D I S C O N T I N U O U S DEMAND FUNCTIONS: ESTIMATION AND PRICING

Arnoud V. den Boer
University of Amsterdam

N. Bora Keskin
Duke University

Rotterdam
May 24, 2018

Dynamic pricing and learning:

- Learning optimal selling price from accumulating sales data

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- Cont. armed MAB, observing demand $d(p)$ and reward $r(p) = p \cdot d(p)$

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- Learning optimal selling price from accumulating sales data
- Cont. armed MAB, observing demand $d(p)$ and reward $r(p) = p \cdot d(p)$
- Standard assumption: $d(\cdot)$ is continuous

Nous admettons que la fonction $F(p)$ qui exprime la loi de la demande ou du débit est une fonction **continue***...*

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Is the assumption of continuous demand functions reasonable?

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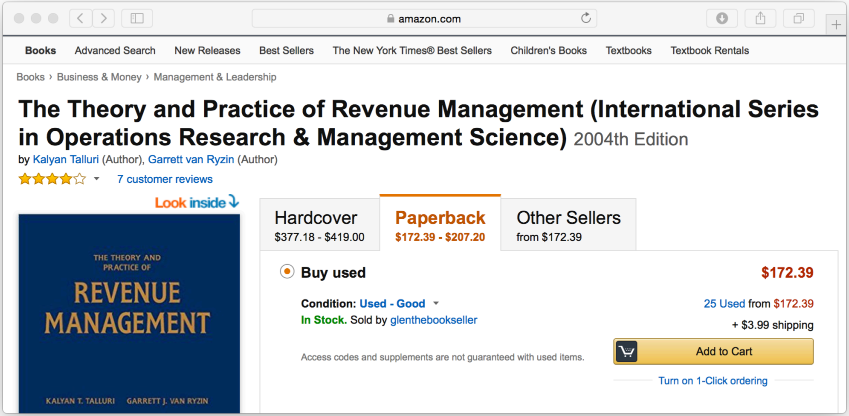
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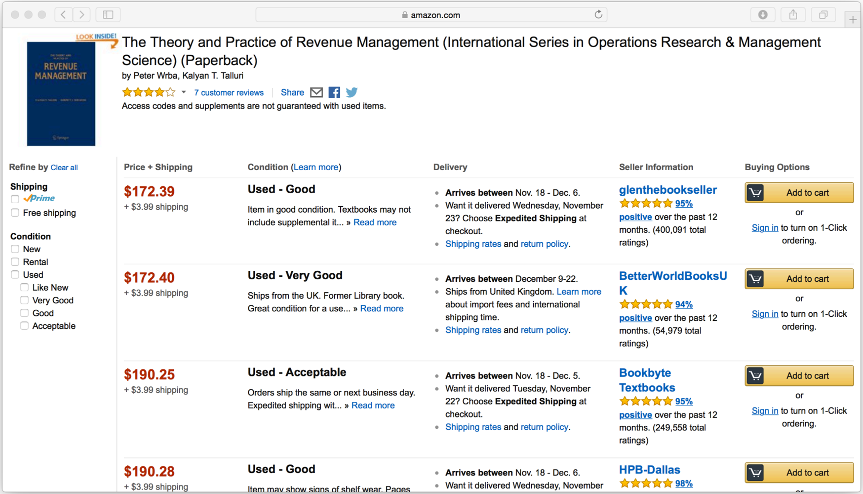
Motivation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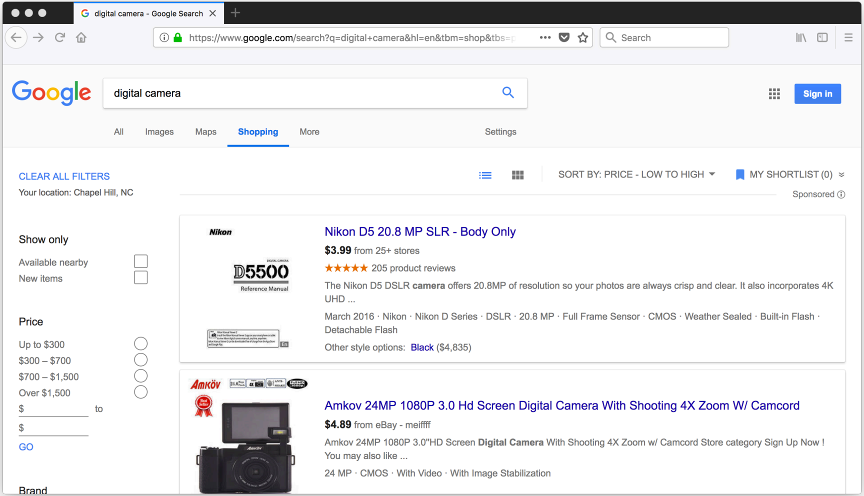
- Rankings in online marketplaces (e.g. Amazon's BuyBox)
- Product search with price thresholds

Motivation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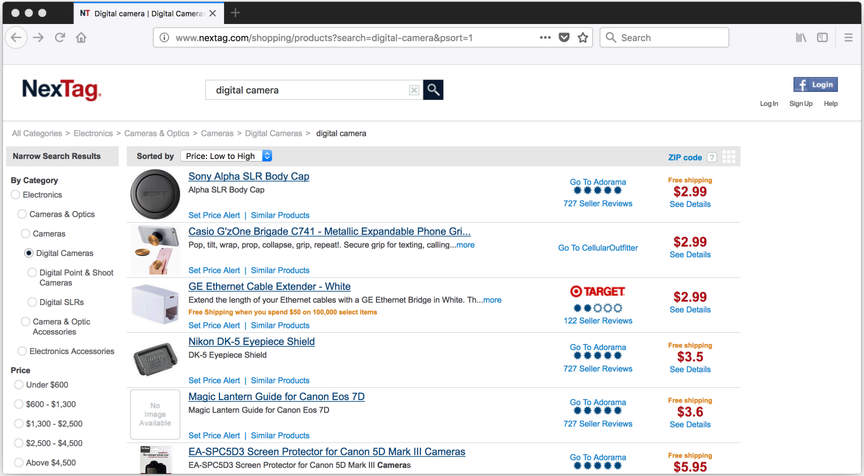
Motivation



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- Many online applications challenge Cournot's continuity assumption

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- Many online applications challenge Cournot's continuity assumption
- Not treated in dynamic pricing or MAB literature

Central questions

1. Is there a substantial cost of neglecting demand discontinuities in dynamic pricing and learning?

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2. If yes, how to implement estimation and pricing in the presence of demand discontinuities?

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- **Model uncertainty:**

  unknown demand parameters $\theta_n = (\alpha_n, \beta_n) \quad (n = 0, 1, \ldots, N)$

  unknown discontinuity points $\kappa_n \quad (n = 1, \ldots, N)$

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- **Pricing policy:** $\pi = (p_1, p_2, \ldots)$ non-anticipating

Performance

- Revenue loss in T periods relative to a clairvoyant

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  - Regret or "revenue loss due to demand model uncertainty"

$$\Delta_\pi(T, \boldsymbol{\kappa}, \boldsymbol{\theta}) \;=\; \sum_{t=1}^{T} \mathbb{E}_\pi \left\{ \sup_{p \in [\underline{p}, \overline{p}]} \{R(p, \boldsymbol{\kappa}, \boldsymbol{\theta})\} - R(p_t, \boldsymbol{\kappa}, \boldsymbol{\theta}) \right\}$$

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- **Objective:** choose π to minimize

$$\mathcal{R}_\pi(T) \;=\; \sup \left\{ \Delta_\pi(T, \boldsymbol{\kappa}, \boldsymbol{\theta}) \,:\, \boldsymbol{\kappa} \in \mathcal{K},\, \boldsymbol{\theta} \in \Theta \right\}$$

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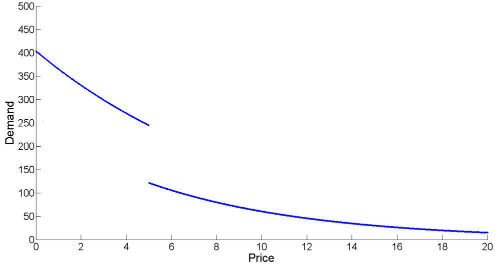
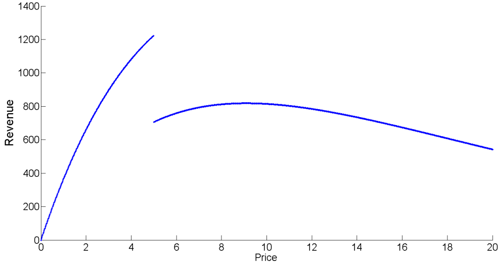
2. If yes, how to implement estimation and pricing in the presence of demand discontinuities?

Cost of ignoring demand discontinuities





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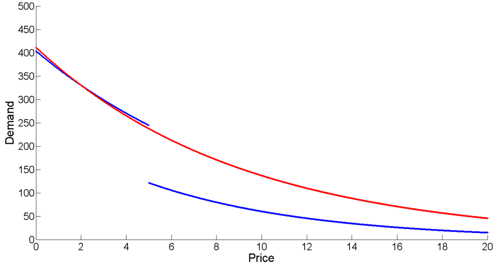

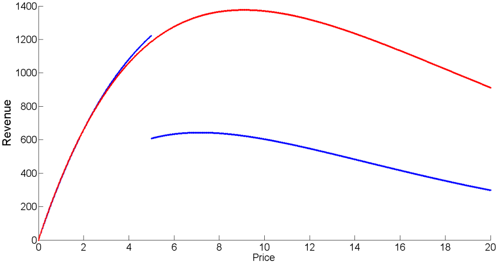

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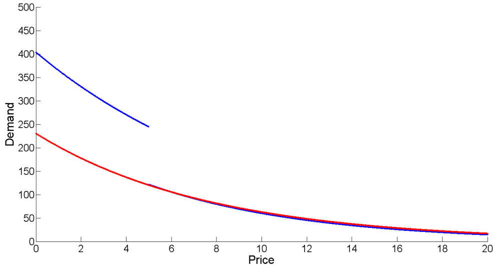

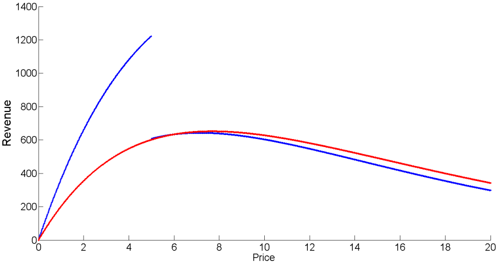

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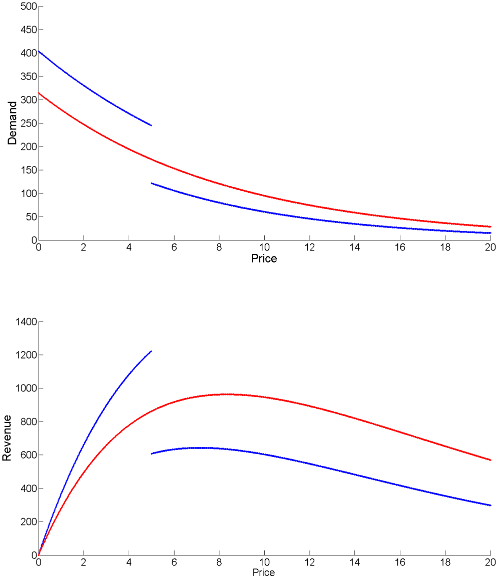
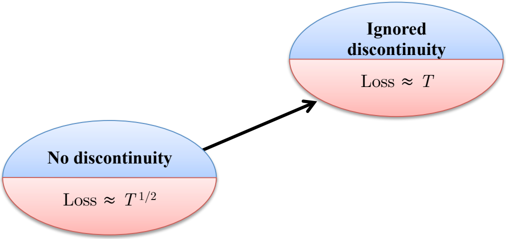


Cost of ignoring demand discontinuities

No discontinuity

Loss $\approx T^{1/2}$

Cost of ignoring demand discontinuities



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Estimating a discontinuous demand function

- **Two-step maximum likelihood estimation:**
  - Log-likelihood function

$$\mathcal{L}_t : (\varkappa, \boldsymbol{\vartheta}) \;\mapsto\; \sum_{s=1}^{t} \sum_{n=0}^{N} \left(d_s \vartheta_n \cdot (1, p_s) - e^{\vartheta_n \cdot (1, p_s)}\right) \mathbb{I}\{\kappa_n < p_s \le \kappa_{n+1}\}$$

$$\hat{\boldsymbol{\theta}}_t(\varkappa) = \arg\max_{\boldsymbol{\vartheta}} \{\mathcal{L}_t(\varkappa, \boldsymbol{\vartheta})\}$$

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  - <u>Step 1 (discontinuity estimation)</u>

$$\hat{\boldsymbol{\kappa}}_t = \arg\max_{\boldsymbol{\varkappa}} \left\{ \mathcal{L}_t\big(\boldsymbol{\varkappa}, \hat{\boldsymbol{\theta}}_t(\boldsymbol{\varkappa})\big) \right\}$$

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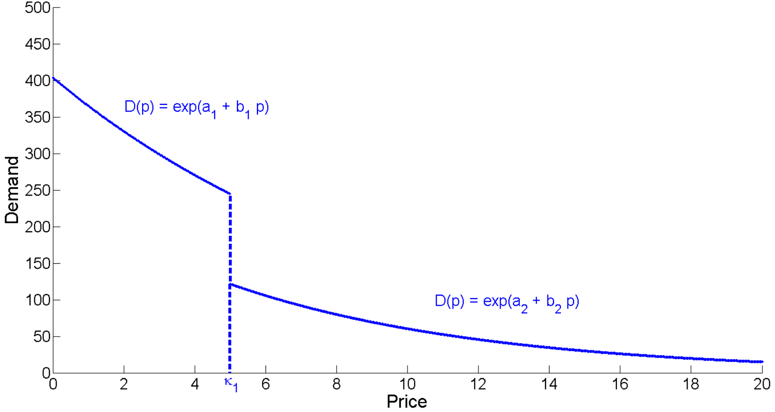
  - <u>Step 2 (demand parameter estimation)</u>

$$\hat{\boldsymbol{\theta}}_t = \hat{\boldsymbol{\theta}}_t(\hat{\boldsymbol{\kappa}}_t)$$

Estimating a discontinuous demand function



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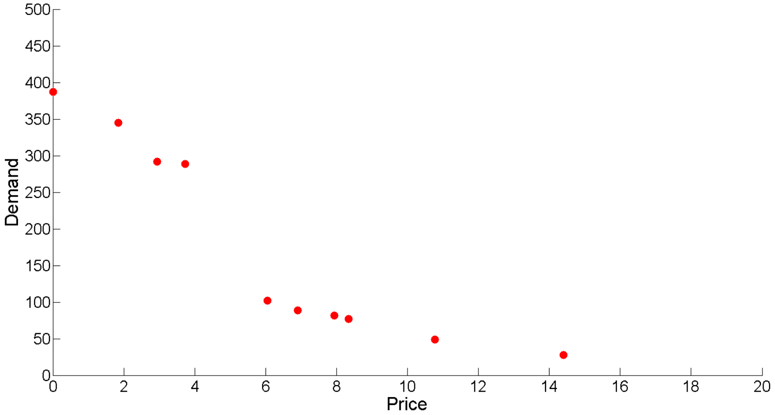

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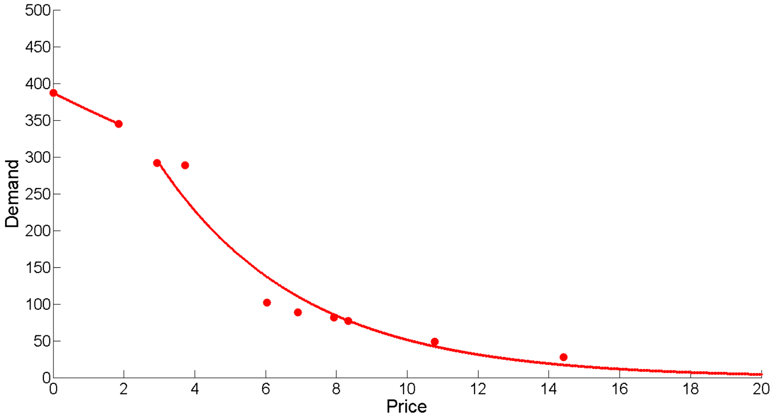
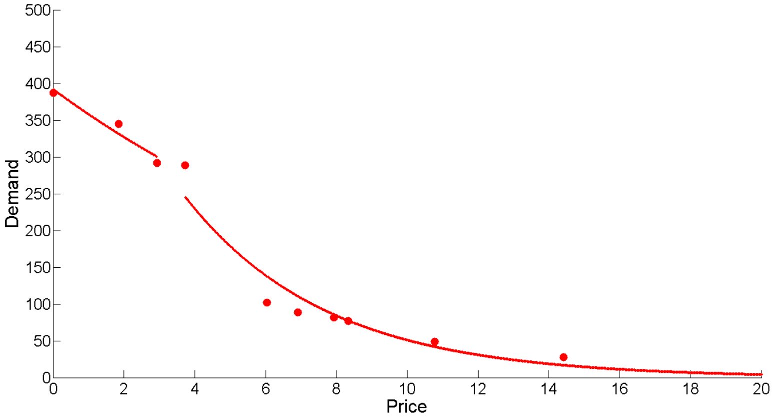


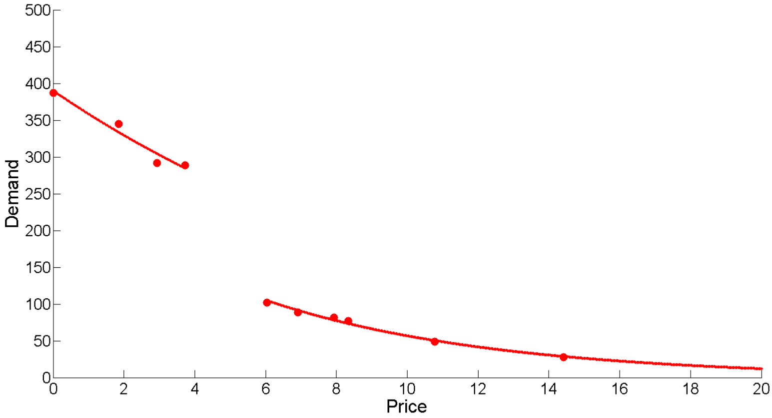
$$p_{(2)} \leq \hat{\kappa}_1 < p_{(3)}$$

Estimating a discontinuous demand function



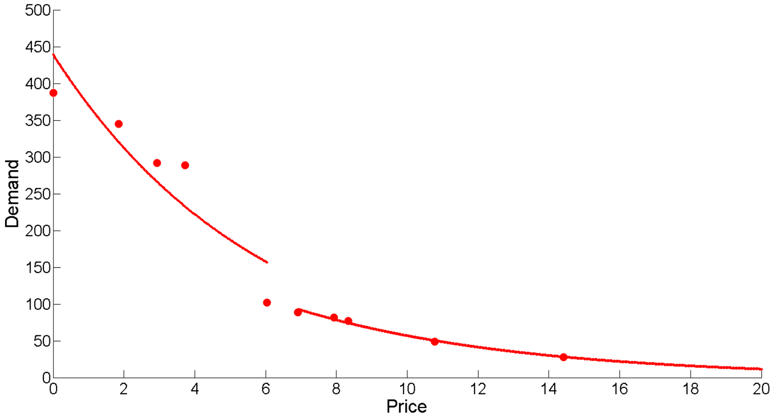
$$p_{(3)} \leq \hat{\kappa}_1 < p_{(4)}$$

Estimating a discontinuous demand function



$$p_{(4)} \leq \hat{\kappa}_1 < p_{(5)}$$

Estimating a discontinuous demand function



$$p_{(5)} \leq \hat{\kappa}_1 < p_{(6)}$$

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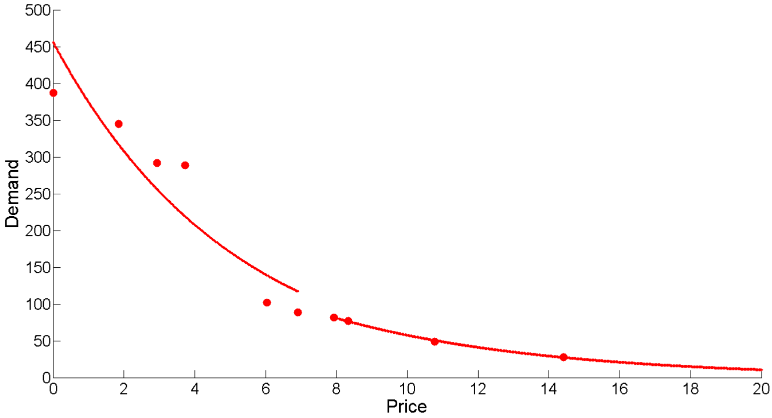
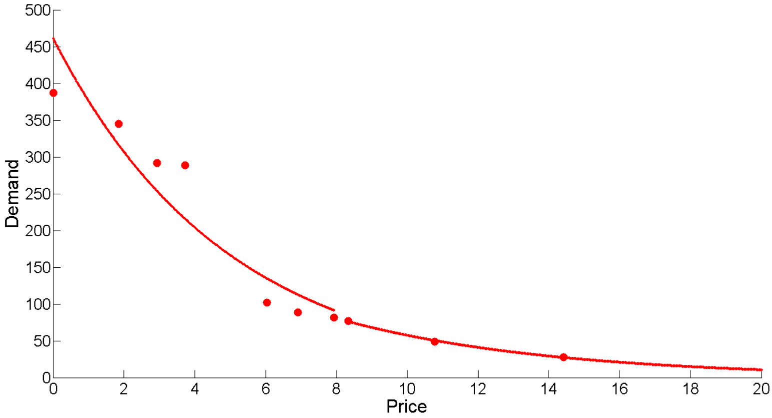


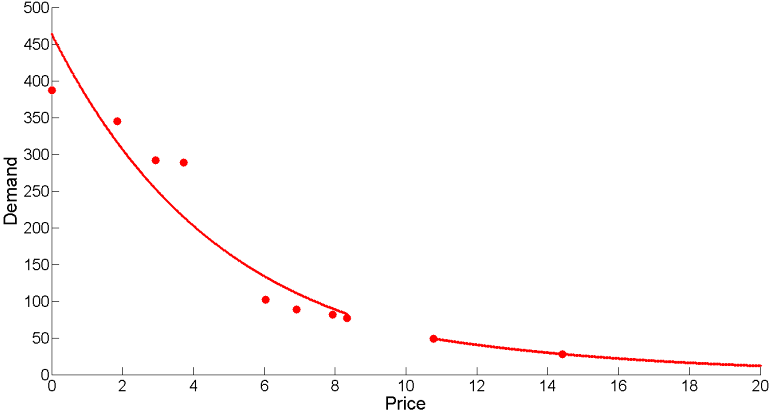
$$p_{(6)} \leq \hat{\kappa}_1 < p_{(7)}$$

Estimating a discontinuous demand function



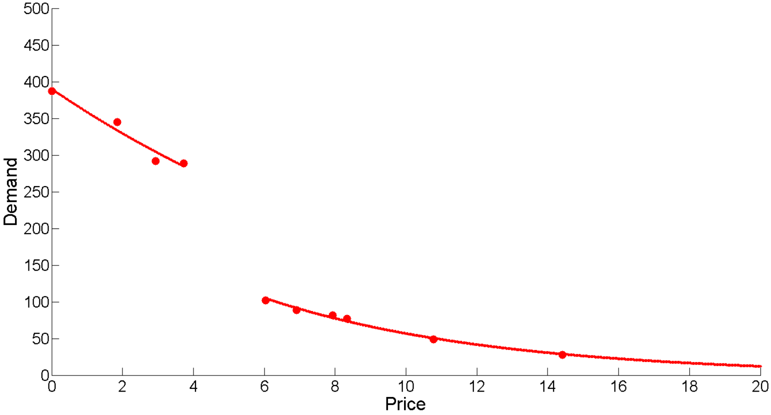
$$p_{(7)} \leq \hat{\kappa}_1 < p_{(8)}$$

Estimating a discontinuous demand function



$$p_{(8)} \leq \hat{\kappa}_1 < p_{(9)}$$

Estimating a discontinuous demand function



Highest likelihood if $p_{(4)} \leq \hat{\kappa}_1 < p_{(5)}$.

Designing a near-optimal policy

Discontinuity estimation and pricing policy π

Time horizon $\{1, \ldots, T\}$.

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(2) [Estimate] Compute $\hat{\boldsymbol{\kappa}}$ and $\hat{\boldsymbol{\theta}}$.

(3) [Exploit] Based on $\hat{\boldsymbol{\kappa}}$ and $\hat{\boldsymbol{\theta}}$, use the estimated optimal price in the remaining $T - M$ periods, but a factor $\log(M)/M$ away from the estimated discontinuities.

Analysis of estimation errors

Theorem (discontinuity estimation error)

There exist constants $M_1, z_1, \gamma_1 > 0$ *such that, if* $M \geq M_1$*, then*

$$\mathbb{P}_\pi \left\{ |\hat{\kappa}_n - \kappa_n| > \frac{z_1 \log M}{M} \text{ for some } n = 1, \ldots, N \right\} \; \leq \; \frac{\gamma_1}{M}.$$

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Theorem (parameter estimation error)

There exist constants $M_2, z_2, \gamma_2 > 0$ *such that, if* $M \geq M_2$*, then*

$$\mathbb{P}_\pi \left\{ \|\hat{\theta}_n - \theta_n\|^2 > \frac{z_2 \log M}{M} \text{ for some } n = 0, 1, \ldots, N \right\} \leq \frac{\gamma_2}{M}.$$

Sufficient condition for good performance

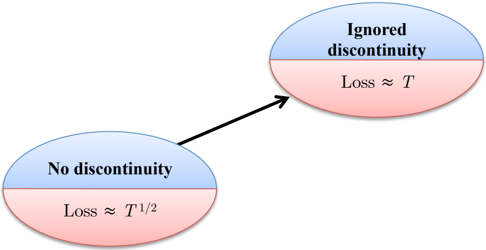
Theorem (upper bound on regret)

There exists a constant $C > 0$ *such that, if* $M = \lceil \sqrt{T} \, \rceil$, *then*

$$\mathcal{R}_\pi(T) \;\leq\; C\sqrt{T}\log T$$

for all $T \geq 4(N+1)^2$.

Summary of results



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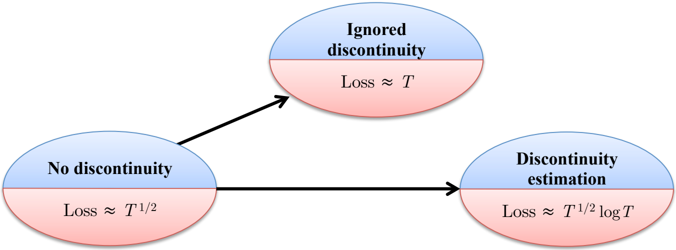
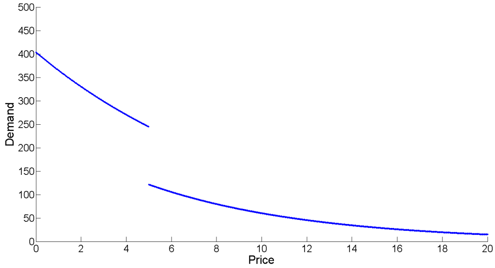
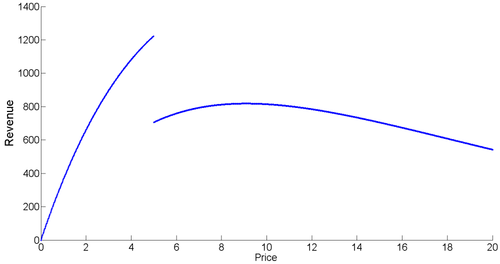


Some intuition





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THE END