohort:										
1911–1915	8.00	(2.11)	-9.47	(-3.55)	1.03	(0.35)	12.74	(5.09)	2.06	(0.92)
1916–1920	14.23	(3.86)	-11.61	(-4.54)	0.29	(0.09)	13.97	(5.07)	1.79	(0.76)
1921–1925	17.76	(4.75)	-13.74	(-5.32)	-2.26	(-0.65)	15.12	(5.11)	4.92	(1.68)
1926–1930	20.17	(5.25)	-16.40	(-6.53)	-2.84	(-0.77)	18.48	(5.76)	5.72	(1.85)
1931–1935	25.27	(6.54)	-18.78	(-6.93)	-5.94	(-1.56)	16.72	(5.49)	5.91	(1.98)
1936–1940	28.79	(7.45)	-22.52	(-8.44)	-8.55	(-2.21)	14.18	(4.80)	6.47	(2.16)
1941–1945	31.50	(9.66)	-20.74	(-9.92)	-11.26	(-2.81)	9.26	(2.98)	9.16	(2.42)
1946–1950	35.92	(14.07)	-22.17	(-14.75)	-13.93	(-3.34)	6.48	(1.97)	10.81	(2.48)
1951–1955	36.75	(16.66)	-21.57	(-17.05)	-22.64	(-4.96)	-0.89	(-0.27)	11.23	(2.29)
Correctly predicted (%):										
Both outcomes	75.68		85.20		81.11		85.07		92.17	
Owner only	77.24		74.62		83.85		60.66		64.51	
Not owner only	73.56		89.52		77.24		89.84		94.37	
Observations	18,353		18,651		39,457		38,031		38,080	

Table 11: Optimal Portfolio with Supplementary Health Insurance

Panel A reports the optimal health and mortality delta by age, implied by a life-cycle model with the preference parameters in Table 7. Panel B reports a portfolio of short-term life insurance, deferred annuities, short-term health insurance, and bonds that replicates the optimal health and mortality delta. Short-term policies have maturity of two years, and the income payments of deferred annuities start at age 65. Panel C reports the cost of the optimal portfolio in thousands of 2005 dollars, averaged across the health distribution at the given age. The reported estimates are for males in good health at age 51, born 1936 to 1940 in the Health and Retirement Study.

	Age						
	51	59	67	75	83	91	99
Panel A: Optimal health and mortality delta (thousands of 2005 dollars)							
Health delta	4	-9	-20	-27	-24	4	88
Mortality delta	135	-31	-116	-162	-191	-210	-209
Panel B: Optimal portfolio (units)							
Short-term life insurance	135	23	0	0	0	0	0
Deferred annuity	0	9	16	28	45	72	107
Short-term health insurance	0.98	0.00	0.06	0.52	0.77	0.82	0.85
Bond	63	167	182	175	168	161	154
Panel C: Cost of the optimal portfolio (thousands of 2005 dollars)							
Short-term life insurance	4	1	0	0	0	0	0
Deferred annuity	0	48	101	135	143	128	93
Short-term health insurance	1	0	0	2	8	22	47
Bond	61	161	175	168	161	155	148
Total cost	65	210	276	305	312	305	288

Table 12: Optimal Portfolio with Life Insurance and Annuities Only

Panel A reports the optimal health and mortality delta by age, implied by a life-cycle model with the preference parameters in Table 7. Panel B reports a portfolio of short- and long-term life insurance, short- and long-term annuities, and bonds that replicates the optimal health and mortality delta. Short-term policies have maturity of two years, while long-term policies mature at death. Panel C reports the cost of the optimal portfolio in thousands of 2005 dollars, averaged across the health distribution at the given age. The reported estimates are for males in good health at age 51, born 1936 to 1940 in the Health and Retirement Study.

	Age						
	51	59	67	75	83	91	99
Panel A: Optimal health and mortality delta (thousands of 2005 dollars)							
Health delta	4	-9	-20	-27	-24	4	88
Mortality delta	135	-31	-116	-162	-191	-210	-209
Panel B: Optimal portfolio (units)							
Short-term life insurance	93	33	0	0	0	0	0
Long-term life insurance	113	0	0	0	0	126	4,194
Short-term annuity	0	0	3	39	96	224	629
Long-term annuity	0	7	15	22	23	0	0
Bond	-7	157	182	175	168	34	-4,040
Panel C: Cost of the optimal portfolio (thousands of 2005 dollars)							
Short-term life insurance	3	1	0	0	0	0	0
Long-term life insurance	69	0	0	0	0	113	3,851
Short-term annuity	0	0	3	34	80	159	320
Long-term annuity	0	57	98	103	71	0	0
Bond	-7	151	175	168	161	33	-3,883
Total cost	65	210	276	305	312	305	288

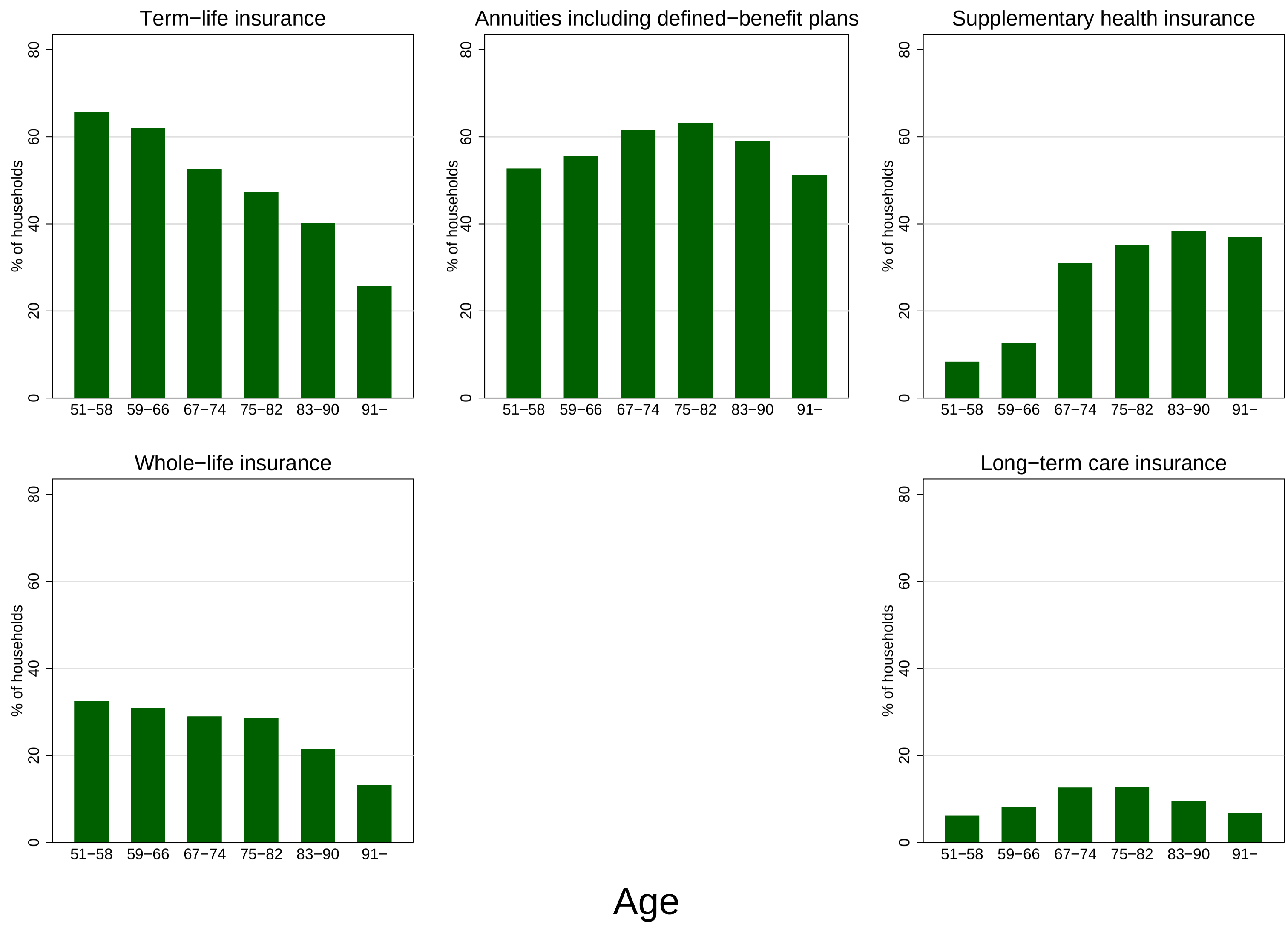

Figure 1: Ownership Rate of Health and Longevity Products
Term-life insurance refers to individual and group policies that have only a death benefit. Whole-life insurance refers to policies that build cash value, from which the policyholder can borrow or receive cash upon surrender of the policy. Supplementary health insurance includes Medigap insurance and refers to any coverage that is not government, employer-provided, or long-term care insurance. The sample consists of males aged 51 and older in the Health and Retirement Study for the period 1992 to 2006.

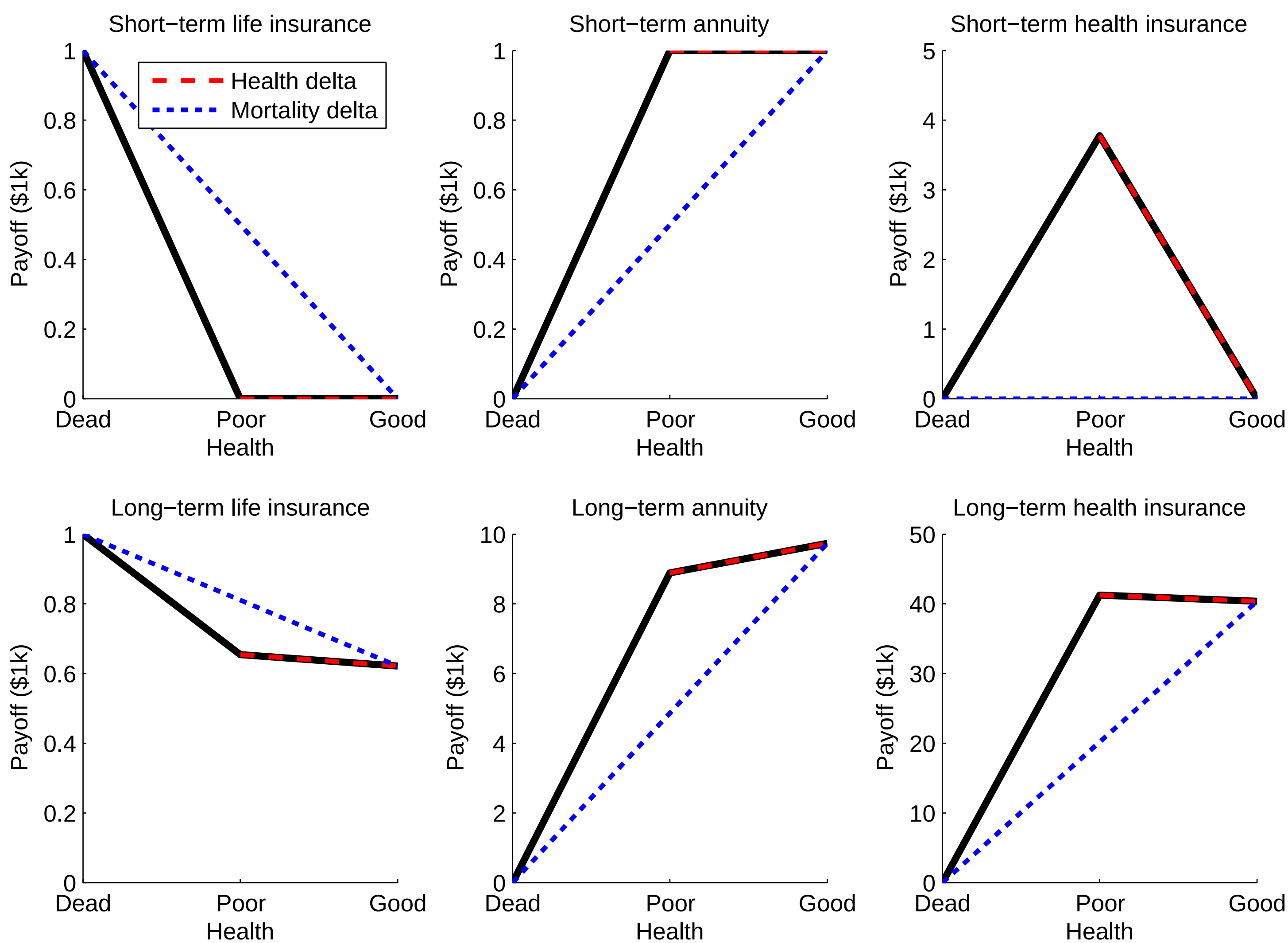

Figure 2: Health and Mortality Delta for Health and Longevity Products

This figure reports the health and mortality delta for life insurance, annuities, and health insurance. The solid line represents the payoff of each policy for the three possible health states in two years, reported in thousands of 2005 dollars. Health delta is minus the slope of the dashed line, normalizing the horizontal distance between good and poor health as one. Mortality delta is minus two times the slope of the dotted line, normalizing the horizontal distance between good health and death as two. Short-term policies have maturity of two years, while long-term policies mature at death. The reported estimates are for males at age 51, born 1936 to 1940 in the Health and Retirement Study.

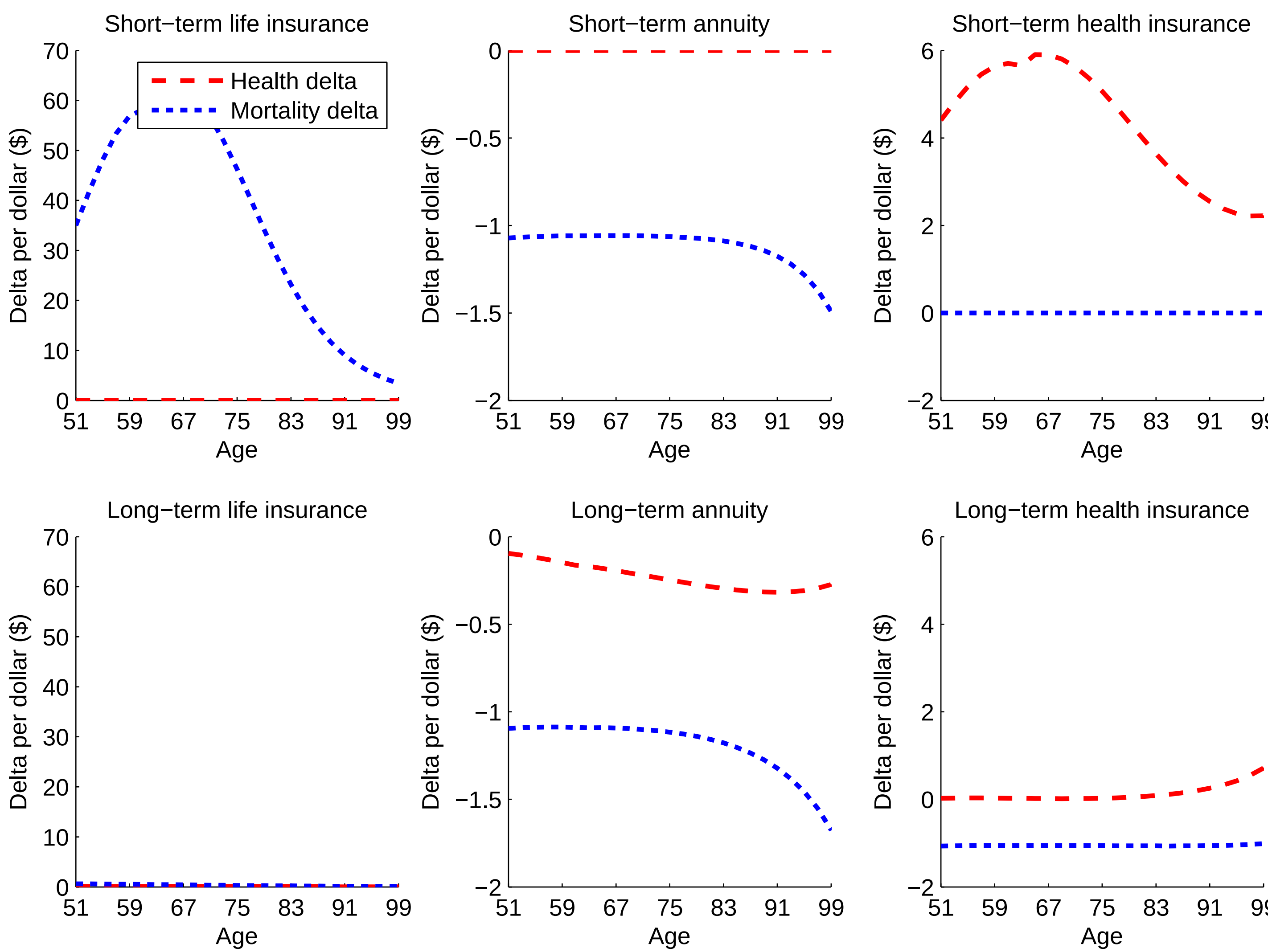

Figure 3: Health and Mortality Delta for Health and Longevity Products over the Life Cycle
This figure reports the health and mortality delta per dollar investment for life insurance, annuities, and health insurance. Short-term policies have maturity of two years, while long-term policies mature at death. The reported estimates are for males in good health at the given age, born 1936 to 1940 in the Health and Retirement Study.

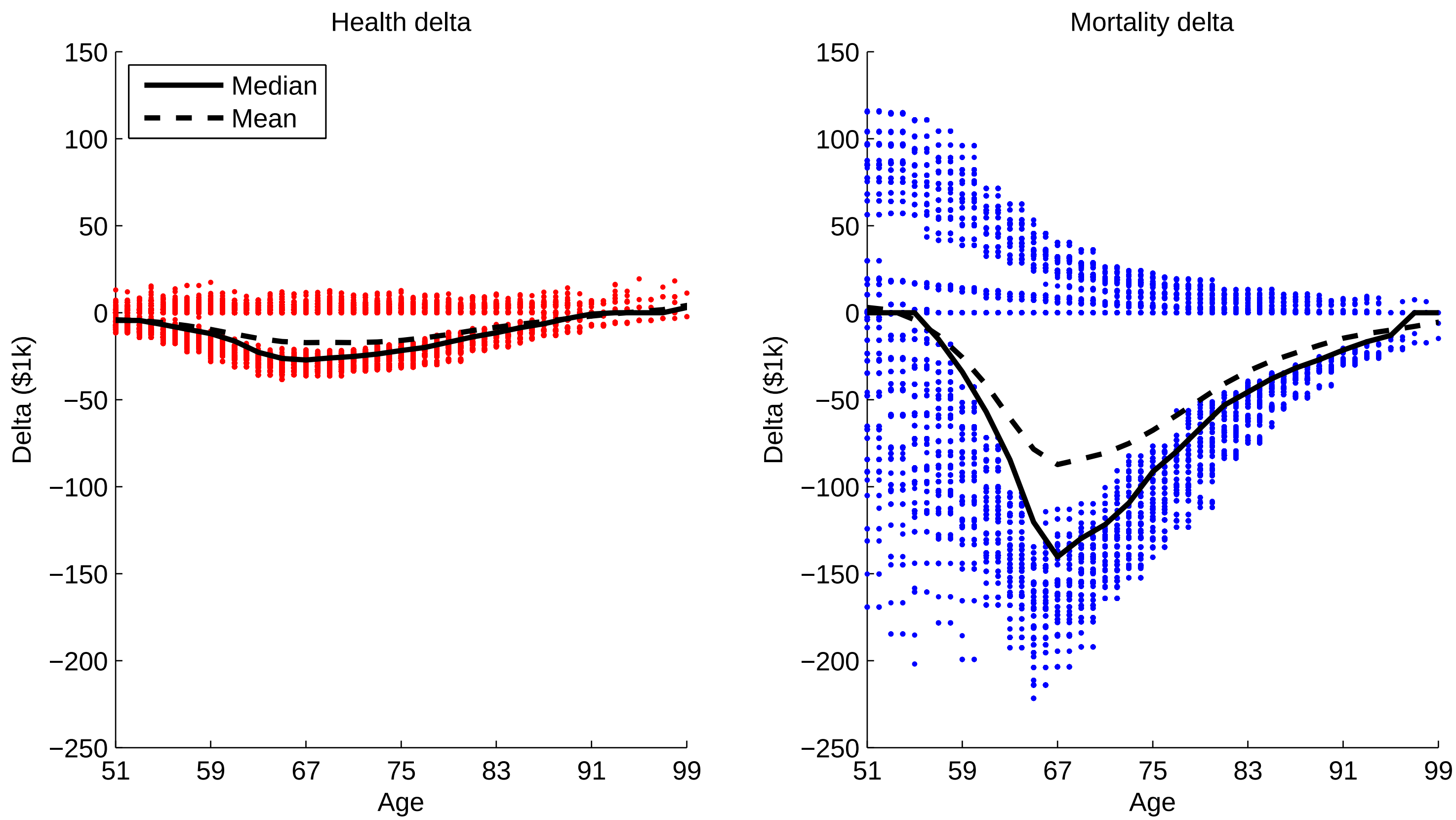

Figure 4: Health and Mortality Delta Implied by the Observed Household Portfolios
This figure reports the median and mean health and mortality delta by age, smoothed around a plus and minus two-year window. Each dot represents one of 32,778 household-interview observations in the Health and Retirement Study for the period 1992 to 2006.

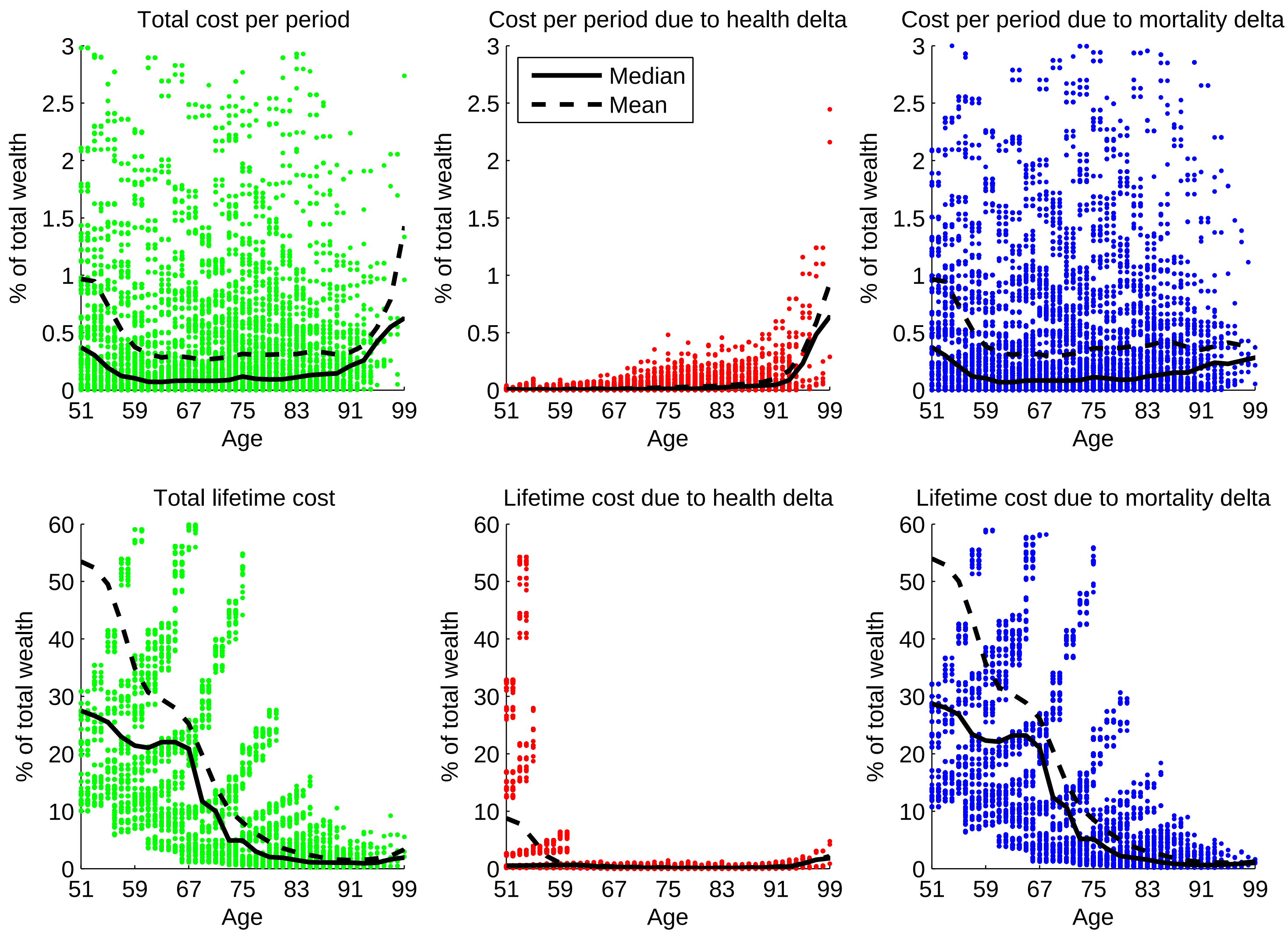

Figure 5: Welfare Cost of the Observed Health and Mortality Delta

This figure reports the median and mean welfare cost by age, smoothed around a plus and minus two-year window and expressed as a percentage of total wealth. The welfare cost for each household is measured by the deviations of the observed health and mortality delta from the optimal health and mortality delta. The lifetime cost is based on the probability of future ownership of health and longevity products, implied by the probit model in Table 10. Each dot represents one of 32,778 household-interview observations in the Health and Retirement Study for the period 1992 to 2006.

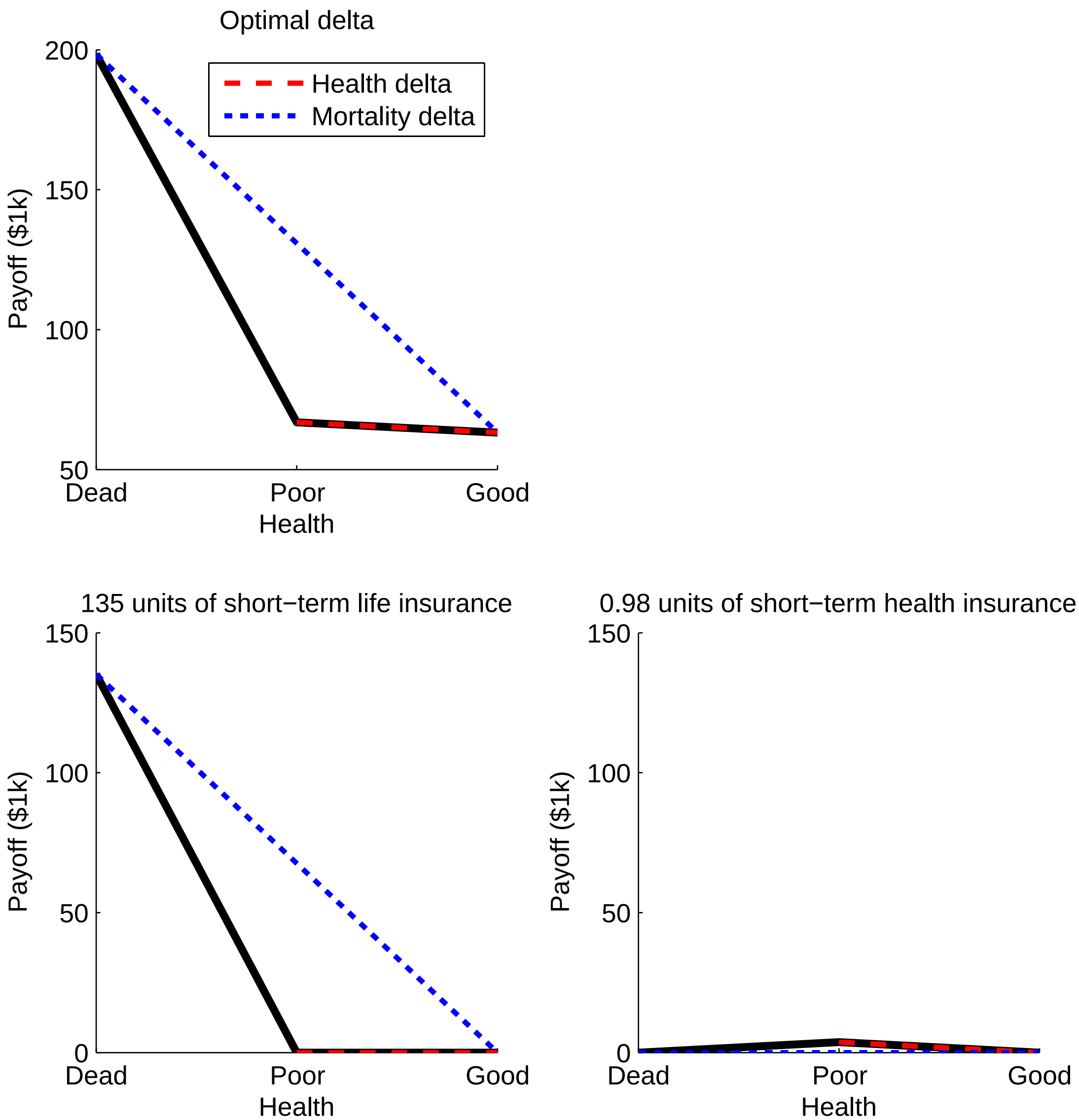

Figure 6: Replicating the Optimal Health and Mortality Delta

This figure illustrates how a portfolio that includes short-term life insurance and health insurance replicates the optimal health and mortality delta. The solid line is the payoff of the portfolio for the three possible health states in two years, reported in thousands of 2005 dollars. Health delta is minus the slope of the dashed line, normalizing the horizontal distance between good and poor health as one. Mortality delta is minus two times the slope of the dotted line, normalizing the horizontal distance between good health and death as two. The sum of health (mortality) delta for short-term life insurance and health insurance equals the optimal health (mortality) delta. The reported estimates are for males in good health at age 51, born 1936 to 1940 in the Health and Retirement Study.

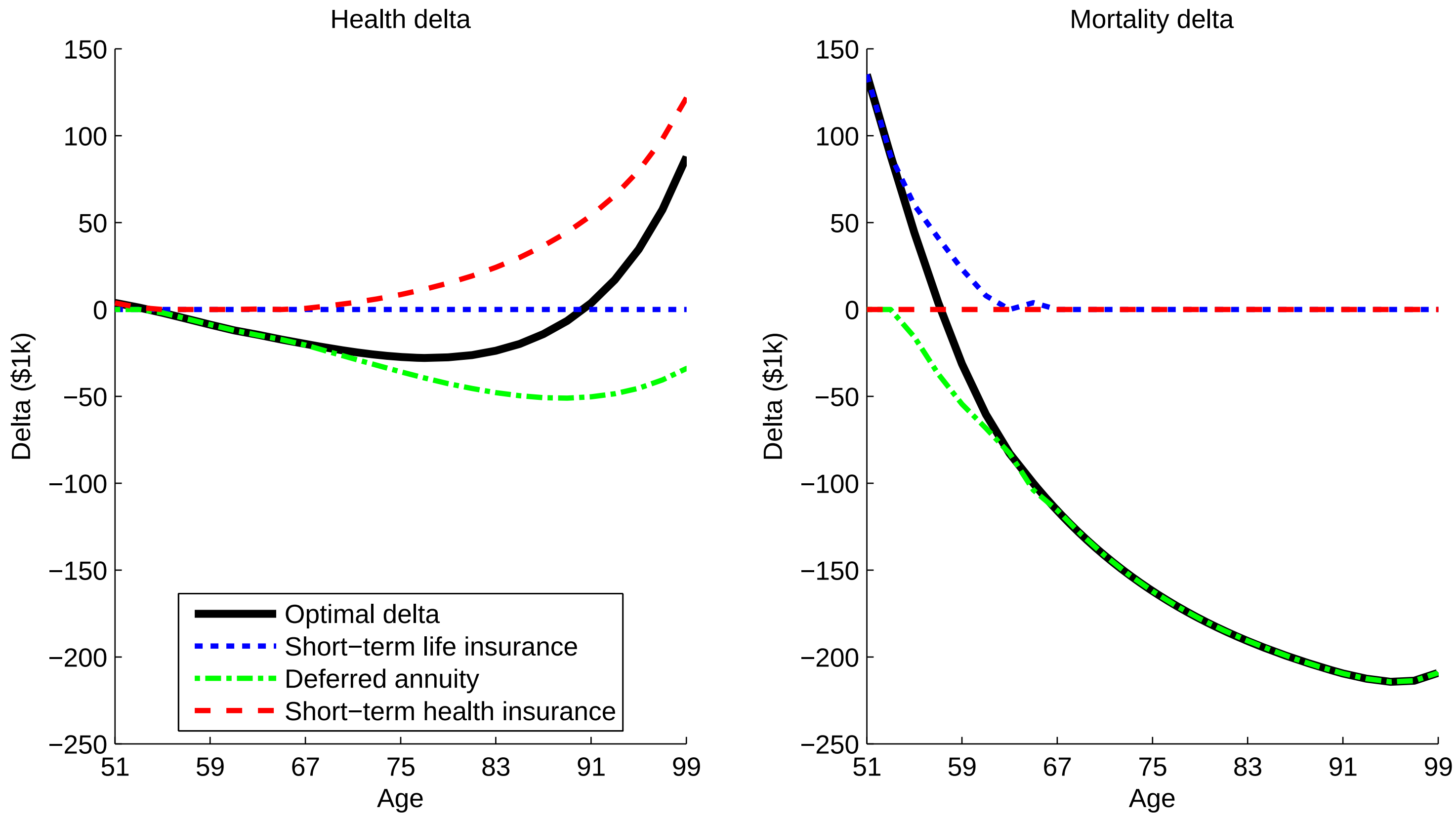

Figure 7: Optimal Health and Mortality Delta over the Life Cycle
The sum of health (mortality) delta for short-term life insurance, deferred annuities, and short-term health insurance equals the optimal health (mortality) delta at each age. Short-term policies have maturity of two years, and the income payments of deferred annuities start at age 65. The reported estimates are for males in good health at age 51, born 1936 to 1940 in the Health and Retirement Study.