Information Asymmetries, Common Factors, and International Portfolio Choice

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JEL Classification: D82, G11, G12, G15

Keywords: Information Asymmetry, REE Models, Home Bias

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Abstract

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1 Introduction

Economists have long been interested in how markets aggregate disperse pieces of information. This paper contributes to our understanding of the role of information in determining asset prices by proposing a rational expectations equilibrium model with multiple assets, common factors, and investors.[1] The model considers two different components of each asset's payoffs (dividends). The first component is asset-specific while the second component is a function of common (economy-wide) factors. We envisage a generalized information structure that allows an agent to have information about all, some, or none of the asset-specific components. The same agent may also have information about all, some, or none of the common factors.

The model produces closed-form solutions for both asset prices and the holdings of individual agents. The solutions, in turn, allow for a straightforward analysis of equilibrium prices as well as the ability to compare and contrast results with exiting models. For example, when all agents have full information about assets' future payoffs, prices in our model converge to those in the Sharpe-Lintner-Mossin Capital Asset Pricing Model (or "CAPM"). We show that an asset's price today equals it's expected future payoff minus a discount (risk premium) that is proportional to the covariance of the asset's payoff with other assets' payoffs.

When all agents do not have full information about future payoffs, an asset's price equals its expected future payoff minus the CAPM (full information) discount and minus an additional discount called the "information risk premium".[2] In our model, an asset's information risk premium can be related to both the asset-specific component of its payoffs and the common factors. The information risk premium is a function of how many agents have information about the asset's future payoffs, how important common factors are to the payoffs, which agents have information about the common factors, and whether these agents have information about *other* assets' payoffs.

By introducing non-public information about underlying factors, we offer the ability to link a rich variety of information structures with asset prices in ways not previously possible. In existing models, such as Admati (1985), agents who receive information about assets' total

[1]See Brunnermeier (2001) for a review of existing models. Early examples of rational expectations equilibrium models are Grossman (1976), Grossman and Stiglitz (1980) and Hellwig (1980). Multi-asset models include Admati (1985) and Kodres and Pritsker (2002), while dynamic models include Wang (1993) and Bacchetta and van Wincoop (2006). Brennan and Cao (1997) provide a dynamic, multi-asset rational expectations model.

[2]Such cases arise when some agents have less than full information (asymmetric information). Alternatively, all agents can have an equal, but less than full, measure of information. Both cases give rise to information risk premia.

payoffs are unable to separate factor information from asset-specific information even if an underlying factor structure exists.[3] To understand how factor information is different from information about correlated asset payoffs, consider agents who receive positive information about a single asset's (total) payoff. The agents are unable to tell if the payoff comes from good news about underlying factors or good news about the asset. If the agents receive information about two assets, they are still in the same predicament. They have two pieces of information (which may be correlated) but at least three unknowns quantities: two asset-specific quantities and quantities related to underlying factors.

We present two numerical analyses of the model in order to highlight the role of factor information. First, we study international portfolio choice and the well-known "home bias puzzle".[4] Investors and assets in the model are partitioned into groups (nationalities and national stock markets). We mimic existing papers and assume agents receive superior information about the asset-specific component of their home country's assets.[5] We next assume that a few investors have superior information about the common factors. One can think of these investors as being located in a major financial center such as New York City. Analysts working for large investment banking houses or mutual funds synthesize and produce information about factors such as short-term interest rates, commodity prices, and global shipping costs. Information about these factors plays an important role when estimating the future payoffs of many different assets. Fund managers with access to this research use the information when choosing their portfolios.

A simple two-country, two-asset, single-factor numerical analysis confirms that high levels of information about the asset-specific components of payoffs lead residents of one country to overweight assets from their own country. We can calculate the fraction (weight) of each investor's portfolio that is invested in domestic and foreign assets. We can also calculate the weight of a given country's assets in the world market portfolio. In this way, we are able calculate whether a particular investor is over- or underweighs a given asset (relative to the asset's weight in the world market portfolio). Our numerical results regarding portfolio choice parallel existing studies of home bias as investors place up to 30% more of their wealth

[3] Admati (1985) notes the difficulties in extending her model to one with a factor structure.

[4] Over the past two decades, home bias has spawned a large literature. A search for the term "Home Bias" in the title or abstract yields 222 papers from EconLit and 177 papers from SSRN. Classic articles on home bias include French and Poterba (1991), Cooper and Kaplanis (1994), and Teser and Werner (1995).

[5] Gehrig (1993) presents a related two-country model of home bias. Brennan and Cao (1997) study investment flows (changes in holdings) and information asymmetries. In their model, investors with less information (foreigners) update priors about future payoffs more heavily than investors with more information (locals). Van Nieuwerburgh and Veldkamp (2006) use a rational expectations model to justify the persistence of the home bias when investors initially have a small information advantage.

in the domestic asset than world market capitalizations indicate.[6]

More importantly, we consider cases where investors are asymmetrically informed about common factors. A contribution of this paper is to show that low levels of asset-specific information and high levels of information about common (cross-border) factors can lead investors in one country to overweigh assets from other countries. The phenomenon is called "reverse home bias". Our numerical example provides intuition behind the existence of reverse home bias. Consider a high-tech computer company located in France. French investors may speak the same language as the CEO, know people who work at the company, and have immediate access to information released by the company. However, the company's future dividends are likely to be sensitive to the world-wide demand for high-tech equipment. Sophisticated investment funds (say in the United States) with skills in analyzing world hardware prices may have superior information about the French company's prospects—even if the funds are located far from Europe. In such cases, U.S. investment funds may require a smaller information risk premium for holding the stock than French investors require for holding the same stock. A smaller information risk premium implies a higher willingness to pay, which translates into increased ownership by U.S. investment funds.

In the second numerical example, we show the sensitivity of an asset's price to information about the asset-specific component of its payoffs can be negative. In other words, good news about future payoffs can result in an asset's price *falling*. Such a phenomenon is first shown in Admati (1985) where results rely on a high, positive correlation between payoffs and non-informational demand (liquidity) shocks. By contrast, we obtain the result even after allowing liquidity shocks to be uncorrelated in our two-country, two-asset, single-factor analysis.

The two numerical examples help to shed light on general properties of equilibrium prices in our model. For example, an asset is less risky from the perspective of a single agent if the agent has precise information about its future payoffs. The asset is more risky if the agent must glean information from equilibrium prices. When common factors are considered, a single asset may no longer be viewed as low-risk even if the agent has information about the asset-specific component of payoffs. As the component related to common factors becomes more prominent in payoffs, asset-specific information becomes less valuable. Thus, the price

[6]French and Poterba (1991) document that American investors allocate about 84% of their wealth in domestic stocks although the weight of the American stocks in the world market portfolio is only about 50% (an overweighing of 34%.) Using 1997 data, Ahearne, Griever, and Warnock (2004) show that 89.9% of US portfolio holdings are allocated to US stocks even though these stocks comprise 48.3% of the world market portfolio (overweight by 41.6%.) Chan, Covrig and Ng (2005) show that the degree of home bias in other countries is generally greater then 30%.

of a given asset is sensitive to how many agents have information about the asset-specific component of payoffs, how many agents have information about factors that affect the asset's payoffs, how sensitive the asset's payoffs are to the common factors, and what other information the agents have.

Our paper extends existing theoretical work on information structures and risk premia. For example, Easley and O'Hara (2004) present a multi-asset model that focuses on the role of public and private signals in determining a firm's cost of capital. Private signals in their model are received only by a group of informed investors as in Grossman and Stiglitz (1980). In our model, it is possible for different groups of investors to have information about different groups of the securities. In this way, investors can be asymmetrically informed without introducing a strict information hierarchy.[7] Bacchetta and van Wincoop (2006) argue in favor of structures with a "[broad] dispersion of information."

We end the paper with an empirical, cross-sectional analysis that complements the numerical analysis of international portfolio choice. We use Thomson Financial International Mutual Fund Holdings data. The dataset consists of US$720 billion of cross-border holdings (positions) in 5,781 stocks from 21 developed countries. We create two proxy variables: one for the degree of asset-specific information about a stock and the other for the degree of factor information. Cross-border holdings increase as asset-specific information decreases and the holdings increase as factor information increases. Our results continue to hold after controlling for variables that have, in the past, been used as proxies for familiarity (the size of the company and the number of equity analysts following the company). Our results also hold after controlling for a firm's leverage—another variable that has been found to explain cross-border holdings.[8] A number of robustness checks give consistent results. The empirical tests are motivated by implications of our model and highlight how information about common factors plays a significant role in international portfolio choice.

[7] Our approach incorporates an aspect of models which endow all agents with small pieces of information about risky assets payoffs—see Grossman (1976), Hellwig (1980), and Admati (1985). Easley and O'Hara (2004) contains an excellent review of information structures and existing papers. Two recent working papers also address information and home bias. Van Nieuwerburgh and Veldkamp (2006a) study information acquisition and dynamic learning. The authors show that small home-country information advantages can persist as investors may choose to specialize in obtaining information about home-country assets. Like our paper, Albuquerque, Bauer, and Schneider (2006) model local and global information (the papers were developed independently.) Investors in their model receive signals about future payoffs. There is a single global signal which conveys information about the sum of all payoffs.

[8] Our results complement a number of existing papers including: Kang and Stulz (1997) who look at foreign holdings of Japanese stocks, Dahlquist, Pinkowitz, Stulz, and Williamson (2003) who study how corporate governance affects home bias, Ahearne, Griever, and Warnock (2004) who study information quality and cross-border investment, and Chan, Covrig, and Ng (2005) who study international mutual fund allocations. The latter paper concludes that stock market development and variables linked to familiarity explain the majority of home bias.

The paper proceeds as follows. Section 2 presents our model, notation, and assumptions. We provide closed-form solutions for equilibrium prices, holdings, and information risk premia. Section 3 numerically analyses prices and holdings as functions of parameters in the model. Section 4 empirically studies cross-border mutual fund holdings. The final section concludes.

2 Model

The model has I investors indexed $i = 1, \ldots, I$ who trade at date 0 and consume at date 1. Each agent i can invest his initial wealth, w_i^0, in a riskless asset and J risky assets indexed $j = 1, \ldots, J$. The riskless interest rate is denoted r_f and we define $R \equiv (1 + r_f)$. For simplicity, we normalize the price of the riskless asset to one. Each risky asset j pays a liquidating dividend $\tilde{P}_j^1$ at date 1. The vector of final payoffs $\tilde{P}^1 = (\tilde{P}_1^1, \ldots, \tilde{P}_J^1)'$ is generated by a K-factor linear process:

$$\tilde{P}^1 = \tilde{\theta} + \mathbf{B}\tilde{f} + \tilde{\varepsilon} \tag{1}$$

The vector $\tilde{\theta} = (\tilde{\theta}_1, \ldots, \tilde{\theta}_J)'$ is the asset-specific component of payoffs, the vector $\tilde{f} = (\tilde{f}_1, \ldots, \tilde{f}_K)'$ contains the K common factors, and $\mathbf{B}$ is a $J \times K$ matrix of factor loadings. The remaining part of each asset's final payoff, $\tilde{\varepsilon} = (\tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_J)'$, is unknown to all investors and referred to as residual uncertainty. We assume that $\tilde{\theta}$, $\mathbf{B}\tilde{f}$, and $\tilde{\varepsilon}$ are jointly multivariate normal and independent. We further assume that $\tilde{f}$ and $\tilde{\varepsilon}$ have mean zero. Since $\tilde{\theta}$ is the asset-specific component, we assume its covariance matrix is diagonal and denoted $\mathbf{T}$.[9] For tractability, we assume that the covariance matrix of $\tilde{f}$ is the identity matrix. The covariance matrix of $\mathbf{B}\tilde{f}$ is $\mathbf{L}$, with $\mathbf{L} = \mathbf{B}\mathbf{B}'$. Finally, the covariance matrix of $\tilde{\varepsilon}$ is denoted $\mathbf{\Sigma}$.

As is common in rational expectations equilibrium models, we introduce some noise in the form of random supply shocks. This addition is necessary to make equilibrium prices partially (not perfectly) revealing and can be justified by agents trading assets for non speculative reasons such as liquidity purposes. The per-capita supply of risky assets is defined as the realization of a random vector $\tilde{z}$. The vector $\tilde{z}$ is independent and jointly normally distributed along with the other variables in the model and has a covariance matrix denoted $\mathbf{Z}$. In order to insure the existence and uniqueness of the date 0 equilibrium price vector, $\tilde{P}^0$, we assume that $\mathbf{\Sigma}$, $\mathbf{T}$, and $\mathbf{Z}$ are regular matrices.

[9] This assumption is not necessary to solve the model. However, it enables us to distinguish factors that affect a single asset from common factors that affect two or more assets.

We assume all agents have an exponential utility function: $U(\tilde{w}_i^1) = -e^{-a\tilde{w}_i^1}$, where $\tilde{w}_i^1$ is the wealth of investor i on date 1. The utility function has a constant absolute risk aversion[10] with coefficient $a > 0$ which is the same for all agents. Let X_i be investor i's vector of holdings of the risky assets. Investor i's final wealth is:

$$\tilde{w}_i^1 = w_i^0 R + X_i'(\tilde{P}^1 - R\tilde{P}^0) \tag{2}$$

2.1 Information Structure and Notation

To facilitate linking our model to international holdings data, we partition investors, assets, and common factors into groups. A group of investors can be thought of a nationality (French investors, Japanese investors, etc.) The I investors in our model are partitioned into N non-overlapping groups. Each group of investors represents a fraction, λ_n, of the total number of investors (I) in the market such that $\sum_{n=1}^{N} \lambda_n = 1$.

Asset-Specific Information The J securities are partitioned into N non-overlapping groups. A group of securities can be thought of as comprising a country's equities (French stocks, Japanese stocks, etc.) We define the set of all assets as S. The set of assets in group n contains J_n risky assets and is denoted S_n. Thus, $\bigcup_{n=1}^{N} S_n = S$ and $\forall(n_1, n_2)$, $n_1 \neq n_2$, $S_{n_1} \cap S_{n_2} = \oslash$. We assume there are an equal number (N) of securities groups and investors groups to ensure that each security has at least one investor with specific information about that security. A single investor i in group n knows the realization of the asset-specific component, θ_j, of each asset j in the set S_n. For any asset j not in S_n, investor i only knows the distribution of $\tilde{\theta}_j$ but he does not know its realization.

Information About the Common Factors We assign the K common factors into N groups denoted F_n, with $n = 1, \ldots, N$. The set F_n contains K_n common factors. A single investor i in group n knows the realization of each common factor $\tilde{f}$ in the set F_n. For any factor not in F_n, the investor only knows the distribution of $\tilde{f}$ but not its realization. For tractability purposes of the model, we assume that two groups of investors do not have information about the same common factor.

Chen et al. (1986) document nine macroeconomic risk factors affecting stock returns. We therefore envisage the number of common factors to be much less than the number of assets, $K \ll J$. If the number of common factors is less than the number of investor groups, then

[10] This assumption is common in rational expectations models and ensures that an investor's demand for the risky asset is independent of his initial wealth.

$K < N$, some F_n sets will not contain any common factors ($K_n = 0$), and the corresponding investor group will not be informed about any of the common factors.

Notation The information structure of our model implies that investors belonging to the same group n possess the same private information (for asset-specific components and for factors), they face the same optimization problem, and they optimally choose identical portfolios. In this sense they can be said to be identical. We use the following terms interchangeably (and a bit loosely): "investor i from group n", "investor group n", and "investor n". In order to simplify the notation, we write the payoffs of the risky assets as:

$$\tilde{P}^1 = \mathbf{C}\tilde{\eta} + \tilde{\varepsilon} \tag{3}$$

Where, $\tilde{\eta} = \begin{pmatrix} \tilde{\theta}' & \tilde{f}' \end{pmatrix}'$ is a $J + K$ column vector and $\mathbf{C}$ is a $J \times (J + K)$ block-diagonal matrix consisting of a $J \times J$ identity matrix, $\mathbf{I}_J$, and the matrix $\mathbf{B}$. The variance-covariance matrix of $\tilde{\eta}$ is $\mathbf{Q} = \begin{pmatrix} \mathbf{T} & 0 \\ 0 & \mathbf{I}_K \end{pmatrix}$ where $\mathbf{I}_K$ is the identity matrix of order K.

Definition 2.1. For each investor n, we define the diagonal matrix $\mathbf{D}_n$ of order $J + K$ with $\mathbf{D}_n(j, j) = 1$ if investor n knows the realization of the j^{th} random variable in $\tilde{\eta}$ and $\mathbf{D}_n(j, j) = 0$ otherwise. The j^{th} random variable represents an asset-specific component of stock j's payoffs if $j \leq J$, and a common factor otherwise.

Definition 2.2. We define $\mathbf{D} \equiv \sum_{n=1}^{N} \lambda_n \mathbf{D}_n$. The matrix $\mathbf{D}$ plays an important role in our model as each element on the main diagonal represents the proportion of investors who know the realization of the corresponding random variable in the vector $\tilde{\eta}$.

Definition 2.3. For each investor group n, the matrix $\mathbf{M}_n$ is obtained by eliminating all the null rows of $\mathbf{D}_n$. Consequently, the number of rows of $\mathbf{M}_n$ is equal to $J_n + K_n$, which represents the number of asset-specific and common factors about which investor n is informed. If investor n does not receive any private information, $\mathbf{D}_n$ becomes the null matrix and $\mathbf{M}_n$ cannot be defined. It is straightforward that $\mathbf{M}'_n \mathbf{M}_n = \mathbf{D}_n$ and $\mathbf{M}_n \mathbf{M}'_n = \mathbf{I}_{J_n + K_n}$, where $\mathbf{I}_{J_n + K_n}$ is the identity matrix of order $J_n + K_n$.

Under these definitions, the private information received by investor n consists of the realization of the random vector $\mathbf{M}_n \tilde{\eta}$. As in Admati (1985), equilibrium prices also reveal some information to investors beyond their own private information. Consequently, each investor n maximizes his expected utility of consumption conditional on the realization of his private information and on the observation of the public information in the form of prices at date 0.

2.2 Equilibrium Prices

We seek a closed-form solution for prices at date 0 within the class functions that are linear in our information variable $\tilde{\eta}$ and supply variable $\tilde{z}$. The form of the solution implies investors assume prices are a linear function of private signals and noise. In equilibrium, this hypothesis is verified. The date 0 price vector is:

$$\tilde{P}^0 = \mathbf{A}_0 + \mathbf{A}_1\tilde{\eta} - \mathbf{A}_2\tilde{z} \tag{4}$$

where the dimensions of the matrix $\mathbf{A}_0$ is $J \times 1$, the matrix $\mathbf{A}_1$ is $J \times (J + K)$, and the matrix $\mathbf{A}_2$ is $J \times J$. We suppose that $\mathbf{A}_2$ is regular. Under these assumptions, investor n's demand is:

$$X_n = a^{-1}\mathbf{V}_n^{-1}\left(E_n\left[\tilde{P}^1\right] - R\tilde{P}^0\right) \tag{5}$$

Equation (5) gives an expression for agent n's holdings at date 0—please see Appendix A for additional details. The expression $E_n[\tilde{P}^1] = E[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0]$ gives the expected prices of the risky assets at date 1 from investor n's point of view (i.e. conditional on his information set). $\mathbf{V}_n = Var[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0]$ represents the conditional return variance of $\tilde{P}^1$ from investors n's point of view. By equating the supply and the aggregate demand of the N groups of investors, $\left(\sum_{n=1}^N \lambda_n X_n = z\right)$, it follows:

$$\sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1}\left(E_n\left[\tilde{P}^1\right] - R\tilde{P}^0\right) - az = 0 \tag{6}$$

Joint normality implies that the distribution of prices, conditional on investor n's private and public information, is also multi-variate normal with the following expectation:

$$\begin{aligned} E_n\left[\tilde{P}^1\right] &= E\left[\tilde{P}^1|\mathbf{M}_n\tilde{\eta},\tilde{P}^0\right] \\ &= \mathbf{B}_{0n}+\mathbf{B}_{1n}\mathbf{M}_n\tilde{\eta}+\mathbf{B}_{2n}\tilde{P}^0 \end{aligned} \tag{7}$$

where the dimension of the matrix $\mathbf{B}_{0n}$ is $J \times 1$, $\mathbf{B}_{1n}$ is $J \times (J_n + K_n)$, and $\mathbf{B}_{2n}$ is $J \times J$ respectively. Equations (4), (6), and (7) imply the system to be solved has the following form (please see Appendix B):

$$\begin{aligned} a\mathbf{A}_2^{-1}\mathbf{A}_0 &= \sum_{n=1}^{N}\lambda_n\mathbf{V}_n^{-1}\mathbf{B}_{0n} \\ a\mathbf{A}_2^{-1}\mathbf{A}_1 &= \sum_{n=1}^{N}\lambda_n\mathbf{V}_n^{-1}\mathbf{B}_{1n}\mathbf{M}_n \\ a\mathbf{A}_2^{-1} &= \sum_{n=1}^{N}\lambda_n\mathbf{V}_n^{-1}\left(R\mathbf{I}_J-\mathbf{B}_{2n}\right) \end{aligned} \tag{8}$$

As shown in Appendix C, the matrices $\mathbf{B}_{1n}$, $\mathbf{B}_{2n}$ and $\mathbf{V}_n$ can be written as functions of the matrices $\mathbf{A}_1$ and $\mathbf{A}_2$. The system is a fixed point problem in a $2J^2 + JK + J$ Euclidian space. To obtain a solution for $\tilde{P}^0$, we define the matrix $\mathbf{U} \equiv \mathbf{A}_2^{-1}\mathbf{A}_1$. We also introduce the function $g(\mathbf{G}) = \sum_{n=1}^{N}\mathbf{D}_n\mathbf{G}\mathbf{D}_n$, where $\mathbf{G}$ is a matrix of order $J + K$. The function $g(\cdot)$ transforms a matrix $\mathbf{G}$ into a N-block diagonal matrix whose block elements are the same as the elements of the matrix $\mathbf{G}$.

Definition 2.4. We define a "g-matrix" to be any square matrix $\mathbf{G}$ of order $J+K$ which satisfies $g(\mathbf{G}) = \mathbf{G}$. This means that $\mathbf{G}$ is an N-block diagonal matrix, the size of block n is equal to the number of specific and common factors known by investor n.

Define $\mathbf{\Psi} \equiv Var\left[\tilde{\eta}|\tilde{P}^0\right]$ i.e., the variance-covariance matrix of $\tilde{\eta}$ conditional on observing the equilibrium price vector at date 0. The matrix $\mathbf{\Psi}$ is endogenously defined and represents the variance of $\tilde{\eta}$ from the point of view of an investor who does not posses any private information but only observes the equilibrium price vector. The following lemma gives an analytical solution for $\mathbf{U}$.

Lemma 2.1. If $(\mathbf{\Psi}^{-1} + \mathbf{C}'\mathbf{\Sigma}^{-1}\mathbf{C})$ is a g-matrix, then the closed-form solution for $\mathbf{U}$ is:

$$\mathbf{U} = a^{-1}\mathbf{\Sigma}^{-1}\mathbf{C}\mathbf{D} \tag{9}$$

Proof: See Appendix D.

For the particular case of Lemma (2.1), $\mathbf{U}$ is not a function of the coefficients $\mathbf{B}_{0n}$, $\mathbf{B}_{1n}$, and $\mathbf{B}_{2n}$. Therefore, to determine $\mathbf{A}_0$, $\mathbf{A}_1$, and $\mathbf{A}_2$, we must first compute the matrix $\mathbf{\Psi}$ as a function of $\mathbf{U}$. In this way, the variance-covariance matrices of any investor group, $\mathbf{V}_n$, can be written as a function of $\mathbf{\Psi}$:

$$\mathbf{V}_n \;=\; \mathbf{\Sigma} + \mathbf{C}\mathbf{\Psi}\mathbf{C}' - \mathbf{C}\mathbf{\Psi}\mathbf{M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{\Psi}\mathbf{C}' \tag{10}$$

Where $\mathbf{\Psi}_n = \mathbf{M}_n\mathbf{\Psi}\mathbf{M}_n'$. Also, $\mathbf{\Psi} = \mathbf{Q} - \mathbf{Q}\mathbf{U}'\mathbf{M}^{-1}\mathbf{U}\mathbf{Q}$ and $\mathbf{M} = \mathbf{U}\mathbf{Q}\mathbf{U}' + \mathbf{Z}$. The following theorem gives a closed-form solution for the equilibrium price vector at date 0.

Theorem 2.1. Under the conditions of Lemma (2.1), there exists a closed form solution for Equation (6) within the class of linear functions of $\tilde{\eta}$ and $\tilde{z}$. The solution can be written as, $\tilde{P}^0 = \mathbf{A}_0 + \mathbf{A}_1\tilde{\eta} - \mathbf{A}_2\tilde{z}$, where $\mathbf{A}_2$ is a regular matrix and:

$$\mathbf{A}_0 \;=\; \frac{1}{R}\Big(\big(\mathbf{C} - R\mathbf{A}_1\big)E[\tilde{\eta}] + \big(R\mathbf{A}_2 - a\mathbf{V}_N\big)E[\tilde{z}]\Big) \tag{11}$$

$$\mathbf{A}_1 \;=\; \frac{1}{R}\big(\mathbf{C}\mathbf{Q}\mathbf{C}' + \mathbf{\Sigma} - \mathbf{V}_N\big)\big(\mathbf{C}\mathbf{D}\mathbf{Q}\mathbf{C}'\big)^{-1}\mathbf{C}\mathbf{D} \tag{12}$$

$$\mathbf{A}_2 \;=\; \frac{1}{R}a\big(\mathbf{C}\mathbf{Q}\mathbf{C}' + \mathbf{\Sigma} - \mathbf{V}_N\big)\big(\mathbf{C}\mathbf{D}\mathbf{Q}\mathbf{C}'\big)^{-1}\mathbf{\Sigma} \tag{13}$$

Proof: See Appendix E.

The matrix $\mathbf{V}_N = (\sum_{n=1}^{N}\lambda_n\mathbf{V}_n^{-1})^{-1}$ represents the variance-covariance matrix of $\tilde{P}^1$ for the "average" investor in the market. The precision of $\mathbf{V}_N$ equals the weighted mean of each group's precisions where the weights are proportional to the number of agents in each group. From Equation (10), it is straightforward to show that $\mathbf{V}_N$ can be written as:

$$\mathbf{V}_N = (\mathbf{\Sigma} + \mathbf{C}\mathbf{\Psi}\mathbf{C}')(\mathbf{I}_J + \mathbf{\Sigma}^{-1}\mathbf{C}\mathbf{D}\mathbf{\Psi}\mathbf{C}')^{-1} \tag{14}$$

Thus we have provided a closed-form solution for prices at date 0. The solution takes the form shown in (4) with constant values shown in (11), (12), and (13). Holdings of investors in group n are given in Equation (5).

2.3 Risk Premia

We analyze the relationship between information structures and asset prices. Our analysis produces a closed-form expression for the information risk premium (the difference between asset prices when all agents are fully informed and asset prices when at least some agents are not fully informed.) Rearranging Equation (6) gives a general expression for prices at date 0:[11]

$$E\left[\tilde{P}^0\right] = \frac{1}{R}\left(E\left[\tilde{P}^1\right] - a\mathbf{V}_N E\left[\tilde{z}\right]\right) \tag{15}$$

Equation (15) shows that asset prices at date 0 are less than the value of expected future payoffs.[12] The total risk premium (price discount) is given by the expression $a\mathbf{V}_N E\left[\tilde{z}\right]$. The risk premium depends on risk aversion (a) and the market's "average" uncertainty about future payoffs ($\mathbf{V}_N$).

Full Information: We consider the case where all investors in the market are informed about all asset-specific components and all factors. In this case, all investors belong to the same group ($\lambda = 1$), $\mathbf{D}$ is the identity matrix, and $\mathbf{V}_N = \mathbf{\Sigma}$. It can be shown that asset prices reduce to the Sharpe-Lintner-Mossin Capital Asset Pricing Model (CAPM) with:

$$E\left[\tilde{P}^0\right] = \frac{1}{R}\left(E\left[\tilde{P}^1\right] - a\mathbf{\Sigma} E\left[\tilde{z}\right]\right) \tag{16}$$

For a given stock j, we can express the CAPM in terms of both prices and returns—with

[11]This section analyzes the relationship between model parameters $\{r_f, a, \lambda_1, \ldots, \lambda_N, \mathbf{B}, \mathbf{T}, \mathbf{\Sigma}, \mathbf{Z}\}$ and ex-ante equilibrium prices. We do this by taking expectations over the random variables in the model $\{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\}$. An alternative methodology involves drawing a set of random variables $\{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\}$ and calculating asset prices at date 0. Repeating this process converges to the same expected values as the number of draws goes to infinity. By taking expectations, we solve for prices and holdings before agents receive private information. Such solutions are sometimes referred to as *ex-ante*. Appendix F provides the closed-form solution investor n's ex-ante holdings at date 0.

[12]Assuming assets are expected to be in positive net supply: $E\left[\tilde{z}\right] > 0$.

the second expression being more familiar to financial economists:

$$
\begin{aligned}
E\left[\tilde{P}_j^0\right] &= \frac{1}{R}\Big(E\left[\tilde{P}_j^1\right] - aCov\left[\tilde{P}_j^1, \tilde{P}_m^1\right]\Big) \\
E\left[\tilde{r}_j\right] &= r_f + \beta_{j,m}\Big(E\left[\tilde{r}_m\right] - r_f\Big)
\end{aligned}
$$

Where $\tilde{r}_j \equiv \frac{\tilde{P}_j^1 - E\left[\tilde{P}_j^0\right]}{E\left[\tilde{P}_j^0\right]}$ and $\beta_{j,m} \equiv \frac{Cov[\tilde{r}_j, \tilde{r}_m]}{Var[\tilde{r}_m]}$. The return on the market is $\tilde{r}_m \equiv \frac{\tilde{P}_m^1 - E\left[\tilde{P}_m^0\right]}{E\left[\tilde{P}_m^0\right]}$. As the supply is unknown by agents, we consider its expectation so that the payoff of the market (which contains all assets) is: $\tilde{P}_m^1 = \sum_{j=1}^{J} \tilde{P}_j^1 E\left[\tilde{z}_j\right]$. Please see Appendix G for details.

Information Risk Premium: We define the "information risk premium" (or "*IRP*") as the difference between the risk premia shown in Equations (15) and (16). The *IRP* represents the amount an asset's price at date 0 is below its expected future value solely due to agents not having full information about future payoffs.

$$
\begin{aligned}
IRP &\equiv a\mathbf{V}_N E\left[\tilde{z}\right] - a\mathbf{\Sigma} E\left[\tilde{z}\right] \\
&= a(\mathbf{V}_N - \mathbf{\Sigma})E\left[\tilde{z}\right] \\
&= a\Big(\mathbf{C}\left(\mathbf{I}_{J+K} - \mathbf{D}\right)\mathbf{\Psi}\mathbf{C}'\left(\mathbf{I}_J + \mathbf{\Sigma}^{-1}\mathbf{C}\mathbf{D}\mathbf{\Psi}\mathbf{C}'\right)^{-1}\Big)E\left[\tilde{z}\right]
\end{aligned} \tag{17}
$$

In a single-asset model with no factor structure, the information risk premium is proportional to the difference between the market's average uncertainty about future payoffs (V_N) and residual uncertainty about the same payoffs (Σ). This difference is a signal-to-noise measure. When the difference is small, investors have a lot of information about future payoffs, the *IRP* is low, and prices are high. Note that $IRP \geq 0$ as the market is always bounded in its assessment of future payoffs by $\mathbf{\Sigma}$.

In a multi-asset model with uncorrelated residual uncertainties and no factor structure, the single-asset intuition discussed in the paragraph above continues to hold. The diagonal matrix ($\mathbf{V}_N - \mathbf{\Sigma}$) represents a series of signal-to-noise differences.

In a multi-asset model with correlated residual uncertainties and/or a factor structure, the information risk premium can be driven by both the asset-specific component of payoffs and common factors. The matrix ($\mathbf{V}_N - \mathbf{\Sigma}$) can still be roughly interpreted as signal-to-noise differences. However, the matrix is no longer diagonal which means that covariance terms affect the *IRP*. Section 3 now turns to numerically investigating the effects of the covariance terms.

3 Numerical Analysis of Prices and Holdings

In this section we numerically analyze equilibrium prices and holdings. Our analysis highlights how the introduction of a factor structure extends results beyond what is possible when simply allowing payoffs to be correlated across assets. The analysis also helps understand the role of covariance terms in Equation (17).[13] Sections 3.1, 3.2, and 3.3 focus on international portfolio choice and the home bias puzzle. Section 3.4 shows how the sensitivity of an asset's price to information about the asset-specific component of its payoffs can be negative.

The numerical analysis considers a simple setting with two assets (an American stock and a French stock), a single factor, and two groups of investors (American people and French people). Each investor group has specific information about their home country's stock. All investors have a risk aversion coefficient of $a = 1.00$. Payoffs of the American (A) and French (F) assets follow from Equation (1):

$$\tilde{P}_A^1 = \tilde{\theta}_A + \mathbf{B}_A \tilde{f} + \tilde{\varepsilon}_A$$
$$\tilde{P}_F^1 = \tilde{\theta}_F + \mathbf{B}_F \tilde{f} + \tilde{\varepsilon}_F$$

The asset-specific component of both payoffs is $E\left[\tilde{\theta}\right] = 1.00$, the realization for both assets is $\theta = 1.00$, the expected supply of both assets is $E\left[\tilde{z}\right] = 0.01$, and the supply realization is $z = 0.01$. We assume that the variance-covariance matrices $\mathbf{Z}$ and $\mathbf{\Sigma}$ are both equal to the identity matrix. The variance-covariance matrix $\mathbf{T}$ is proportional to the identity matrix. We measure the degree of information advantage about θ (the asset-specific component of payoffs) by the matrix $\mathbf{T}$ (i.e. by the diagonal elements of $\mathbf{T}$).[14] We vary the degree of information advantage about the asset-specific components of payoffs from 0 to 10 for both groups of assets/investors.

[13] Covariance terms also play a role in the equilibrium holdings of individual investors. An explicit expression for these holdings, as well as an economic interpretation, is given in Appendix F.

[14] Fundamentally, the information asymmetry about an asset should be measured by the corresponding element of the matrix $\mathbf{\Psi}$. However, as seen in the model section, $\mathbf{\Psi}$ is an endogenous matrix. All else being equal, an increase in the matrix $\mathbf{T}$ corresponds to an increase in the matrix $\mathbf{\Psi}$, and vice-versa. This is due to the fact that an increase in the variance of asset-specific information corresponds to an increase in the asymmetric information surrounding this asset. We note that in this section the matrix U is obtained numerically and not by using the expression in Equation (9).

The realization of the common factor is known by only one of the two investor groups (assume the Americans have information about the common factor). In the calibration, the expected factor realization is $E\left[\tilde{f}\right] = 0$ and the variance is $Var\left[\tilde{f}\right] = 1$. The degree of information advantage about the common factor is equal to the square of the factor loading. We vary the factor loading for the French asset, $\mathbf{B}_F$ from 0 to 10. In this analysis, the factor loading for the American asset is ten times lower than that of the French asset.

3.1 Asset Prices

We calculate asset prices at date 0. The value of the world market portfolio is equal to the value of the American asset plus the value of the French asset. In Figure 1, the x-axis shows different degrees of information advantage about the asset-specific components of payoffs. For x-axis values greater than zero, the American investors have increasingly valuable information about the American asset and the French investors have increasingly valuable information about the French asset.

[**Insert Figure 1 About Here**]

The top graph line (thick red) assumes the common factor plays no role in determining asset payoffs. The line starts at \$19.79 when there is no asset-specific asymmetry and drifts down very slightly to \$19.68 where there are high levels of asymmetry. The "flatness" of this line comes from the fact that the American investors have information about the American asset and the French investors have information about their asset. As information asymmetry increases, each group of investors increases the value they place on their own country's assets and decreases the value they place on the other country's assets. There is little change in overall asset prices.

Figure 1 also depicts the role of the common factor. In this analysis, the American investors have information about the common factor. As price of the French asset becomes more sensitive to the common factor, the French investors' asset-specific information becomes less valuable. The bottom graph line (thin, purple, with "O" markings) represents the highest degree of information advantage about the common factor. The right-hand side of Figure 1 represents situations when asset-specific information is normally valuable. However, as the common factor becomes more important, the French asset becomes less valuable, and its information risk premium increases. The net result is that the price of the world market portfolio falls.

[Insert Figure 2 About Here]

Figure 1 shows the value of the world market portfolio decreases when there is high asset-specific information asymmetry and high information asymmetry about the common factor. The same information structure causes the weight of the American asset to increase. Figure 2 shows changes in the composition of the world market portfolio. As can be seen, the relative value of the American asset increases (and the relative value of the French asset falls) as we move from left to right across Figure 2. In this figure, the top graph line (thin, purple, with "O" markings) represents the highest levels of asymmetry about the common factor.

3.2 Home Bias and Portfolio Holdings

We calculate the weight of the American asset in the American investors' portfolios (W^A_{Amer}) and the weight of the American asset in the world portfolio (W^A_{World}). Weights are calculated using market values. To measure the existence of a home bias, we define a variable "Diff^A" such that a value of $\mathit{Diff}^A > 0$ indicates the existence of home bias.

$$\mathit{Diff}^A \equiv W^A_{Amer} - W^A_{World} \tag{18}$$

The top line in Figure 3 plots the degree of American home bias (i.e., the value of Diff^A) for different levels of asset-specific information asymmetry. When there is no information advantages about the common factor, results are symmetric and we obtain the same graph when we measure home bias of the French investors.

[Insert Figure 3 About Here]

The top graph line slopes upward indicating that higher levels of asset-specific information asymmetry lead investors to increase the weight of their home country assets (i.e., higher home bias). In other words, an increase in the degree of information asymmetry about the American asset encourages the American investors to allocate a higher part of their wealth to the American asset and to deviate from the world market portfolio. For high levels of information asymmetry, Diff^A approaches 30%, in accordance with existing empirical results for U.S. investors. Note that the empirical degree of home bias for investors from other parts of the world (Japan, etc.) has been found to be even higher than 30%.

3.3 Reverse Home Bias

We consider the role of common factors and again examine portfolio holdings. Figure 3 depicts three additional graph lines for three different levels of information asymmetry about the common factor. The figure illustrates home bias, no bias, and reverse home bias. For low levels of information asymmetry about the common factor, the American investors always have a preference for the American asset ($Diff^A > 0$). The top graph line (thick and red) shows the lowest levels of information asymmetry about the common factor. For high values of information asymmetry about the common factor, the American investors may choose to overweight the French asset ($Diff^A < 0$) in their portfolio. The bottom graph line (thin, purple, with "O" markings) represents the highest levels of asymmetry about the common factor. The finding of reverse home bias is due to the relative informational advantage of the American investors when considering the French asset compared to the American asset.

The economic intuition behind reverse home bias is: i) the French asset's payoff is sensitive to the common factor and today's price of the French asset is affected by large information asymmetries, ii) the American investors are informed about the common factor. The informational advantage of the American investors about the French asset factor may overwhelm their informational disadvantage vis-a-vis the specific component of the French asset's payoff. In such cases, the information risk premium required by the American investors for holding the French asset can be lower than the information risk premium required by the French investors for holding the French asset. This implies that the American investors overweigh the French asset and, thus, the French investors overweigh the American asset.

3.4 Price Anomalies

In this section, we show the sensitivity of an asset's price to information about the asset-specific component of its payoffs can be negative. In other words, good news about future payoffs can result in an asset's price *falling*. We continue to use a simple two-country, two-asset, single-factor numerical analysis (the same American and French stocks used in the last example.) Liquidity shocks remain uncorrelated across assets while the correlation is now 0.50 across residual uncertainties.

[**Insert Figure 4 About Here**]

Figure 4 plots the price reaction of the French asset in response to realization of its asset-specific component of payoffs (θ_F). The X-axis considers different levels of information asymmetry about each of the assets (American investors have information about the American asset and French investors have information about the French asset.) The figure plots four different lines—each corresponding to a different level of information asymmetry about the common factor. The bottom graph line (thin, purple, with "O" markings) represents the highest levels of asymmetry about the common factor. As the figure shows, low levels of asymmetry about the asset-specific component of payoffs and high levels of asymmetry about the common factor lead to anomalous price reactions. In such cases, a realization of θ_F above its expected value (good news) corresponds to a fall in the price of the French stock price. The results extend the Admati (1985) results by showing anomalous price behavior can exist without requiring highly correlated liquidity shocks.

4 Data and Empirical Analysis

We empirically analyze a cross-section of international mutual fund holdings. Our goal is to explore broad implications of the model presented in Section 2. The unit of analysis is a publicly listed company and we measure the fraction of shares outstanding held by foreigners. Our approach complements many existing studies that measure the fraction of investors' portfolios allocated to home-country versus foreign assets.

We start with the hypothesis that sophisticated money managers may generate and/or possess information about economy-wide factors. If this hypothesis is true, our model predicts that foreign holdings should increase (home bias should decrease) as factors become more prominent in a stock's payoffs. Under the assumption that investors have an information advantage about stocks from their own countries, we predict that cross-border holdings decrease as the asset-specific component of payoffs becomes more prominent in a stock's returns.

The empirical tests in this section parallel the numerical analysis from Section 3 and Figure 3 in particular. We create a proxy variable for different levels of information advantage that may exist about economy-wide factors. We also create a proxy variable for different levels of information advantage that may exist about the asset-specific component of payoffs. Our proxy variables are constructed in such a manner that the factor component is not mechanically (and negatively) related to the asset-specific component. We now describe the data and proxy variables.

Holdings Data: We obtain international mutual fund holdings on December 31, 2002 from the same source used by Chan, Covrig, and Ng (2005). For a single security, the data consist of the number shares held by domestic mutual funds and the number of shares held by foreign mutual funds. We consider listed stocks from all 21 developed countries except Canada and the United States.[15] The main dataset contains 10,292 different securities which are identified by Sedol number.

[**Insert Table 1 About Here**]

Table 1, Panel A provides a list of the 21 countries along with the number of stocks from each country. More stocks are Japanese (2,676) than from any other country. There are 1,973 stocks from the U.K., 744 from Germany, down to 56 stocks from Portugal. For each stock in our sample, we calculate the fraction of total shares held by foreign mutual funds in our dataset:[16]

$$\Omega_j = \frac{\text{\# Shares Held by Foreign Funds}}{\text{\# of Shares Outstanding}} \tag{19}$$

Return Data: We obtain up to 60 months of individual stock return data from Datastream (dividends included) starting July 1997 and ending June 2002. Returns are lagged by at least six months (from the Dec-2002 holdings date) in order to separate the holdings measure from our proxy variable used to measure the asset-specific component of returns (described below). The return series may be denominated in a currency other than US dollars (USD). Therefore, we also obtain monthly exchanges rates in order to convert to a base currency (USD). Datastream has available Sedol numbers and sufficient return data for 7,553 stocks as shown in the second column of Table 1, Panel A. For each stock, Datastream includes an associated sector code. There are 38 sectors which are listed in Table 1, Panel B. For each sector, we obtain the monthly return of a US dollar index. We also obtain the monthly returns of the MSCI World Market Index denominated in US dollars.

Firm Characteristics: For each stock, as of December 31, 2002, we obtain the number of

[15] The data we obtain are aggregated at the stock level and do not include holdings of US and Canadian stocks. Note that approximately 71% of the mutual funds are located in Canada and the United States. Since we are interested in cross-board holdings, excluding stocks from these two countries is not likely affect results.

[16] If funds hold the world market portfolio, the measures Ω_j should be equal across stocks. Thus, a high value of Ω_j indicates reduced home bias.

shares outstanding, the share price, the number of analysts covering the stock, sales, and the ratio of the book value of debt to total assets. Together with the holdings and return data, our final sample consists of 5,781 stocks. The stocks in our final sample are tabulated by country (Table 1, Panel A) and by sector (Panel B).

4.1 Proxy Variables

Asset-Specific Information: We create a proxy variable to measure the level of a stock's asset-specific information. The following time-series regression is estimated for each stock j in our sample that has at least 20 months of return data.

$$r_{j,t} = \alpha + \beta_w r_{w,t} + \beta_k r_{k,t} + \varepsilon_{j,t} \tag{20}$$

$$\textit{Asset Specific}_j \equiv 1 - R_j^2$$

Above, $r_{j,t}$ is the return on stock j in month t and $r_{w,t}$ is the return on the world market portfolio. The return $r_{k,t}$ is from a global sector index where k is determined by the sector of stock j. Our proxy variable $\textit{Asset Specific}_j$ is defined as one minus the fit from Equation (20). A high value of $\textit{Asset Specific}_j$ indicates that asset-specific information plays a large role in determining asset j's prices. A low value of $\textit{Asset Specific}_j$ indicates that economy-wide factors (and information about these factors) plays a large role in determining j's prices.

Factor Information: We create a proxy variable for the information advantage mutual fund managers may have about common factors. Fund managers in our sample are primarily located in North America. Therefore, we note the industry (k) for each international stock j in our sample. We set the value of the proxy variable equal to the average number of analysts per United States stock in industry k. Our measure is created without using any information about the number of analysts who actually cover non-U.S. stocks. In addition, our measure has an advantage over counting the total number of analysts because stocks from industries with many U.S. firms do not necessarily receive higher values.[17] The average value of our proxy variable is 6.41 with a $[5.16, 7.39]$ inter-quartile range. A high average number of

[17]From Datastream, we match 2,537 U.S. securities with a primary listing on one of the three three major U.S. exchanges with the I/B/E/S database and download the number of analysts as of Dec-2002. The three U.S. exchanges are The New York Stock Exchange, American Stock Exchange, and Nasdaq. "I/B/E/S" stands for the Institutional Brokers Estimate System.

analysts is interpreted as more information being generated about an industry—which in turn gives fund managers a potential information advantage when analyzing overseas stocks from the same industry.

Overview Statistics: Table 2 provides overview statistics of the variables used in this paper. Panel A shows that the average stock has 2.76% of its shares held by foreign mutual funds from our dataset. The 25^{th} percentile of holdings is 0.14% and 75^{th} percentile is 3.22%. The difference between the number of Stock j's shares held by foreign and domestic mutual funds (normalized by shares outstanding) is denoted Ω_j^*, has a -2.00% average value and a $[-3.40\%, 0.39\%]$ interquartile range.

The average value of our proxy variable $\textit{Asset Specific}_j$—the average value of $1 - R_j^2$ from Equation (20)—is 0.85 with a $[0.79, 0.95]$ inter-quartile range. Market capitalizations are highly skewed. The average is USD 2.56 billion with a $[0.04, 0.47]$ inter-quartile range. For this reason, we use the natural log of market capitalization in our cross-sectional regressions. The average log of market capitalization is 18.75 with a $[17.41, 19.97]$ inter-quartile range. The average number of analysts covering the foreign stocks is 4.16 and the average book leverage is 0.57.

[**Insert Table 2 About Here**]

Table 2, Panel B shows that our $\textit{Asset Specific}_j$ measure is negatively cross-sectionally correlated with the natural log of market capitalization (-0.25 correlation coefficient) and negatively correlated with number of analysts (-0.37 coefficient).

4.2 Cross-Border Holdings and Double Sort Results

We compare the empirical relationship between mutual fund holdings and information proxies with the theoretical relationship shown in Figure 3. We sort the 5,781 stocks into quartiles by our proxy variable for asset-specific information. We also sort the stocks into quartiles by our proxy variable for factor information.

Table 3 shows the average foreign holdings ($\bar{\Omega}$) for the four combinations where the sort variable is either “low” (bottom 25%) or “high” (upper 25%). When there are low-levels of information about common factors and high-levels of asset-specific information, the average cross-border holding is 0.0141 of shares outstanding. Low-levels of cross-border holdings correspond to situations when home bias is high—see the upper-right hand side of Figure 3 for a graphical example.[18]

[Insert Table 3 About Here]

When there are high-levels of information about common factors and low-levels of asset-specific information, the average cross-border holdings is 0.0507 of shares outstanding. High-levels of cross-border holdings correspond to situations when home bias is low and reverse home bias is possible—see the lower-left hand side of Figure 3 for a graphical example.

Table 3 shows that when information about common factors is "low", a decrease in asset-specific information leads to an increase of 0.0217 in cross-border holdings. When information about common factors is "high", a decrease in asset-specific information leads to an increase of 0.0348 in cross-border holdings. These increases are both statistically significant as t-statistics are 6.25 and 5.72 respectively. The increases are also economically significant and are approximately equal to holdings increasing from their 25^{th} to its 75^{th} percentile—see Table 2, Panel A for an overview of the holdings quartiles.

4.3 Cross-Border Holdings and Regression Results

We use regression analysis to test whether our proxy variable for asset-specific information ($\textit{Asset Specific}_j$) is inadvertently picking up effects known to influence cross-border holdings. Our results are shown in Table 4 and we start with the following, cross-sectional regression:

$$\Omega_j = \gamma_0 + \gamma_1 \left(\textit{Asset Specific}_j\right) + \nu_j$$

The coefficient of interest is γ_1. Table 4, Regression 1 shows the estimated value of γ_1 is -0.0855 with a -11.81 t-statistic. We use robust (White) standard errors to compute t-statistics. Stocks with high levels of asset specific information (i.e., stocks that move less with world and sector indices) have lower levels of cross-border holdings.

[Insert Table 4 About Here]

[18] If the mutual funds in our dataset held the world market portfolio, we should measure similar values of Ω_j across all stocks and thus report similar values of $\bar{\Omega}$ for each of the four bins shown in Table 3.

We expand the basic regression to include variables that have previously been linked to cross-border holdings. The variables include the natural log of equity market value ($lnMC_j$), the number of analysts following the stock $\left(\#\ of\ Analysts_j\right)$, and a measure of leverage $\left(\frac{Debt_j}{Assets_j}\right)$.

$$\Omega_j = \gamma_0 + \gamma_1 \left(Asset\ Specific_j\right) + \gamma_2 \left(lnMC_j\right) + \gamma_3 \left(\#\ of\ Analysts_j\right) + \gamma_4 \left(\frac{Debt_j}{Assets_j}\right) + \nu_j$$

In Table 4, Regression 2 shows the results after including two explanatory variables in the cross-sectional regression. The coefficient on *Asset Specific*$_j$ (γ_1) is -0.0550 with a -3.19 t-statistic. The fit of the regression is higher than Regression 1 and the adjusted R^2 of the cross-sectional regression is 0.1261.[19] Regression 3 includes the number of analysts covering stock j on the right-hand side. The coefficient γ_1 is -0.0177 with a t-statistic of -3.19 and a fit of 0.2131.

The remainder of Table 4 is a series of robustness checks. We test different regression specifications and check if the γ_1 coefficient remains significantly negative. In Table 3, Regression 4, the left-hand side variable is changed to Ω_j^* which is defined using the difference between number of shares held by foreign and local institutions in the numerator of Equation (19). If a stock is particularly attractive to all institutional investors (as opposed to just cross-border investors), Ω_j^* will be low. However, the results in Regression 4 are not materially different from those in Regression 3. We conclude that controlling for possibly (unobserved) characteristics that may be attractive to institutional investors does not affect our results.

A Sales-Based Measure of *Asset Specific*$_j$: We calculate a second measure of *Asset Specific*$_j$ based on sales data as opposed to stock market data. For each company, we obtain a history of annual sales (in US dollars) and calculate annual sales growth. We next calculate each industry's annual sales growth as the equal-weighted average of company sales growths. The number of firms per industry is given in Table 1, Panel B. The new measure of *Asset Specific*$_j$ *(Sales)* is simply one minus the correlation of stock j's annual sales growth with its industry's annual sales growth. We require a company to have six years of sales growth data to be included which reduces our sample to 3,905 stocks.

In Table 3, Regression 5, the γ_1 coefficient is -0.0602 with a -13.69 t-statistic. We continue

[19] The fit shown at the bottom of Table 4 is from the cross-sectional regression—not to be confused with the fit from the time series regression (20) used to construct *Asset Specific*$_j$.

to use robust (White) standard errors when calculating t-statistics. In Regression 6, we include country fixed effects (dummy variables). The γ_1 coefficient is -0.0076 with a -1.98 t-statistic. Controlling for country fixed effects addresses the possibility that unobserved country-level differences are driving results. If, however, country-level differences are related to the information structure, such a regression would not make sense in light of our model. We report results for completeness.

In the final column, Regression 7, we control for the large number of Japanese and UK firms in our sample. We randomly choose 155 of the 2,108 Japanese stocks in our sample and 155 of the 721 UK stocks in the sample (note that 155 is the average number of stocks from other countries in our sample.) The sample size is 1,346 as we have dramatically reduced the number of Japanese and UK stocks used. In addition, some of the 155 stocks that were randomly chosen may not have six years of sales growth data. Regression 7 shows the γ_1 coefficient is -0.0200 with a -3.34 t-statistic.

Our regressions show that cross-border holdings decrease as asset-specific information becomes more important for a given stock's returns. We measure the level of asset-specific information using both stock return data and sales growth data.

4.4 Information about Common Factors and Regressions

We end the empirical analysis by combining our proxy variable for information advantage about common factors with the regression analysis. Table 5, Regression 1a considers only stocks with a low degree of information advantage about common factors (bottom 25%). We regress our cross-border holdings (Ω_j) on a constant and our proxy for asset-specific information ($Asset\ Specific_j$). The γ_1 coefficient is -0.0759 with a -7.63 t-statistic. Regression 1b uses stocks with a high degree of information advantage about common factors (upper 25%). The γ_1 coefficient is -0.1125 with a -4.93 t-statistic.

[**Insert Table 5 About Here**]

The regression results shown in Table 5 match the double sort results shown in Table 3. Specifically, stocks with a high degree of information advantage about common factors are more sensitive to our proxy for asset-specific information ($-0.1125 < -0.0759$). Also, stocks with a high degree of information advantage have higher levels of cross-border holdings—see the estimated constant term γ_0 and note that $0.1253 > 0.0877$). An F-test of coefficient equality across the two groups has a 0.0001 p-value indicating that the proxy for factor information is significantly different from the proxy for asset-specific informaiton.

Table 5, Regressions 2a and 2b give the same general picture even after controlling for stock size and leverage. The γ_1 coefficients become increasingly negative when moving from stocks with low information advantage about the common factor to stocks with high information advantage (-0.0783 $<$ -0.0384). The F-test of coefficient equality again rejects the hypothesis of coefficient equality at the 1.50%-level.

The final set of regressions (5*a and 5*b) again shows the γ_1 coefficients become increasingly negative when moving from stocks with low information advantage about the common factor to stocks with high information advantage (-0.0770 $<$ -0.0484). However, the F-test fails to reject the hypothesis of coefficient equality. These regressions use the difference between foreign and local holdings as the dependent variable (Ω_j^*).

5 Conclusion

This paper proposes a rational expectations equilibrium model in which agents are asymmetrically informed about asset-specific components of payoffs and common factors that also affect payoffs. The model produces closed-form solutions for prices and the holdings of individual agents.

We show that prices in the model collapse to traditional CAPM prices when all agents are fully informed about all payoffs in the economy. An asset's price today equals its discounted future payoffs minus a risk premium. The risk premium is a function of risk aversion and the covariance of the asset's payoffs with the payoffs of all other assets (the market). When agents are asymmetrically informed about the asset's payoffs we show prices are below CAPM values. The discount below CAPM values is called the information risk premium which can (roughly) be thought of as a signal-to-noise measure. When the market has high uncertainty about an asset's future payoffs (relative to any residual uncertainty) the information risk premium is large, and the asset's price is low. A thorough understanding of the information risk-premium depends on the market's uncertainty about the asset's payoffs compared with the market's uncertainty about other assets' payoffs. Thus, we turn to numerically analyzing our model to gain economic understanding.

Our first numerical analysis focuses on international portfolio choice and the home bias puzzle. We consider a simple two-country, two-asset, single-factor setting. Under a typical assumption that American investors have better information about the asset-specific component of American stocks and French investors have better information about the asset-specific component of French stocks, we measure home bias that parallels the existing empirical literature. More interestingly, low levels of asset-specific information and high-level of information asymmetry about the common factor can lead investors from one country to overweigh assets from the other country—a phenomenon called "reverse home bias."

Our second numerical analysis considers a similar two-country, two-asset, single-factor setting. We show the sensitivity of one asset's price to information about the asset-specific component of its payoffs can be negative. In other words, good news about payoffs can correspond to a drop in the asset's price. Introducing a factor structure allows our model to produce this anomalous price behavior even when liquidity shocks are uncorrelated.

We end the paper with an empirical analysis that revisits international portfolio choice. We create two proxy variables. The first measures the degree of information advantage about the asset-specific component of a stock's payoffs. The second measures the degree of information advantage about common factors. We show that both a decrease in the proxy for asset-specific information advantage and an increase in the proxy for factor information lead to greater levels of cross-border holdings. Regression analysis shows our results hold even after controlling for variables that have previously been linked to cross-border holdings (the market capitalization of a firm's equity, the number of analysts following the firm, and the firm's leverage.)

There are a number of potential avenues for future research. First, one could try to extend the model to multiple periods. This would provide expressions for net trading as in Brennan and Cao (1997) as well as suggest empirical tests based on trading (as opposed to holdings) data. Second, one could work to devise methods of empirically identifying different information structures. While no small task, structures could then be used to test relative prices of assets using expressions in this paper. Third, our model may be adapted to better understanding partially segmented markets. In such cases, the "friction" which segments markets is information. One may be able to model groups of investors who face low frictions only when trading securities from their home country, groups of investors who face low frictions when trading securities in a contiguous block of countries (a geographic region), or groups of investors who face very little frictions when investing in any global security. None of the three extensions is likely to be easy—all are potentially interesting.

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Appendix A

The information set of investor n (formally, investor i in group n) consists of the realization of private signals $\mathbf{M}_n\tilde{\eta}$ and of equilibrium prices $\tilde{P}^0$. The equilibrium price vector $\tilde{P}^0$ is a linear function of the information $\tilde{\eta}$ and the supply $\tilde{z}$ with $\tilde{P}^0 = \mathbf{A}_0 + \mathbf{A}_1\tilde{\eta} - \mathbf{A}_2\tilde{z}$. Since, $\tilde{w}_n^1 = w_n^0 R + X_n'(\tilde{P}^1 - R\tilde{P}^0)$ and $\tilde{P}^1$ is a linear function of $\tilde{\eta}$ and $\tilde{\varepsilon}$, it follows that $\tilde{w}_n^1$ joins the multivariate normal distribution of $(\tilde{\eta}, \tilde{\varepsilon}, \tilde{z})$. Consequently, $\tilde{w}_n^1$ is a normal random variable conditional on $\mathbf{M}_n\tilde{\eta}$ and $\tilde{P}^0$. Properties of normal distributions imply that investor n's expected utility can be written as:

$$\begin{aligned} E\left[U(\tilde{w}_n^1)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] &= U\left\{E\left[\tilde{w}_n^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] - \frac{a}{2}Var\left[\tilde{w}_n^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right]\right\} \\ &= U\Big\{E\left[w_n^0 R + X_n'(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] - \frac{a}{2}Var\left[w_n^0 R + X_n'(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right]\Big\} \end{aligned}$$

Since the utility function is exponential, maximizing this expected utility is identical to maximizing:

$$\begin{aligned} &\max_{X_n}\left\{E\left[w_n^0 R + X_n'(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] - \frac{a}{2}Var\left[w_n^0 R + X_n'(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right]\right\} \\ &= \max_{X_n}\left\{X_n' E\left[\left(\tilde{P}^1 - R\tilde{P}^0\right)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] - \frac{a}{2}X_n' Var\left[(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] X_n\right\} \end{aligned}$$

The equation to be solved is:

$$0 = E\left[(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] - aVar\left[(\tilde{P}^1 - R\tilde{P}^0)|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] X_n \tag{21}$$

This implies that investor n's demand vector is:

$$X_n = a^{-1}Var^{-1}\left[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] \times \left(E\ \left[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] - RP^0\right) \tag{22}$$

Appendix B

From Equations (4), (6), and (7), we have:

$$0 = \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1}\Big(\mathbf{B_{0n}} + \mathbf{B}_{1n}\mathbf{M}_n\tilde{\eta} + \left(\mathbf{B}_{2n} - R\mathbf{I}_J\right)\left(\mathbf{A}_0 + \mathbf{A}_1\tilde{\eta} - \mathbf{A}_2\tilde{z}\right)\Big) - a\tilde{z}$$

By canceling the $\tilde{z}$, $\tilde{\eta}$, and constant terms, it is straightforward to show that:

$$\begin{aligned} a\mathbf{A}_2^{-1}\mathbf{A}_0 &= \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1}\mathbf{B}_{0n} \\ a\mathbf{A}_2^{-1}\mathbf{A}_1 &= \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1}\mathbf{B}_{1n}\mathbf{M}_n \\ a\mathbf{A}_2^{-1} &= \sum_{n=1}^{N} \lambda_n \mathbf{V}_n^{-1}\left(R\mathbf{I}_J - \mathbf{B_{2n}}\right) \end{aligned} \tag{23}$$

Appendix C

The vector $\left(\tilde{P}^{1\prime} \quad \mathbf{M}_n\tilde{\eta}' \quad \tilde{P}^{0\prime} \right)'$ is normally distributed and its var-cov matrix is:

$$Var\left[\left(\tilde{P}^{1\prime} \quad \mathbf{M}_n\tilde{\eta}' \quad \tilde{P}^{0\prime} \right)'\right] = \begin{pmatrix} \mathbf{CQC}' + \mathbf{\Sigma} & \mathbf{CQM}_n' & \mathbf{CQA}_1' \\ \mathbf{M}_n\mathbf{QC}' & \mathbf{M}_n\mathbf{QM}_n' & \mathbf{M}_n\mathbf{QA}_1' \\ \mathbf{A}_1\mathbf{QC}' & \mathbf{A}_1\mathbf{QM}_n' & \mathbf{A}_1\mathbf{QA}_1' + \mathbf{A}_2\mathbf{ZA}_2' \end{pmatrix} \tag{24}$$

The conditional expectation is:

$$E_n\left[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] = E[\tilde{P}^1] + Cov\left[\tilde{P}^1; \begin{pmatrix} \mathbf{M}_n\tilde{\eta} \\ \tilde{P}^0 \end{pmatrix}\right] \times Var^{-1}\begin{bmatrix} \mathbf{M}_n\tilde{\eta} \\ \tilde{P}^0 \end{bmatrix} \times \left(\begin{pmatrix} \mathbf{M}_n\tilde{\eta} \\ \tilde{P}^0 \end{pmatrix} - E\begin{bmatrix} \mathbf{M}_n\tilde{\eta} \\ \tilde{P}^0 \end{bmatrix}\right)$$

Normal distributions give $E_n\left[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] = \mathbf{B}_{0n} + \mathbf{B}_{1n}\mathbf{M}_n\tilde{\eta} + \mathbf{B}_{2n}\tilde{P}^0$. Hence,

$$\begin{pmatrix} \mathbf{B}_{1n} & \mathbf{B}_{2n} \end{pmatrix} = Cov\left[\tilde{P}^1; \begin{pmatrix} \mathbf{M}_n\tilde{\eta} \\ \tilde{P}^0 \end{pmatrix}\right] \times Var^{-1}\begin{bmatrix} \mathbf{M}_n\tilde{\eta} \\ \tilde{P}^0 \end{bmatrix} \tag{25}$$

$$\begin{pmatrix} \mathbf{CQM}_n' & \mathbf{CQA}_1' \end{pmatrix} = \begin{pmatrix} \mathbf{B}_{1n} & \mathbf{B}_{2n} \end{pmatrix}\begin{pmatrix} \mathbf{M}_n\mathbf{QM}_n' & \mathbf{M}_n\mathbf{QA}_1' \\ \mathbf{A}_1\mathbf{QM}_n' & \mathbf{A}_1\mathbf{QA}_1' + \mathbf{A}_2\mathbf{ZA}_2' \end{pmatrix} \tag{26}$$

The variance of returns conditional on n's information is:

$$\begin{aligned} \mathbf{V}_n &= Var\left[\tilde{P}^1|\mathbf{M}_n\tilde{\eta}, \tilde{P}^0\right] \\ &= Var\left[\tilde{P}^1\right] - Cov\left[\tilde{P}^1; \begin{pmatrix} \mathbf{M}_n\tilde{\eta} \\ \tilde{P}^0 \end{pmatrix}\right] \times Var^{-1}\begin{bmatrix} \mathbf{M}_n\tilde{\eta} \\ \tilde{P}^0 \end{bmatrix} \times Cov\left[\begin{pmatrix} \mathbf{M}_n\tilde{\eta} \\ \tilde{P}^0 \end{pmatrix}; \tilde{P}^1\right] \end{aligned} \tag{27}$$

We use Equation (25) to get:

$$\mathbf{V}_n = Var\left[\tilde{P}^1\right] - \begin{pmatrix} \mathbf{B}_{1n} & \mathbf{B}_{2n} \end{pmatrix} Cov\left[\begin{pmatrix} \mathbf{M}_n\tilde{\eta} \\ \tilde{P}^0 \end{pmatrix}; \tilde{P}^1\right]$$

Because $Cov\left[\begin{pmatrix} \mathbf{M}_n\tilde{\eta} \\ \tilde{P}^0 \end{pmatrix}; \tilde{P}^1\right] = \begin{pmatrix} \mathbf{M}_n\mathbf{QC}' \\ \mathbf{A}_1\mathbf{QC}' \end{pmatrix}$ we get:

$$\mathbf{V}_n = \mathbf{CQC}' + \mathbf{\Sigma} - \mathbf{B}_{1n}\mathbf{M}_n\mathbf{QC}' - \mathbf{B}_{2n}\mathbf{A}_1\mathbf{QC}' \tag{28}$$

Appendix D

In order to determine a closed form solution for $\mathbf{U}$, we solve the second equation from the system shown in Equation (8):

$$a\mathbf{A}_2^{-1}\mathbf{A}_1 = a\mathbf{U} = \sum_{n=1}^{N}\lambda_n\mathbf{V}_n^{-1}\mathbf{B}_{1n}\mathbf{M}_n \tag{29}$$

The following properties apply to matrices $\mathbf{D}_n$ and $\mathbf{M}_n$:

P1: $\sum_{n=1}^{N}\mathbf{D}_n = \mathbf{I}_J$

P2: $\forall\ n_1 \neq n_2$: $\mathbf{D}_{n1}\mathbf{D}_{n2} = \mathbf{0}_J$ where $\mathbf{0}_J$ is the null matrix of order J;

P3: $\mathbf{M}_{n1}\mathbf{M}'_{n2} = \mathbf{0}_{Jn1,Jn2}$ where $\mathbf{0}_{Jn1,Jn2}$ is the null matrix of order $J_{n1} \times J_{n2}$;

P4: $\mathbf{D}_n\mathbf{D}_n = \mathbf{D}_n$ and $\mathbf{M}_n\mathbf{M}_n^{-1} = \mathbf{I}_{Jn}$

P5: $\forall\ \mathbf{G_1}, \mathbf{G_2}$: $g(\mathbf{G_1})g(\mathbf{G_2}) = \sum_{n=1}^{N}\mathbf{D}_n\mathbf{G_1}\mathbf{D}_n\mathbf{G_2}\mathbf{D}_n$

P6: $\forall\ \mathbf{G}$: $g(\mathbf{GD}) = g(\mathbf{G})\mathbf{D} = \mathbf{D}g(\mathbf{G})$

There are three matrices key to obtaining a closed form solution for $\mathbf{U}$:

$$\begin{aligned}
\mathbf{M} &= \mathbf{UQU'} + \mathbf{Z} \\
\mathbf{\Psi} &= Var\left[\tilde{\eta}|\tilde{P}^0\right] = \mathbf{Q} - \mathbf{QU'M^{-1}UQ} \\
\mathbf{\Psi}_n &= \mathbf{M}_n\mathbf{\Psi}\mathbf{M}_n'
\end{aligned}$$

We first solve Equation (26) for $\mathbf{B}_{1n}$ and $\mathbf{B}_{2n}$. The two equations to be solved are:

$$\mathbf{B}_{1n}\left(\mathbf{M}_n\mathbf{QM}_n'\right) + \mathbf{B}_{2n}\left(\mathbf{A}_1\mathbf{QM}_n'\right) = \mathbf{CQM}_n' \tag{30}$$

$$\mathbf{B}_{1n}\left(\mathbf{M}_n\mathbf{QA}_1'\right) + \mathbf{B}_{2n}\left(\mathbf{A}_1\mathbf{QA}_1' + \mathbf{A}_2\mathbf{ZA}_2'\right) = \mathbf{CQA}_1' \tag{31}$$

Using $\mathbf{M} = \mathbf{UQU'} + \mathbf{Z}$, we obtain $\mathbf{A}_1\mathbf{QA}_1' + \mathbf{A}_2\mathbf{ZA}_2' = \mathbf{A}_2\mathbf{MA}_2'$. This implies:

$$\begin{aligned}
&\mathbf{B}_{1n}(\mathbf{M}_n\mathbf{Q}) + \mathbf{B}_{2n}(\mathbf{A}_2\mathbf{MA}_2')\mathbf{A}_1' = \mathbf{CQ} \\
\Leftrightarrow\quad &\mathbf{B}_{2n}\mathbf{A}_2\mathbf{MU}'^{-1} = \mathbf{CQ} - \mathbf{B}_{1n}\mathbf{M}_n\mathbf{Q} \\
\Leftrightarrow\quad &\mathbf{B}_{2n} = (\mathbf{C} - \mathbf{B}_{1n}\mathbf{M}_n)\mathbf{QU'M}^{-1}\mathbf{A}_2^{-1}
\end{aligned}$$

In a second step, we solve Equation (30):

$$\begin{aligned}
&\mathbf{B}_{1n}(\mathbf{M}_n\mathbf{QM}_n') + (\mathbf{C} - \mathbf{B}_{1n}\mathbf{M}_n)\mathbf{QU'M}^{-1}\mathbf{A}_2^{-1}(\mathbf{A}_1\mathbf{QM}_n') = \mathbf{CQM}_n' \\
\Leftrightarrow\quad &\mathbf{B}_{1n}\mathbf{M}_n(\mathbf{QM}_n' - \mathbf{QU'M}^{-1}\mathbf{UQM}_n') = (\mathbf{CQ} - \mathbf{CQU'M}^{-1}\mathbf{UQ})\mathbf{M}_n' \\
\Leftrightarrow\quad &\mathbf{B}_{1n}\mathbf{M}_n(\mathbf{Q} - \mathbf{QU'M}^{-1}\mathbf{UQ})\mathbf{M}_n' = \mathbf{C}(\mathbf{Q} - \mathbf{QU'M}^{-1}\mathbf{UQ})\mathbf{M}_n' \\
\Leftrightarrow\quad &\mathbf{B}_{1n}\mathbf{M}_n\mathbf{\Psi}\mathbf{M}_n' = \mathbf{C}\mathbf{\Psi}\mathbf{M}_n' \\
\Leftrightarrow\quad &\mathbf{B}_{1n}\mathbf{\Psi}_n = \mathbf{C}\mathbf{\Psi}\mathbf{M}_n' \\
\Leftrightarrow\quad &\mathbf{B}_{1n} = \mathbf{C}\mathbf{\Psi}\mathbf{M}_n'\mathbf{\Psi}_n^{-1}
\end{aligned}$$

We have thus demonstrated that:

$$\begin{aligned}
\mathbf{B}_{1n} &= \mathbf{C}\Psi\mathbf{M}_n'\mathbf{\Psi}_n^{-1} \\
\mathbf{B}_{2n} &= \left(\mathbf{C} - \mathbf{B}_{1n}\mathbf{M}_n\right)\mathbf{QU'M}^{-1}\mathbf{A}_2^{-1}
\end{aligned} \tag{32}$$

By substituting $\mathbf{B}_{1n}$ and $\mathbf{B}_{2n}$ into Equation (28) we obtain the variance-covariance matrix

$\mathbf{V}_n$ as a function of $\mathbf{\Psi}$

$$\begin{aligned}
\mathbf{V}_n &= \mathbf{CQC}' + \mathbf{\Sigma} - \mathbf{B}_{1n}\mathbf{M}_n\mathbf{QC}' - \mathbf{B}_{2n}\mathbf{A}_1\mathbf{QC}' \\
\Leftrightarrow \mathbf{V}_n &= \mathbf{CQC}' + \mathbf{\Sigma} - \mathbf{C\Psi M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{QC}' - (\mathbf{C} - \mathbf{C\Psi M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n)\mathbf{QU}'\mathbf{M}^{-1}\mathbf{A}_2^{-1}\mathbf{A}_1\mathbf{QC}' \\
\Leftrightarrow \mathbf{V}_n &= \mathbf{\Sigma} + \mathbf{C}(\mathbf{Q} - \mathbf{QU}'\mathbf{M}^{-1}\mathbf{UQ})\mathbf{C} - \mathbf{C\Psi M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{QC}' + \mathbf{C\Psi M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{QU}'\mathbf{M}^{-1}\mathbf{UQC}' \\
\Leftrightarrow \mathbf{V}_n &= \mathbf{\Sigma} + \mathbf{C\Psi C}' - \mathbf{C\Psi M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n(\mathbf{Q} - \mathbf{QU}'\mathbf{M}^{-1}\mathbf{UQ})\mathbf{C}' \\
\Leftrightarrow \mathbf{V}_n &= \mathbf{\Sigma} + \mathbf{C\Psi C}' - \mathbf{C\Psi M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{\Psi C}' \qquad (33)
\end{aligned}$$

We use Equation (29) to determine $\mathbf{U}$. Multiplying (29) by $\mathbf{M}_n'$ on the right, we obtain $\lambda_n\mathbf{V}_n^{-1}\mathbf{B}_{1n} = a\mathbf{UM}_n'$. We then multiply this last equation by $\mathbf{V}_n$ on the left and we replace $\mathbf{B}_{1n}$ with its value from (32):

$$\lambda_n\mathbf{C\Psi M}_n'\mathbf{\Psi}_n^{-1} = a\mathbf{V}_n\mathbf{UM}_n' \qquad (34)$$

If we multiply (34) by $\mathbf{M}_n$ on the right and if we sum for $n = 1, \dots N$, we obtain Equation (29). We conclude that Equation (29) is equivalent to Equation (34) for all $n = 1, \dots, N$. If we multiply Equation (34) by $\mathbf{\Psi}_n$ and $\mathbf{M}_n$ on the right and if we replace $\mathbf{V}_n$ with its value in Equation (33) we then obtain:

$$\lambda_n\mathbf{C\Psi D}_n = a\left(\mathbf{\Sigma} + \mathbf{C\Psi C}' - \mathbf{C\Psi M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{\Psi C}'\right)\mathbf{UM}_n'\mathbf{\Psi}_n\mathbf{M}_n$$

If we now sum for $n = 1, \dots, N$ we obtain:

$$\sum_{n=1}^{N}\lambda_n\mathbf{C\Psi D}_n = a\left(\mathbf{\Sigma}\sum_{n=1}^{N}\mathbf{UM}_n'\mathbf{\Psi}_n\mathbf{M}_n + \mathbf{C\Psi C}'\sum_{n=1}^{N}\mathbf{UM}_n'\mathbf{\Psi}_n\mathbf{M}_n - \sum_{n=1}^{N}\mathbf{C\Psi M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{\Psi C}'\mathbf{UM}_n'\mathbf{\Psi}_n\mathbf{M}_n\right)$$

which is equivalent to:

$$\mathbf{C\Psi D} = a\left(\mathbf{\Sigma U}\sum_{n=1}^{N}\mathbf{D}_n\mathbf{\Psi D}_n + \mathbf{C\Psi C}'\mathbf{U}\sum_{n=1}^{N}\mathbf{D}_n\mathbf{\Psi D}_n - \mathbf{C\Psi}\sum_{n=1}^{N}\mathbf{M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{\Psi C}'\mathbf{UM}_n'\mathbf{\Psi}_n\mathbf{M}_n\right)$$

By introducing the function $g(\cdot)$, we obtain:

$$\mathbf{C\Psi D} = a\mathbf{\Sigma U}g(\mathbf{\Psi}) + a\mathbf{C\Psi C'U}g(\mathbf{\Psi}) - a\mathbf{C\Psi}\left(\sum_{n=1}^{N}\mathbf{M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{\Psi C'UM}_n'\mathbf{\Psi}_n\mathbf{M}_n\right) \tag{35}$$

The reader can easily check that (35) is equivalent to (29). We substitute $\mathbf{U}$ in (35) with $\mathbf{U} = a^{-1}\mathbf{\Sigma}^{-1}\mathbf{CD}$ and we have to check the following equality:

$$\mathbf{C\Psi D} = \mathbf{CD}g(\mathbf{\Psi}) + \mathbf{C\Psi C'\Sigma}^{-1}\mathbf{CD}g(\mathbf{\Psi}) - \mathbf{C\Psi}\left(\sum_{n=1}^{N}\mathbf{M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{\Psi C'\Sigma}^{-1}\mathbf{CDM}_n'\mathbf{\Psi}_n\mathbf{M}_n\right) \tag{36}$$

Thanks to Lemma 2.1, $\mathbf{\Psi}^{-1} + \mathbf{C'\Sigma}^{-1}\mathbf{C}$ is a g-matrix which means by definition, that $g(\mathbf{\Psi}^{-1} + \mathbf{C'\Sigma}^{-1}\mathbf{C}) = \mathbf{\Psi}^{-1} + \mathbf{C'\Sigma}^{-1}\mathbf{C}$. We then replace $\mathbf{C'\Sigma}^{-1}\mathbf{C}$ by $g(\mathbf{\Psi}^{-1} + \mathbf{C'\Sigma}^{-1}\mathbf{C}) - \mathbf{\Psi}^{-1}$ in the right term of Equation (36). This enables us to prove the equality in (36). We conclude $\mathbf{U} = a^{-1}\mathbf{\Sigma}^{-1}\mathbf{CD}$ represents a solution for $\mathbf{U}$.

Appendix E

We replace $\mathbf{B}_{2n}$ in the first equation of (8) with its value given in (32). We then obtain $\mathbf{A}_2$. We eliminate the $\mathbf{B}_{1n}$ coefficients ($n = 1, \ldots, N$) using the second equation in (8). We then directly obtain $\mathbf{A}_1$ from the expression for $\mathbf{U}$. In order to determine $\mathbf{A}_0$, we replace the following in the third equation of (8).

$$\mathbf{B}_{0n} = (\mathbf{I} - \mathbf{B}_{1n}\mathbf{M}_n - \mathbf{B}_{2n}\mathbf{A}_1)\,E\left[\tilde{\theta}\right] - \mathbf{B}_{2n}\left(\mathbf{A}_0 - \mathbf{A}_2 E\left[\tilde{z}\right]\right)$$

The reader can easily check that the matrix $\mathbf{A}_2$ is regular and it follows from Equation (13) that $\mathbf{A}_2 = a\mathbf{A}_1(\mathbf{CD})^{-1}\mathbf{\Sigma}$. The matrices $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{\Sigma}$ are, by definition, regular matrices. Moreover, using the properties of positive definite matrices, $\mathbf{A}_1$ appears to be a regular matrix.

Appendix F

We analyze the relationship between model parameters $\{r_f, a, \lambda_1, \ldots, \lambda_N, \mathbf{B}, \mathbf{T}, \mathbf{\Sigma}, \mathbf{Z}\}$ and ex-ante equilibrium holdings. We do this by taking expectations over the random variables in the model $\{\tilde{\eta}, \tilde{\varepsilon}, \tilde{z}\}$. Taking expectations of Equation (5) gives the following expression for investor n's holdings:

$$\begin{aligned} E[X_n] &= a^{-1}\mathbf{V}_n^{-1}\left(E\left[\tilde{P}^1\right] - RE\left[\tilde{P}^0\right]\right) \\ &= \mathbf{V}_n^{-1}\mathbf{V}_N E[\tilde{z}] \\ &= \left(\mathbf{\Sigma} + \mathbf{C\Psi C'} - \mathbf{C\Psi M}_n'\mathbf{\Psi}_n^{-1}\mathbf{M}_n\mathbf{\Psi C'}\right)^{-1}(\mathbf{\Sigma} + \mathbf{C\Psi C'})\left(\mathbf{I}_J + \mathbf{\Sigma}^{-1}\mathbf{CD\Psi C'}\right)^{-1}E[\tilde{z}] \end{aligned} \tag{37}$$

In a single-stock world with no factors, investor n's holdings depends on the ratio of the market's uncertainty about the future payoff ($\mathbf{V}_N$) to his uncertainty about the same payoff ($\mathbf{V}_n$). The higher the investor's uncertainty relative to the market, the lower the ratio, and the lower the weight of the asset in his portfolio.

In a multi-asset framework with uncorrelated residual uncertainty and no factors, the matrices ($\mathbf{V}_N$) and ($\mathbf{V}_n$) are diagonal. The term $\mathbf{V}_n^{-1}\mathbf{V}_N$ represents a series of uncertainty ratios. The same intuition described in the paragraph above holds.

In a multi-asset model with correlated residual uncertainties and/or a factor structure of payoffs, thinking about $\mathbf{V}_n^{-1}\mathbf{V}_N$ as a ratio of two uncertainty measures provides rough intuition only. However, the product of the two matrices includes covariance terms relating to assets' payoffs. Investor n's holdings of a specific asset now depends on his uncertainty about the asset's payoffs, his uncertainty about other assets' payoffs, and others' uncertainty about all assets (including the specific asset in question). Section 3 numerically analyzes equilibrium prices and holdings in an effort to better understand the role of the covariance terms.

Appendix G

We demonstrate that when all investors are informed about all asset-specific components and factors, prices reduce to the Sharpe-Lintner-Mossin Capital Asset Pricing Model (CAPM). We subtract R times Equation (4) from Equation (3) and take expectations to get:

$$E\left[\tilde{P}^1\right] - RE\left[\tilde{P}^0\right] = (\mathbf{C} - RA_1)E[\tilde{\eta}] + R\mathbf{A}_2E[\tilde{z}] - R\mathbf{A}_0$$

Equations (11), (12), and (13) enable us to write:

$$\begin{aligned} R\mathbf{A}_1 &= (\mathbf{CQC'}) + \mathbf{\Sigma} - \mathbf{V}_N)(\mathbf{CDQC'})^{-1}\mathbf{CD} &= \mathbf{C} \\ R\mathbf{A}_2 &= a(\mathbf{CQC'}) + \mathbf{\Sigma} - \mathbf{V}_N)(\mathbf{CDQC'})^{-1}\mathbf{\Sigma} &= a\mathbf{\Sigma} \\ R\mathbf{A}_0 &= (\mathbf{C} - R\mathbf{A}_1)E[\tilde{\eta}] + (R\mathbf{A}_2 - a\mathbf{V}_N)E[\tilde{z}] &= 0 \end{aligned}$$

Combining these results gives the CAPM expressed in prices:

$$E\left[\tilde{P}^0\right] = \frac{1}{R}\left(E\left[\tilde{P}^1\right] - a\mathbf{\Sigma}E\left[\tilde{z}\right]\right) \tag{38}$$

We can express the same result in terms of covariance and expected returns—a form more familiar to financial economists. As all investors are informed, they know the realization η of $\tilde{\eta}$. Therefore, $\tilde{P}^1 = C\eta + \tilde{\varepsilon}$ and $Var\left[\tilde{P}^1\right] = \Sigma$:

$$\begin{aligned} a\mathbf{\Sigma}E\left[\tilde{z}\right] &= a\,Var\left[\tilde{P}^1\right]E\left[\tilde{z}\right] = a\,Cov\left[\tilde{P}^1,\tilde{P}^1\right]E\left[\tilde{z}\right] = a\,Cov\left[\tilde{P}^1,\left(\tilde{P}^1\right)'E\left[\tilde{z}\right]\right] \\ &= a\,Cov\left[\tilde{P}^1,\tilde{P}^1_m\right] \end{aligned}$$

Here, $\tilde{P}^1_m$ is the payoff of the market portfolio (the one that contains all the assets) divided by the number of investors (sinse $\tilde{z}$ has been defined as the supply per investor). As the supply is unknown by the agents in the market, we consider the expectations of the supply, rather than the supply itself. Using Equation (38) and the above result gives: $E\left[\tilde{P}^1\right] - RE\left[\tilde{P}^0\right] = a\mathbf{\Sigma}E\left[\tilde{z}\right] = aCovE\left[\tilde{P}^1,\tilde{P}^1_m\right]$. For asset j, we get: $E\left[\tilde{P}^1_j\right] - RE\left[\tilde{P}^0_j\right] = aCov\left[\tilde{P}^1_j,\tilde{P}^1_m\right]$. Dividing the last result by $E\left[\tilde{P}^1_j\right]$, we obtain:

$$\begin{aligned} & \frac{E\left[\tilde{P}^1_j\right] - RE\left[\tilde{P}^0_j\right]}{E\left[\tilde{P}^0_j\right]} = \frac{a\,Cov\left[\tilde{P}^1_j,\tilde{P}^1_m\right]}{E\left[\tilde{P}^0_j\right]} \\ \Leftrightarrow\quad & \frac{E\left[\tilde{P}^1_j\right] - E\left[\tilde{P}^0_j\right] - (R-1)E\left[\tilde{P}^0_j\right]}{E\left[\tilde{P}^0_j\right]} = \frac{a\,Cov\left[\tilde{P}^1_j,\tilde{P}^1_m\right]}{E\left[\tilde{P}^0_j\right]} \\ \Leftrightarrow\quad & \frac{E\left[\tilde{P}^1_j\right] - E\left[\tilde{P}^0_j\right] - r_f E\left[\tilde{P}^0_j\right]}{E\left[\tilde{P}^0_j\right]} = \frac{a\,Cov\left[\tilde{P}^1_j - E\left[\tilde{P}^0_j\right],\tilde{P}^1_m\right]}{E\left[\tilde{P}^0_j\right]} \\ \Leftrightarrow\quad & \frac{E\left[\tilde{P}^1_j\right] - E\left[\tilde{P}^0_j\right]}{E\left[\tilde{P}^0_j\right]} - r_f = a\,Cov\left[\frac{\tilde{P}^1_j - E\left[\tilde{P}^0_j\right]}{E\left[\tilde{P}^0_j\right]},\tilde{P}^1_m\right] \\ \Leftrightarrow\quad & E\left[\tilde{r}_j\right] - r_f = a\,Cov\left[\tilde{r}_j,\tilde{P}^1_m\right] \end{aligned}$$

Where $\tilde{r}_j = \frac{\tilde{P}_j^1 - E\left[\tilde{P}_j^0\right]}{E\left[\tilde{P}_j^0\right]}$. If we multiply $E\left[\tilde{P}_j^1\right] - RE\left[\tilde{P}_j^0\right] = aCov\left[\tilde{P}_j^1, \tilde{P}_m^1\right]$ by z_j and sum from $j = 1, \ldots, J$ we get: $E\left[\tilde{r}_m\right] - r_f = aCov\left[\tilde{r}_m, \tilde{P}_m^1\right]$. The market return is the value weighted average of individual stock returns:

$$\tilde{r}_m = \frac{\tilde{P}_m^1 - E\left[\tilde{P}_m^0\right]}{E\left[\tilde{P}_m^0\right]} \tag{39}$$

If we divide both sides of the equation $E\left[\tilde{r}_j\right] - r_f = aCov\left[\tilde{r}_j, \tilde{P}_m^1\right]$ by the equation $E\left[\tilde{r}_m\right] - r_f = aCov\left[\tilde{r}_m, \tilde{P}_m^1\right]$ we get:

$$\begin{aligned}
E\left[\tilde{r}_j\right] - r_f &= \frac{a\,Cov\left[\tilde{r}_j, \tilde{P}_m^1\right]}{a\,Cov\left[\tilde{r}_m, \tilde{P}_m^1\right]}\left(E\left[\tilde{r}_m\right] - r_f\right) = \frac{Cov\left[\tilde{r}_j, \frac{\tilde{P}_m^1 - E\left[\tilde{P}_m^0\right]}{E\left[\tilde{P}_m^0\right]}\right]}{Cov\left[\tilde{r}_m, \frac{\tilde{P}_m^1 - E\left[\tilde{P}_m^0\right]}{E\left[\tilde{P}_m^0\right]}\right]}\left(E\left[\tilde{r}_m\right] - r_f\right) \\
&= \frac{Cov\left[\tilde{r}_j, \tilde{r}_m\right]}{Cov\left[\tilde{r}_m, \tilde{r}_m\right]}\left(E\left[\tilde{r}_m\right] - r_f\right) = \frac{Cov\left[\tilde{r}_j, \tilde{r}_m\right]}{Var\left[\tilde{r}_m\right]}\left(E\left[\tilde{r}_m\right] - r_f\right) \\
&= \beta_{j,m}\left(E\left[\tilde{r}_m\right] - r_f\right)
\end{aligned}$$

Here, $\beta_{j,m} = \frac{Cov\left[\tilde{r}_j, \tilde{r}_m\right]}{Var\left[\tilde{r}_m\right]}$ and we thus obtain the traditional CAPM expressed in terms of returns.

Table 1
Sample Size

The table shows the number of stocks in our data. Panel A sorts stocks by country. Panel B sorts the final sample of 5,781 stocks by industry. The table is based on cross-border holdings on December 31, 2002. Holdings data are from Thompson Financial. Price data are from Datastream.

Panel A: Number of Stocks by Country

	Country	Holdings Data	Holdings and Price Data	Holdings and X-Sec. Data	Holdings, Prices, and X-Sec. Data
1	Australia	695	587	307	293
2	Austria	91	78	65	60
3	Belgium	191	166	99	95
4	Denmark	162	124	111	97
5	Ireland	72	48	37	34
6	Finland	143	122	109	97
7	France	686	567	497	448
8	Germany	744	642	507	467
9	Greece	299	242	117	110
10	Hong Kong	196	160	157	150
11	Italy	302	243	219	189
12	Japan	2,676	2,370	2,216	2,108
13	Netherlands	198	137	117	109
14	New Zealand	79	70	41	41
15	Norway	178	106	107	88
16	Portugal	56	46	34	30
17	Singapore	274	233	214	200
18	Spain	687	121	118	105
19	Sweden	326	226	209	174
20	Switzerland	264	198	184	165
21	United Kindom	1,973	1,067	794	721
	TOTAL STOCKS	**10,292**	**7,553**	**6,259**	**5,781**

Table 1
Sample Size

Panel B: Industry Break-Down

Industry		Num. of Stocks	Industry		Num. of Stocks
1	Aerospace & Defense	23	20	Industrial Metals	100
2	Auto & Parts	160	21	Industrial Trans.	149
3	Banks	225	22	Leisure Goods	70
4	Beverages	76	23	Life Insurance	23
5	Chemicals	234	24	Media	219
6	Construction	372	25	Mining	64
7	Electricity	54	26	Mobile Telecom.	33
8	Electronic Equip.	315	27	Nonlife Insur.	66
9	Equity Investments	55	28	Oil & Gas Producers	51
10	Fixed Line Telecom.	26	29	Oil Equip. & Srvcs	18
11	Food & Drug Retail	79	30	Personal Goods	190
12	Food Producers	224	31	Pharm. & Biotech.	154
13	Forestry & Paper	47	32	Real Estate	252
14	General Financial	197	33	Software Services	474
15	General Indus.	110	34	Support Services	253
16	General Retailers	293	35	Tech. Equipment	246
17	Health Equipment	110	36	Tobacco	8
18	Household Goods	159	37	Travel & Leisure	230
19	Industrial Engin.	384	38	Utilities	38

Total Number of Stocks: 5,781

Table 2
Overview Statistics

The table shows the overview statistics for the main variables in our empirical analysis. Panel A shows each variable's cross-sectional mean, standard deviation, 25th, 50th, and 75th percentiles for the 5,781 stocks. Panel B shows correlation coefficients of the variables. "Foreign Holdings (Ω_j)" is the number of shares held by foreign funds divided by shares outstanding. "Foreign – Domestic Holdings (Ω^*_j)" is the number of shares held by foreign funds minus shares held by domestic fund all divided by shares outstanding. "*Asset Specific* $_j$" is a measure of the idiosyncratic part of stock *j*'s returns. The table is based on cross-border holdings on December 31, 2002. Holdings data are from Thompson Financial. Price data are from Datastream.

Panel A: Cross-Sectional Statistics

	Units	Mean	Stdev	25th Ptile	50th Ptile	75th Ptile
Foreign Hold (Ω_j)	%	2.76	5.19	0.14	0.59	3.22
For-Dom Hold (Ω^*_j)	%	(2.00)	8.43	(3.40)	(0.36)	0.39
Proxy: Asset Specific $_j$	--	0.85	0.13	0.79	0.89	0.95
Market Capitalization $_j$	$ bn	2.56	79.29	0.04	0.11	0.47
ln(Market Cap $_j$)	ln($)	18.75	1.96	17.41	18.55	19.97
Num. of Analysts $_j$	--	4.16	6.44	0.00	1.00	5.00
Leverage $_j$	--	0.57	0.36	0.39	0.57	0.73

Panel B: Correlation of Variables

	Ω_j	Ω^*_j	Asset Spec. $_j$	MktCap $_j$	ln(MktCap $_j$)	Num. Anal $_j$	Lev $_j$
Foreign Hold (Ω_j)	1.00						
For-Dom Hold (Ω^*_j)	0.52	1.00					
Proxy: Asset Specific $_j$	(0.21)	(0.11)	1.00				
Market Capitalization $_j$	0.03	(0.03)	(0.06)	1.00			
ln(Market Cap $_j$)	0.33	0.15	(0.25)	0.12	1.00		
Num. of Analysts $_j$	0.46	0.23	(0.37)	0.08	0.65	1.00	
Leverage $_j$	(0.01)	0.30	0.04	(0.00)	0.04	0.04	1.00

Table 3
Cross-Border Holdings and Double Sort Results

The table shows average cross-border holdings as a fraction of shares outstanding. We sort stocks into quartiles along two dimensions. The first sort uses our proxy for the information advantage about the asset specific component of a stock's returns. The second sort uses our proxy for the information advantage of foreign investors with respect to common factors. The table is based on cross-border holdings on December 31, 2002. Holdings data are from Thompson Financial.

		Info Advantage Common Factors			
		Low	**High**	**Diff**	*(T-stat)*
Info Advantage Asset Specific Components	**High**	0.0141	0.0159	0.0018	*(0.65)*
	Low	0.0358	0.0507	0.0149	*(2.34)*
	Diff *(T-stat)*	0.0217 *(6.25)*	0.0348 *(5.72)*		

Table 4
Cross-Border Holdings and Regression Results

The table shows cross-sectional regression results. The dependent variable in Regressions 1 – 3 is Ω_j which is the ratio of shares held by foreign funds divided by shares outstanding. The dependent variable in Regressions 4 – 7 is Ω_j^* which is the difference between shares held by foreign funds and domestic funds, all divided by shares outstanding. "*Asset Specific*$_j$" is our proxy for the information advantage about the asset specific component of a stock's returns. "*Asset Specific*$_j$ *(Sales)*" is a based on sales growth data instead of returns (methodology described in the text.) The table is based on cross-border holdings on December 31, 2002. Holdings data are from Thompson Financial. Price data are from Datastream.

	Reg. 1	Reg. 2	Reg. 3	Reg. 4	Reg. 5	Reg. 6	Reg. 7
Depend. Var.	Ω_j	Ω_j	Ω_j	Ω_j^*	Ω_j^*	Ω_j^*	Ω_j^*
Asset Specific $_j$	-0.0855	-0.0550	-0.0177	-0.0179			
	(-11.81)	*(-7.88)*	*(-3.19)*	*(-2.10)*			
Asset Spec $_j$ (Sales)					-0.0602	-0.0076	-0.0200
					(-13.69)	*(-1.98)*	*(-3.34)*
ln(MktCap$_j$)		0.0079	0.0015	-0.0002	-0.0011	0.0017	0.0010
		(23.76)	*(2.44)*	*(-0.29)*	*(-0.95)*	*(1.56)*	*(0.66)*
Num. Analysts$_j$			0.0033	0.0029	0.0032	0.0016	0.0025
			(12.18)	*(10.18)*	*(9.27)*	*(5.72)*	*(5.59)*
Leverage$_j$		-0.0025	-0.0037	0.0061	0.0110	0.0051	-0.0053
		(-0.86)	*(-1.29)*	*(2.81)*	*(2.12)*	*(1.16)*	*(-0.73)*
Constant	0.1006	-0.0724	0.0036	-0.0164	0.0159	Country	-0.0293
	(15.49)	*(-7.95)*	*(0.29)*	*(-1.06)*	*(0.75)*	Fix. Effects	*(-097)*
# Obs	5,781	5,781	5,781	5,781	3,095	3,095	1,346
Fit	0.0428	0.1261	0.2131	0.0544	0.1334	0.4636	0.0776

Table 5
Proxy for Factor Information and Regression Results

The table shows pairs of cross-sectional regression results. The first regression in the pairs considers stocks with low information advantages vis-à-vis the common factor (bottom 25%). The second regression in the pair considers stocks with high information advantages (upper 25%). The dependent variable in the first two pairs of regressions is Ω_j which is the ratio of shares held by foreign funds divided by shares outstanding. The dependent variable in third pair of regressions is Ω_j^* which is the difference between shares held by foreign funds and domestic funds, all divided by shares outstanding. The table is based on cross-border holdings on December 31, 2002. Holdings data are from Thompson Financial. Price data are from Datastream.

	Reg. 1a & 1b		**Reg. 2a & 2b**		**Reg. 5a* & 5b***	
	Info Advantage Common Factor		**Info Advantage Common Factor**		**Info Advantage Common Factor**	
	Low	**High**	**Low**	**High**	**Low**	**High**
Depend. Var.	Ω_j	Ω_j	Ω_j	Ω_j	Ω_j^*	Ω_j^*
Asset Specific $_j$	-0.0759	-0.1125	-0.0384	-0.0783	-0.0484	-0.0770
(T-stat)	*(-7.63)*	*(-4.93)*	*(-4.28)*	*(-3.32)*	*(-3.43)*	*(-3.31)*
ln(MktCap$_j$)			0.0084	0.0074	0.0045	0.0050
(T-stat)			*(12.51)*	*(12.65)*	*(4.56)*	*(5.85)*
Leverage$_j$			-0.0079	-0.0077	0.0194	0.0049
(T-stat)			*(-1.98)*	*(-1.87)*	*(3.05)*	*(0.69)*
Constant	0.0877	0.1253	-0.0970	-0.0403	-0.0736	-0.0477
(T-stat)	*(10.18)*	*(5.99)*	*(-6.71)*	*(-1.43)*	*(-3.30)*	*(-1.56)*
# Obs	1,445	1,445	1,445	1,445	1,445	1,445
Fit	0.0466	0.0570	0.1587	0.1131	0.0300	0.0373
F-Stat (Coef Diffs)	8.86		3.09		1.07	
(P-value)	*(0.0001)*		*(0.0149)*		*(0.3677)*	

Figure 1
World Market Capitalization

The figure shows the level of world market capitalization for different levels of information advantages vis-à-vis the asset-specific components of payoffs and the common factor related to payoffs. We consider a case with two assets (an American stock and a French stock) and two groups of investors (American people and French people.) Payoffs are generated by a one-factor linear model. Investors have asset-specific information about the asset from their home country. The American investors have information about the common factor. The X-axis represents different levels of information advantage about the home-country assets. The four curves (labeled 0, 2, 4, 10) represent four levels of information advantage (for the American investor) vis-à-vis the common factor. The bottom graph line (thin, purple, with ``O" markings) represents the highest levels of information advantage about the common factor. Details of the numerical analysis are given in the text.

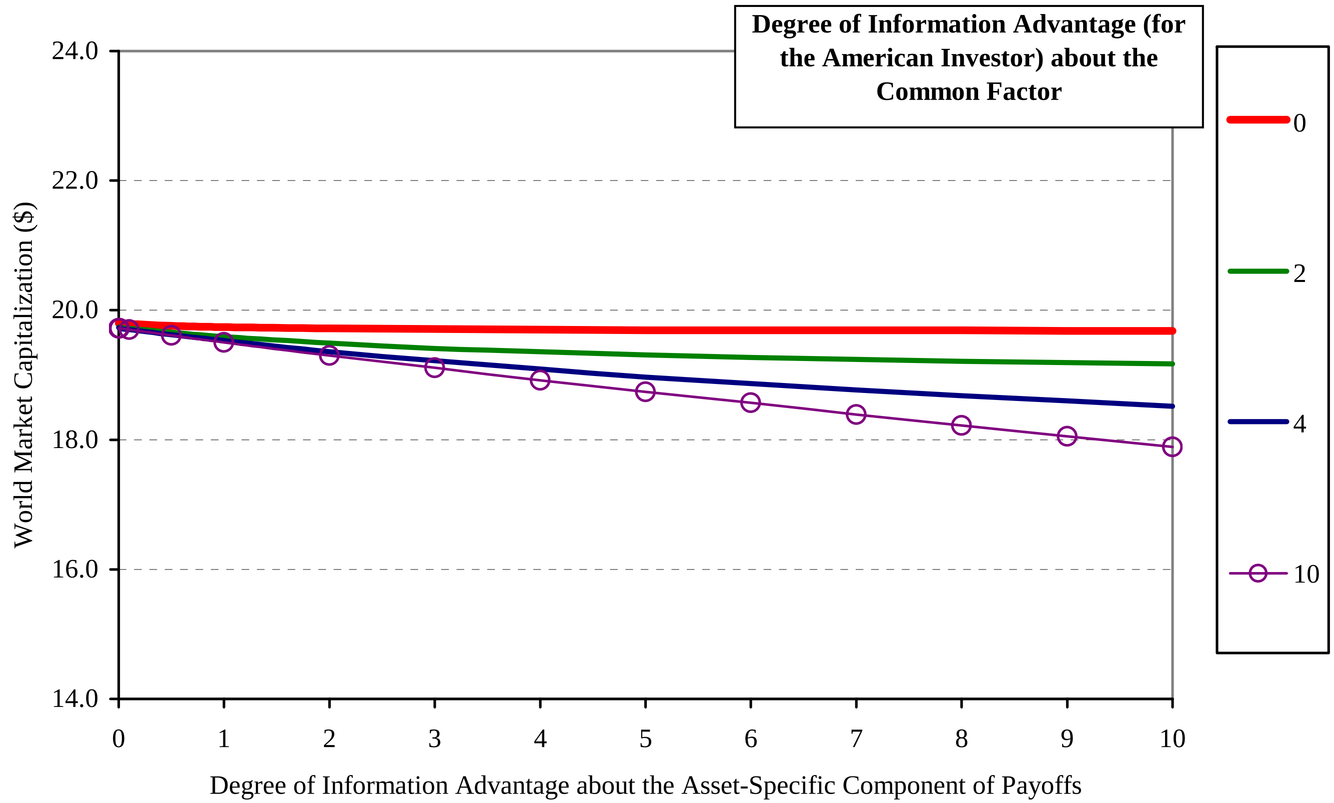

Figure 2
Weight of American Assets in the World Market Portfolio

The figure shows the weight of American assets in the world market portfolio for different levels of information advantages vis-à-vis the asset-specific components of payoffs and the common factor related to payoffs. We consider a case with two assets (an American stock and a French stock) and two groups of investors (American people and French people.) Payoffs are generated by a one-factor linear model. Investors have asset-specific information about the asset from their home country. The American investors have information about the common factor. The X-axis represents different levels of information advantage about the home-country assets. The four curves (labeled 0, 2, 4, 10) represent four levels of information advantage (for the American investor) vis-à-vis the common factor. The top graph line (thin, purple, with ``O" markings) represents the highest levels of information advantage about the common factor. Details of the numerical analysis are given in the text.

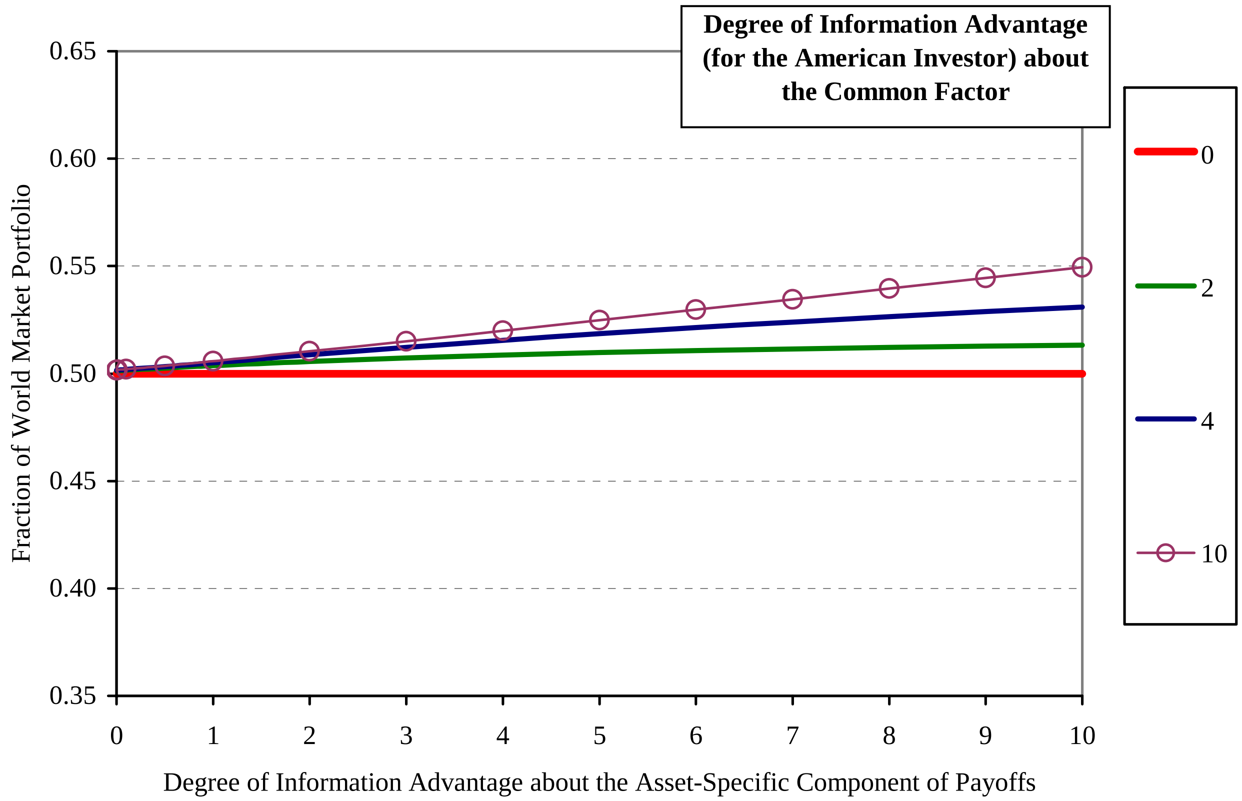

Figure 3
Home Bias and Reverse Home Bias

The figure shows the degree of home bias in the American investor's portfolio for different levels of information advantages vis-à-vis the asset-specific components of payoffs and the common factor related to payoffs. We consider a case with two assets (an American stock and a French stock) and two groups of investors (American people and French people.) Investors have asset-specific information about the asset from their home country. The American investors have information about the common factor. The X-axis represents different levels of information advantage about the home-country asset. The four curves (labeled 0, 2, 4, 10) represent four levels of information asymmetry about the common factor. The bottom graph line (thin, purple, with ``O" markings) represents the highest levels of asymmetry about the common factor. Details of the numerical analysis are given in the text.

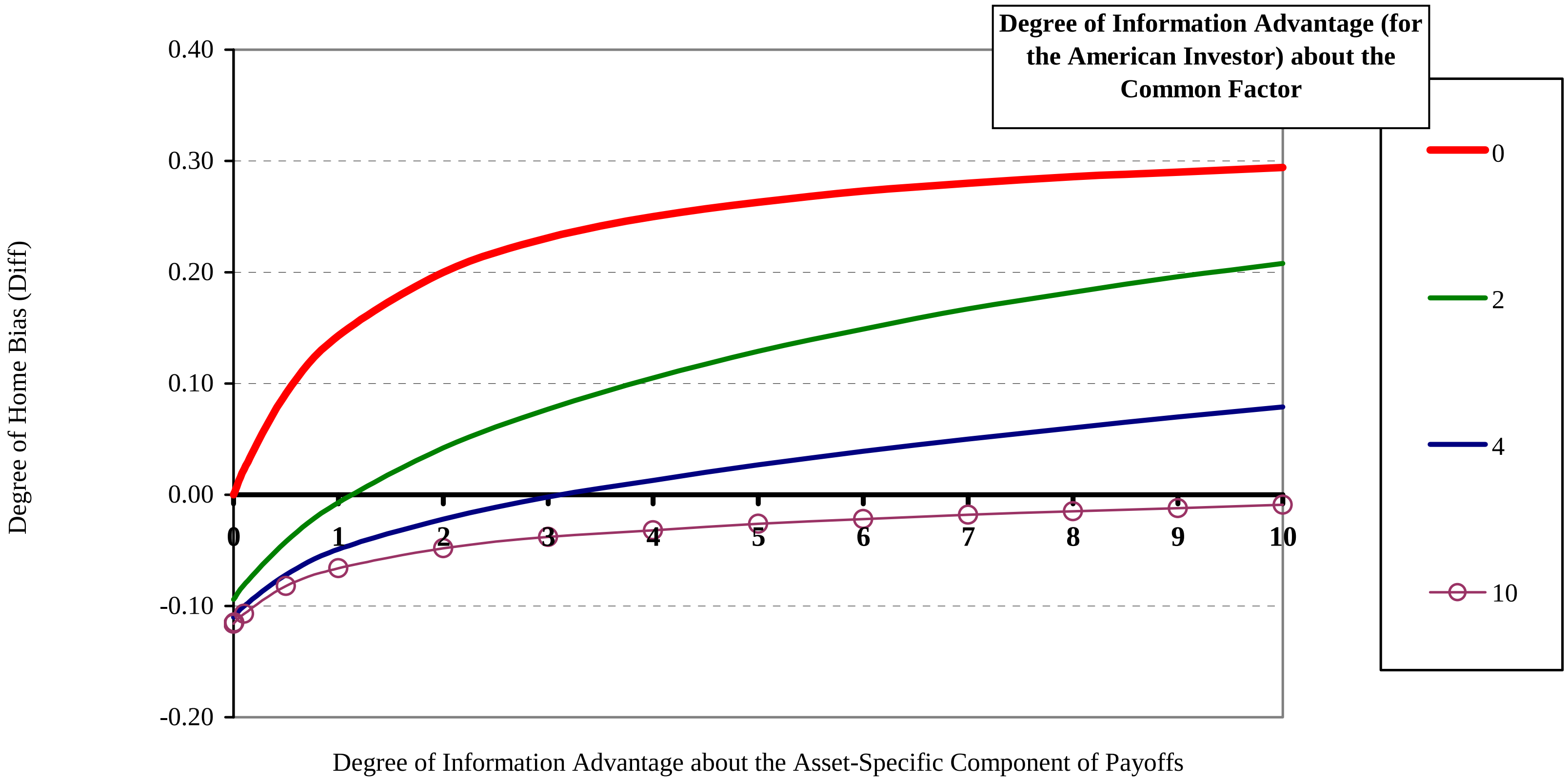

Figure 4
Anomalous Price Behavior

The figure shows the price reaction of the French stock to changes in the asset-specific component of the French asset's payoffs. A negative value indicates times when good news about future payoffs is followed by drops in the French asset's price. The numerical analysis considers two assets with two groups of investors and one common factor as described in the text. The X-axis shows the degree of information asymmetry about each asset. The four lines show price reactions for four different levels of information asymmetry about the common factor. The bottom graph line (thin, purple, with ``O" markings) represents the highest levels of asymmetry about the common factor.