KNOWHOW, CORE COMPETENCIES, AND THE CHOICE BETWEEN MERGING, ALLYING, AND BUYING ASSETS

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Abstract

We characterize the conditions under which two firms choose to (i) merge, (ii) form an alliance, or (iii) trade assets. For that purpose, we distinguish between the firms' assets, their knowhow, and their core competencies. We show that a merger is chosen when the two firms have similar core competencies. When one firm has markedly higher core competencies than the other, that firm operates the assets separately if it also has markedly higher knowhow. Finally, an alliance is chosen when the firm with markedly higher core competencies has markedly lower knowhow.

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1 Introduction

The late nineteen-nineties saw a very large number of mergers, alliances, and asset sales.[1] These three mechanisms for combining, to a greater or lesser extent, part or all of the assets of two or more firms have been extensively studied.[2] However, there thus far appears to be relatively little work on the determinants of a firm's choice among these three mechanisms. This is the object of the present paper. We wish to understand under what circumstances and for what reasons a firm will choose, say, to merge with another firm rather than enter into an alliance with that firm or simply to purchase one or more of the tangible or intangible assets of that firm.

More generally, we answer the following questions in the present paper: When do two firms merge, ally, or trade assets? If assets are traded, which assets are traded and which assets are not? Which firm buys the assets that are traded and which firm sells? If an alliance is entered into, which assets are contributed to the alliance by the allying firms? Is an alliance dissolved? When? Which partner in the alliance buys out the other at dissolution? When a merger involves two firms with different organizational forms, does one form eventually supersede the other? Which form? When?

In order to answer these questions, we develop a model that allows us to capture what we believe are the main forces at work in a context such as the one we consider, and to analyze how the balance of these forces is altered by a merger, an alliance, or the sale of one or more assets. We distinguish between a firm's one or many assets, the firm's knowhow, and the core competencies of the firm. The firm uses its knowhow and its core competencies to add value to its assets.

Knowhow and core competencies are part of what Maksimovic and Phillips (2002) refer to as a firm's skills. We draw the following distinction between knowhow on the one hand and core competencies on the other. Consider two firms that jointly operate an asset within an alliance, or a single merged firm within which the two organizations corresponding to the original merging firms jointly operate that same asset. Some skills can be combined for the purpose of jointly operating the asset. Other skills cannot. Skills that are combined can be observed; they can therefore be acquired. Skills that are not combined can neither

[1] Dyer, Kale, and Singh (2004) report that, over the six-year period 1996-2001, American firms announced 74,000 acquisitions and 57,000 alliances. Maksimovic and Phillips (2001) consider only manufacturing plants reallocated over the longer period 1974-1992. They find that 17,729 plants were reallocated through mergers and acquisitions, whereas 17,571 plants were reallocated through asset sales. Asset sales differ from mergers and acquisitions in that the selling firm remains in existence in the former but not the latter case.

[2] We give only a few references for the sake of brevity. For merger and acquisitions, see for example Andrade, Mitchell, and Stafford (2001) and Gorton, Kahl, and Rosen (2005). For alliances, see for example Hauswald and Hege (2002) and McConnell and Nantell (1985). For asset transfers, see for example Eisfeldt and Rampini (2004) and Maksimovic and Phillips (2001).

be observed nor acquired. We call the first set of skills knowhow and the second core competencies.[3]

Assets can be traded in isolation. Knowhow and core competencies cannot, because they are in the nature of tacit knowledge. Tacit knowledge is known but cannot be explained (Polanyi, 1966). It is therefore difficult to codify explicitly. Instead, it is embedded in the organization of the firm, the informal processes and procedures that structure and coordinate the workings of the firm, and the culture and the spirit that animate and underpin these. Consequently, knowhow and core competencies can only be traded as part and parcel of the firm

Mergers, alliances, and asset sales combine assets, knowhow, and organizational capital in different ways. In an asset sale, the buyer of the asset uses its own knowhow and organizational capital to add value to the asset it has bought. The buyer of the asset has no access to the knowhow and core competencies of the seller.

Alliances differ from asset sales in that they provide the partners in the alliance with access to each other's knowhow. Assets operated jointly within the alliance have direct access to the entirety of the partners' combined knowhow. Assets operated separately without do not. Instead, assets operated separately by a given partner have indirect access to the partners' combined knowhow through the knowhow the given partner acquires in the course of joint operations within the alliance. Partners in an alliance enjoy learning externalities throughout the life of the alliance.

There are, however, costs as well as benefits to the alliance. The first cost stems from double moral hazard. Because the two partners share the benefits from the alliance, their incentive to contribute resources to the alliance is commensurate to their share of these benefits, and likely falls short of the resource contributions that would maximize the total benefits from the alliance.

The second cost is perhaps less familiar. Recall that the knowhow and the core competencies of the partners in the alliance are embedded in the partners' organizations. Combining knowhow necessarily requires getting organizations to work together. But different organizations that work together generally embed different core competencies which, unlike knowhow, cannot be combined. As a result, in an alliance, the partner whose organization embeds the higher core competencies is somewhat penalized by having to work together with the partner whose organization embeds the lower core competencies: this is the price to be paid for the former partner's access to the latter partner's knowhow. The difference between core competencies gives rise to the second cost of the alliance.[4] We refer to this cost as the cost

[3]This appears to be in accordance with the concept of core competencies as introduced by Prahalad and Hamel (1990).

[4]The problems encountered in the attempt by large, well-established Ciba Geigy and small, fledgling

of CCD (core competencies difference).

Mergers serve to combine within the same firm knowhow that was originally separate in two different firms. All assets are operated jointly in a merger; all assets therefore have direct access to the entirety of the merging firms' combined knowhow.

There is no cost of double moral hazard in a merger. This is because a single management team makes resource allocation decisions in a merger, rather than the two management teams of the two separate parent firms in an alliance. This is an advantage of a merger over an alliance. A disadvantage of the merger over the alliance is the greater cost of CCD in a merger. The reason is that the alliance can be made to involve only a subset of assets, whereas the merger involves all assets. In other words, alliances may involve divisions of a firm, whereas mergers generally involve entire firms. This means that the cost of CCD can be limited to a subset of assets in an alliance, whereas it necessarily extends to all assets in a merger.[5]

Which of a merger, an alliance, or an asset sale is chosen by the two firms naturally depends on the relative costs and benefits of the three forms of organizations. A merger is chosen when the two firms have similar core competencies: there is little cost of CCD in a merger in such case, no double moral hazard, and access to combined knowhow for all assets. When one firm has markedly higher core competencies than the other, that firm operates the assets separately if it also has markedly higher knowhow: there is little to be gained and much to be lost from joint operations in such case. Finally, an alliance is chosen when the firm with markedly higher core competencies has markedly lower knowhow: the alliance then serves to confine the cost of CCD to the single asset operated jointly, yet makes accessible to the assets operated separately the knowhow acquired from joint operations within the alliance.

We show that joint operations, whether within an alliance or a merged firm, always must come to an end. The intuition is as follows. The benefit of joint operations is in combining knowhow. As knowhow gradually is acquired over the period of joint operations, the benefits of combining knowhow correspondingly decrease. There comes a point in time at which the benefit of joint operations–combining knowhow–becomes smaller than its costs–double moral hazard for the alliance and CCD for both the alliance and the merger.

Joint operations within an alliance end with the buyout of one partner by the other.

Alza jointly to develop advanced drug delivery systems (Doz, 1996) are an example of the high costs of the difference between core competencies.

[5]Note that we do not explicitly include acquisitions in our analysis. This is because we view an acquisition as akin either to a merger or an asset sale. An acquisition is akin to an asset sale when the acquiror imposes its own structure and organization on the acquiree. In so doing, the acquiror foregoes the knowhow and the core competencies embedded in the acquiree's structure and organization, but avoids the costs of CCD. An acquisition is akin to a merger when the acquiror conserves the acquiree's structure and organization.

Joint operations within a merged firm end with the superseding of one organizational form by the other; the two organizational forms are those of the original merging firms. An asset henceforth is operated with the knowhow and the core competencies of a single firm, one of the original two firms. The knowhow of that firm nonetheless has been increased by the process of knowhow acquisition that has taken place during the period of joint operations, be that within an alliance or a merged firm.

Which partner buys out the other in an alliance, and which organizational form supersedes the other in a merger depend on the allying firms' combined knowhow and core competencies in an alliance, and on the merging firms' core competencies alone in a merger. In the latter case, joint operations within the merged firm do not end before the original firm with higher core competencies has acquired enough knowhow to become the best user of the assets, in case it was not so at the time of the merger. In an alliance, joint operations may end before the allying firm with higher core competencies has acquired enough knowhow to become best user, if the cost of double moral hazard makes such acquisition prohibitively expensive.

We show that an asset that is operated jointly within an alliance is always 'sacrificed,' in the sense that the asset itself would more profitably be operated separately without the alliance than jointly within. The intuition is as follows. Suppose that an asset is more profitably operated jointly within the alliance. Then, the benefit of joint operations within the alliance–access to combined knowhow–dominates the costs of joint operations–double moral hazard and CCD. This implies that the benefit of access to combined knowhow dominates the cost of CCD. This is true for all assets: access to knowhow and CCD pertain to knowhow and core competencies, respectively; both are firm-wide characteristics.[6] As a result, all assets should be operated jointly, i.e., the two original firms should merge. Thus, if one asset is operated more profitably within an alliance than separately without, a merger dominates the alliance.

Now consider the case where no asset whatsoever is more profitably operated jointly within the alliance than separately without. Somewhat counter-intuitively, it is precisely in such a case that an alliance may dominate a merger. Operating a single asset jointly within the alliance confines the cost of CCD to that single asset. This has a value when, unlike in the previous case, the cost of CCD is not necessarily dominated by the benefit of access to combined knowhow. The alliance may be chosen over separate operations because it makes accessible to the assets operated separately without the alliance the knowhow acquired through joint operations within.

There are three main implications to this finding. The first implication is that two firms entering into an alliance will seek to put into the alliance as few assets as possible, subject to

[6]This is unlike double moral hazard which pertains to asset-specific resource contributions.

the requirement that the joint operation of these assets makes possible the desired acquisition of knowhow. The second is that alliances are more likely to be entered into by large firms than by small firms: large firms have more separately operated assets that can profit from the knowhow acquired in the alliance; they also have a wider pool of assets from which to choose the asset or assets to be 'sacrificed.' The third implication is that an evaluation of the alliance that would fail to account for the benefits for the operations without the alliance of the knowhow acquired within invariably would suggest that the alliance destroys value.

To summarize, we provide the following answers to the questions we have asked. (i) Two firms merge when they have similar core competencies. (ii) One firm buys the assets of the other when the first firm dominates the second both in core competencies and in knowhow. Finally, (iii) the two firms ally when one firm dominates in core competencies and the other in knowhow. (iv) As few assets as possible are contributed to an alliance. Ideally, a single asset–the least valuable asset that suffers the least from double moral hazard–is contributed. (v) Both mergers and alliances come to an end. (vi) An alliance ends with the buyout of one partner by the other. Whether the buying partner is that with higher core competencies or higher knowhow–when these differ–depends on the cost of double moral hazard: (vii) the higher the cost of double moral hazard, the more likely it is that the alliance is dissolved before the partner with higher core competencies has acquired enough knowhow to become the best user of the asset within the alliance. (viii) A merger ends with the superseding of one organizational form by the other; the two organizational forms are those of the original merging firms. (ix) The merger always lasts long enough for the original merging firm with higher core competencies to see its organizational form dominate.

As noted above, relatively few papers have compared mergers, alliances, and asset sales. Villalonga and McGahan (2005) have compared acquisitions, alliances, and divestures. Margsiri, Mello, and Ruckes (2005) have compared growth via acquisition and organic growth. Dyer, Kale, and Singh (2004) have compared alliances and mergers. Balakrishnan and Koza (1993), Hennart and Reddy (1997), and Reuer and Koza (2000) have compared joint ventures, a form of alliance, with a form of acquisition that has generally been akin to an asset sale under our classification. Pisano (1989) and Oxley (1997) have compared various forms of alliances. All these papers have provided numerous valuable insights. Many have considered various issues that we do not, such as asymmetric information for example. However, none has explicitly examined the acquisition of knowhow, which plays a central role in our analysis.[7]

Our paper is in the line of a series of papers which have applied continuous-time techniques to problems of corporate finance.[8] This literature was given its early impetus by Leland

[7]The same holds true of what is probably the foremost theory of vertical and lateral integration, that of asset specificity (see Williamson, 1985; Grossman and Hart, 1986; and Hart and Moore, 1990).

[8]For a nice survey of continuous time finance, see Sundaresan (2000).

(1994). Bernardo and Chowdhry (2002) study the pattern of firm growth. Lambrecht (2004) and Morellec and Zhdanov (2005) examine the timing of takeovers. Lambrecht and Myers (2005), examine a firm's closure policy under the threat of takeover. Leland and Skarabot (2005) compare mergers and separate operations with an emphasis on the tax benefits of debt. Hackbarth and Morellec (2006) study stock returns in mergers and acquisitions. Finally, Morellec and Zhdanov (2006) study the financing of takeovers.

Our finding that joint operations within a merged firm must come to an end recalls Fluck and Lynch's (1999) finding that mergers are followed by divestures.[9] Unlike the firms of Fluck and Lynch (1999), however, ours have no wealth constraint and need not engage in mergers for the sole purpose of overcoming such constraints.

From a modeling standpoint, our model builds on Habib and Mella-Barral (2006), which incorporates knowhow acquisition in a continuous time corporate finance model. To study the choice between mergers, alliances, and assets sales, we consider multiple assets; render explicit the concept of core competencies; model the learning externalities that benefit assets operated separately when other assets are operated jointly; and identify the subset of assets optimally to be operated separately and those jointly.

The paper proceeds as follows. We present the model in Section 2. We solve the model for the three cases of wholly unrelated, merged, and partially related operations in Section 3, and determine the optimal pattern of operation. We further analyze our solution and derive various comparative statics results in Section 4. We provide some empirical evidence in Section 5. We conclude in Section 6.

2 The Model

2.1 Basic Setting and Objectives

Consider a set $\mathcal{J}$ of assets $j \in \{1; \ldots ; J\}$ that can be owned and operated by one or both of two firms a and b. The set $\mathcal{J}$ can be divided into three non-overlapping subsets $\mathcal{J}_a$, $\mathcal{J}_b$, and $\mathcal{J}_{ab}$, where $\mathcal{J}_i$ consists of the assets that are operated by firm i alone, $i \in \{a; b\}$, whereas $\mathcal{J}_{ab}$ consists of the assets that are operated by the two firms jointly.[10] At the initial date $t = 0$, no asset is operated jointly, i.e., $\mathcal{J}_{ab} = \emptyset$. Any asset can be traded between the two firms at any date $t \geq 0$.

At any date $t \geq 0$, an asset can be operated solely by a single firm or jointly by the two firms. The operation by firm i of asset j yields instantaneous revenue $R_j(.)$ at instantaneous cost $C_{i,j}(.)$ to firm i. All assets may be made worthless by a common exogenous shock at

[9] See also Kaplan and Weisbach (1992).

[10] $\mathcal{J}_a \cup \mathcal{J}_b \cup \mathcal{J}_{ab} = \mathcal{J}$ and $\mathcal{J}_a \cap \mathcal{J}_b = \mathcal{J}_a \cap \mathcal{J}_{ab} = \mathcal{J}_b \cap \mathcal{J}_{ab} = \emptyset$.

some random date $\bar{t}$, modeled as a stopping time with constant intensity $\lambda \in \mathbf{R}^{+*}$.[11]

The focus of the present paper is on the pattern of asset operation by the two firms a and b. An asset can be operated by one firm alone or by the two firms jointly. Joint operation may involve all assets or only a subset of assets.

- One polar case is where the two firms' operations are wholly unrelated. All assets are operated separately by one or the other firm, and no asset is operated jointly by the two firms. We have $\mathcal{J}_{ab} = \emptyset$.
- The opposite polar case is where the two firms' operations have been merged. Within the merged firm, all assets are operated jointly by the two organizations that correspond to the original merging firms. We have $\mathcal{J}_{ab} = \mathcal{J}$. Joint operations within a merged firm may end.
- An intermediate case is where the two firms' operations are partially related by means of an alliance. Some assets are operated jointly within the alliance, and the remaining assets are operated separately by one or the other firm. We have $\mathcal{J}_{ab} \neq \emptyset$, and assets in the remaining subsets, $\mathcal{J}_i$ for $i \in \{a; b\}$, are operated separately. The alliance can be dissolved.[12]

We view the pattern of asset operation as determined by the interplay between three main forces. First, the dynamics of firm knowhow: joint operations (i) yield synergies that arise from the aggregation of the two firms' knowhow and (ii) make possible the acquisition by each firm of part of the other firm's knowhow. Second, the problem of double moral hazard: each of the two firms jointly operating one or more assets within an alliance makes its resource allocation decisions so as to maximize the value of its own stake in the alliance, rather than the entire value of the alliance. The problem of double moral hazard arises neither under separate operation of an asset, nor under merged operations. In the latter case, a single management team makes its resource allocation decisions so as to maximize the value of the entire firm born of the merger of the two original firms. Third, the difficulties that arise from having different organizations work together. Different organizations have different core competencies, and an organization with higher core competencies is somewhat penalized by working together with an organization with lower core competencies. CCD (core competencies difference) affects all assets in a merger, whereas it generally affects only a subset of assets in an alliance. CCD ends with the ending of joint operations.

[11] This means that, at any date $t < \bar{t}$, the probability of liquidation before $t + \Delta$ is approximately $\lambda\Delta$. See Artzner and Delbaen (1990), Lando (1994), and Jarrow and Turnbull (1995) for further details.

[12] Note that an alliance in which *all* assets are operated jointly within the alliance, i.e., such that $(\mathcal{J}_{ab}, \mathcal{J}_a, \mathcal{J}_b) = (\mathcal{J}, \emptyset, \emptyset)$, is not equivalent to a merger. This is because, as will be discussed in the following paragraph, an alliance suffers from a problem of double moral hazard whereas a merger does not.

A comment about terminology is in order at this point. When all assets are operated jointly within a merged firm, it is not quite correct to speak of the assets as operated by the two firms a and b, for a merger combines these firms into a single firm. It is however correct to speak of the asset as operated by two organizations, those corresponding to the original merging firms a and b. A merger in which the assets are operated by the organization of only one of the two merging firms is in fact an asset sale: an organization embeds knowhow; the superseding of an organization is the foregoing of its knowhow. Because the term 'firm' is more natural in some settings and the term 'organization' in others, we shall use both terms in this paper.

2.2 Detailed Specification of the Model

We now turn to the detailed specification of the model. The operation of asset $j \in \mathcal{J}$ at date $t \in [0; \bar{t}]$ yields instantaneous revenue

$$R_j(\mathbf{e}(t)) \;=\; g_j(\mathbf{e}(t))^{\gamma_j} \;, \tag{1}$$

at instantaneous cost to firm $i \in \{a; b\}$ operating the asset alone or jointly with the other firm

$$C_{i,j}(e_{i,j}(t)) \;=\; \frac{e_{i,j}(t)}{\left[h(\mathbf{k}(t))\,\kappa_i\right]^{\frac{1-\gamma_j}{\gamma_j}}} \;, \tag{2}$$

where $\mathbf{e}(t) \equiv (e_{i,j}(t))$ and $e_{i,j}(t) \in \mathbf{R}^+$ denotes the resource contribution made by firm i towards asset j at date t; $\gamma_j \in (0,1)$ indexes the dependence of profits on resources as compared to knowhow and core competencies (see below); $\mathbf{k}(t) \equiv (k_i(t))$ and $k_i(t) \in \mathbf{R}^{+*}$ denotes the knowhow of firm i at date t; and $\kappa_i \in \mathbf{R}^{+*}$ denotes the core competencies of firm i. In words, the revenues obtained from operating an asset and the costs of operating the asset increase in the resources contributed to the asset. Higher skills on the part of a firm contributing resources to the asset decrease the cost of contributing such resources. Higher skills may take the form of higher knowhow or of greater core competencies.[13]

Revenues vary depending on whether the asset is operated separately by a single firm or jointly by the two firms. Specifically, we have

$$g_j(\mathbf{e}(t)) \;=\; \begin{cases} e_{i,j}(t) & \text{if firm } i \text{ operates alone}\;, \\ \left(\delta_a^{\delta_a}\,\delta_b^{\delta_b}\right)^{-1}\; e_{a,j}(t)^{\delta_a}\; e_{b,j}(t)^{\delta_b} & \text{if the two firms operate jointly}\;. \end{cases} \tag{3}$$

The constant $\delta_i \in (0,1)$, which appears only in the case of joint operation of the asset, indexes the dependence of the revenues generated by asset j on firm i's contribution relative

[13]Note that knowhow may change over time, whereas core competencies do not. We return to the distinction between knowhow and core competencies below.

to that of the other firm. We have $\delta_a + \delta_b = 1$. The factor $\left(\delta_a^{\delta_a}\delta_b^{\delta_b}\right)^{-1}$ ensures that no built-in advantage is artificially conferred to either joint or separate operations.[14]

As do revenues, knowhow differs depending on whether the asset is operated separately or jointly. Specifically, we have

$$h(\mathbf{k}(t)) \;=\; \begin{cases} k_i(t) & \text{if firm } i \text{ operates alone }, \\ \bar{k} & \text{if the two firms operate jointly }, \end{cases} \tag{4}$$

where $\bar{k} \in \mathbf{R}^{+*}$ denotes the two firms' aggregate knowhow. We have $\bar{k} \leq k_a(t) + k_b(t)$.[15] To keep the analysis focused, we assume aggregate knowhow remains constant over time. With (4), joint operations yields synergies that arise from the aggregation of the two firms' knowhow. In the absence of such synergies ($k_a(0) = k_b(0) = \bar{k}$), (3) ensures there are no benefits to joint operations.

When two firms are engaged in joint operations, each firm's knowhow changes over time as each firm acquires part of the other firm's knowhow. We write firm i's knowhow at a date t as the sum of its initial knowhow and the knowhow it gained while engaged in joint operations

$$k_i(t) \;=\; k_i(0) \;+\; k_i^{+}(t) \;. \tag{5}$$

We impose a number of conditions on the gain function, $k_i^{+}(t)$. These conditions are intended to capture the following:

(i) A firm's knowhow never decreases; hence, the gain in knowhow is a non-decreasing function of time.

(ii) A firm's knowhow does not jump through time; hence, the gain in knowhow is a continuous function of time.

(iii) A firm's knowhow initially equals $k_i(0)$; hence, the gain in knowhow initially equals zero.

(iv) A firm's knowhow is at most equal to the two firms' combined knowhow, $\bar{k}$; hence, the knowhow gained by a firm is at most equal to the difference between the two firms' combined knowhow and that same firm's initial knowhow.

(v) The acquisition of knowhow is an uncertain process; hence, the gain in knowhow is

[14] For a given total resource contribution, $e_{a,j}(t) + e_{b,j}(t) = \bar{e}_j(t)$, revenues under separate operations equal those under joint operations with optimal contributions by the two firms: when a single firm operating the asset alone contributes resources $\bar{e}(t)$, we have $R_j(\mathbf{e}(t)) = \bar{e}_j(t)^{\gamma_j}$; when firm i jointly operating the asset with the other firm contributes a share δ_i of the same total resource contribution, $\bar{e}(t)$, we also have $R_j(\mathbf{e}(t)) = \bar{e}_j(t)^{\gamma_j}$. Resource contributions in the ratios $e_{a,j}(t)/\bar{e}(t) = \delta_a$ and $e_{b,j}(t)/\bar{e}(t) = \delta_b$ are optimal in that $R_j(\mathbf{e}(t)) = \bar{e}(t)^{\gamma_j}$ for these two ratios whereas $R_j(\mathbf{e}(t)) < \bar{e}(t)^{\gamma_j}$ for any other ratios.

[15] The equality $\bar{k} = k_a(t) + k_b(t)$ corresponds to the extreme case of no overlap between the two firms' knowhow.

an increasing function of a stochastic state variable, $\hat{x}_t$, that reflects how favorable learning conditions are.

(vi) A firm's knowhow is constant under separate operations; hence, the gain in knowhow is constant under separate operations.

A simple functional form that conforms to the preceding conditions is[16]

$$k_i^+(t) \; = \; \left(\bar{k} \; - \; k_i(0)\right) \left[1 \; - \; \frac{1}{\hat{x}_t}\right] \; . \tag{6}$$

In order to satisfy conditions (i), (ii), and (iv), the state variable, $\hat{x}_t$, that reflects learning conditions must be positive, continuous, and non-decreasing through time. Condition (iii) is satisfied by imposing $\hat{x}_0 = 1$. Condition (vi) is satisfied by requiring $\hat{x}_t$ to be constant under separate operations.

We take $\hat{x}_t$ to be the historical maximum of an upwards drifting geometric Brownian motion, x_t, over the intervals of time, $\mathcal{T}_t$, at date t, during which the firms have operated jointly one or more assets

$$\hat{x}_t \equiv \max_{\tau \in \mathcal{T}_t} x_\tau \; , \quad \text{where } x_0 \; = \; 1 \; , \quad dx_t \; = \; \mu \, x_t \, dt \; + \; \sigma \, x_t \, dB_t \; , \tag{7}$$

$(\mu, \sigma) \in \mathbf{R}^{+*} \times \mathbf{R}^{+*}$, and B_t denotes a standard Brownian motion. We further assume that x_t is constant under separate operations.[17]

Unlike knowhow, core competencies can be neither aggregated nor acquired. Thus, some but not all the skills of a firm can be combined and acquired. We call knowhow the skills that can, and core competencies those that cannot.

Throughout, we assume that there are no asymmetries of information, that capital markets are frictionless, that agents are risk neutral, and that they may borrow and lend freely at the constant, risk-free rate of interest, $r_0 \in \mathbf{R}^{+*}$.

To summarize, the following parameters characterize the model: $\{k_i(0); \bar{k}; \kappa_i; \delta_i\}$ for each firm $i \in \{a; b\}$; $\{\gamma_j\}$ for each asset $j \in \mathcal{J}$; $\{\mu; \sigma\}$ for learning conditions; and $\{r_0; \lambda\}$ for external economic conditions.

2.3 Contracts and Institutional Arrangements

Initial payments: Regardless of whether a merger, an alliance, or an asset sale is chosen by the two firms, a payment must generally be made from one firm to the other. In an asset sale, the seller of the asset must be compensated for selling the asset. Such compensation

[16] In addition, the functional form (6) delivers closed form solutions and finite resource contributions.

[17] This ensures that, in accordance with condition (ii), there is no jump in knowhow in case joint operations should start at a date $t > 0$.

generally takes the form of a cash payment. In an alliance, the owner of the asset that is to be operated jointly must be compensated for bringing the asset into the alliance. Furthermore, the firm that is expected to make the greater contribution to the alliance, by virtue of its greater knowhow or core competencies, must be compensated by the other firm for making that greater contribution. The same considerations apply in a merger, but whereas the compensation is generally paid in cash in an alliance, it may be paid in shares in a merger. Whether in cash or in shares, the compensation is likely to be bargained over by the parties. We assume costless bargaining. We adopt the generalized Nash bargaining solution, and denote β_i, $0 \leq \beta_i \leq 1$, the bargaining power of firm i, with $\beta_a + \beta_b = 1$.

Sharing of revenues: We assume the revenues from each individual asset, $R_j(\mathbf{e}(t))$ for $j \in \mathcal{J}$, are both observable and verifiable. In contrast, the resources contributed to the operation of the assets, $\mathbf{e}(t)$, are neither observable nor verifiable. Finally, we assume that the state variable, x_t, is observable but not verifiable. In an alliance, the two firms taking part in the alliance are therefore limited to writing contracts that are conditioned on revenues. We consider contracts that promise each firm a constant share $\phi_{i,j}$, $0 < \phi_{i,j} < 1$, of the revenue from each individual asset operated jointly within the alliance, $R_j(\mathbf{e}(t))$ for $j \in \mathcal{J}_{ab}$, with $\phi_{a,j} + \phi_{b,j} = 1$.[18] In a merger, as the original two firms have become a single firm whose resource allocation is decided by a single management team, revenue shares have no incentive effect. Instead, ownership shares in the merged firm compensate the merging firms for the contributions they have made to the merger. We assume that each merging firm owns a constant share ϕ_i, $0 < \phi_i < 1$, of the merged firm, with $\phi_a + \phi_b = 1$.[19]

Rules for dissolving the alliance: The possibility of dissolving an alliance suggests the need for a contract that sets the terms of dissolution. In particular, the contract must specify the manner in which the value of each asset $j \in \mathcal{J}_{ab}$, hitherto operated jointly within the alliance but now to be operated separately by one or the other firm, must be divided between the two firms. We consider dissolution rules that make the payoffs to each firm an affine function of the value of each asset to the firm that can best use that asset. We impose the constraint that these rules be renegotiation-proof, as regards both the time of dissolution and the price to be paid by one partner to the other at that time. This is in order to avoid the distortion to resource contributions that would otherwise arise from ex post bargaining over time and price.[20] Thus, although our model allows for renegotiation over both the time of dissolution

[18] Note that the inequalities are strict as each firm must be induced to make some non-zero contribution to the operation of the asset.

[19] Were a merging firm to be compensated exclusively with cash, the inequalities would be weak rather than strict. Without loss of generality, we assume that both merging firms own a stake in the merged firm.

[20] Note that there is no such concern in a merger. The end of joint operations within a merged firm is decided upon by the single management of the firm, whereas the end of joint operations within an alliance–the dissolution of the alliance–is negotiated over by the distinct managements of the allied firms.

and the price to be paid at such time, the exit contract we derive is such as to deny the partners the incentives to engage in such renegotiation.[21]

3 Solving the Model

In this section we first examine the value of the assets when the two firms' operations are (i) wholly unrelated, (ii) merged, and (iii) partially related. We then compare these three values for the purpose of determining the pattern of operation chosen at the outset. We further analyze the solutions obtained in Section 4.

Besides being a prelude to Section 4, the present section is important in its own right, for it characterizes the dynamics of joint operations. In particular, it establishes the result that joint operations always must end, to be abandoned for separate operations. The section computes the state in which joint operations end, and identifies the best user of the hitherto jointly operated asset or assets at the end of joint operations. Interestingly, that best user need not be the firm with higher core competencies in an alliance, although it is in a merger.

3.1 One Polar case: Wholly Unrelated Operations

This first section assumes that the separate operation of *all* assets $j \in \mathcal{J}$ is the superior pattern of operation at the date $t_0 \geq 0$. As knowhow is constant under separate operations, these remain the superior form of organization for all $t \geq t_0$.

At date the $t \geq t_0$, the value to firm i of operating asset j separately is

$$V_{i,j,U}(k_i(t_0)) \equiv \max_{e_{i,j}(.)} \left\{ E_t \left[\int_t^{+\infty} \exp^{-r_0(\tau - t)} \; \{R_j(\mathbf{e}(\tau)) - C_{i,j}(e_{i,j}(\tau))\} \; 1_{\{\tau < \bar{t}\}} d\tau \right] \right\}, \qquad (8)$$

where $g_j(\mathbf{e}(\tau))$ in $R_j(\mathbf{e}(\tau))$ equals $e_{i,j}(\tau)$ and $h(\mathbf{k}(\tau))$ in $C_{i,j}(e_{i,j}(\tau))$ equals $k_i(t_0)$. The factor $1_{\{t<\bar{t}\}}$ is a random variable that equals 1 for all dates prior to the event of liquidation and 0 afterwards. Note that in writing (8), we have used the fact that knowhow is constant under separate operations, $k_i(t) \; = \; k_i(t_0)$ for all $t \geq t_0$.

We show in the Appendix that

$$V_{i,j,U}(k_i(t_0)) \;\; = \;\; n_i(\hat{x}_{t_0}) \; \Gamma_j \; , \qquad (9)$$

[21] Our model also allows the two firms to renegotiate their shares of an ongoing alliance. However, as we shall see in Section 3.3, the stationary nature of the problem implies that the firms have no incentive to engage in such renegotiation.

and the instantaneous resource contribution made by firm i to asset j at date $\tau \geq t_0$ is

$$e_{i,j}(\tau) \;=\; n_i(\hat{x}_{t_0})^{\frac{1}{\gamma_j}} \, \gamma_j^{\frac{1}{1-\gamma_j}} \;, \tag{10}$$

$$\text{where} \quad \Gamma_j \;\equiv\; \frac{(1-\gamma_j)\,\gamma_j^{\frac{\gamma_j}{1-\gamma_j}}}{r_0+\lambda} \;, \tag{11}$$

$$\text{and} \quad n_i(\hat{x}_t) \;\equiv\; \left(\bar{k} - \frac{(\bar{k}-k_i(0))}{\hat{x}_t}\right) \kappa_i \;. \tag{12}$$

The factor Γ_j is intended to simplify the notation throughout the paper. It is in effect a perpetuity factor, adjusted to reflect liquidation risk as well as the importance of resources as compared to knowhow and core competencies. Note that Γ_j is decreasing in γ_j.

The operator $n_i(\hat{x}_t)$ is at the heart of the comparisons that follow. From (5) and (6), we have $n_i(\hat{x}_t) = k_i(t)\,\kappa_i$. The operator $n_i(\hat{x}_t)$ therefore indexes the instantaneous profitability at date t of operation by firm i with knowhow $k_i(t)$ and core competencies κ_i. Observe that $n_i(\hat{x}_t)$ does not depend on j: the index of instantaneous profitability is identical for all assets.

We define the two operators

$$m(\hat{x}_t) \;\equiv\; \max_i\{n_i(\hat{x}_t)\} \;, \tag{13}$$

$$\text{and} \quad \tilde{i}(t) \;\equiv\; \arg\max_i\{n_i(\hat{x}_t)\} \;. \tag{14}$$

The first operator, $m(\hat{x}_t)$, denotes the maximum instantaneous profitability at date t. The operator $\tilde{i}(t) \in \{a;b\}$ identifies the firm that can achieve that maximum. We refer to that firm as the best user at date t. From the observation that the index of instantaneous profitability is identical for all assets, we infer that the best user of *one* asset at date t is the best user of *all* assets at that same date.[22]

The constancy of knowhow under separate operations implies that $\tilde{i}(t)$ is constant over time: Constant knowhow ($k_i(t) = k_i(t_0)$ for all $t \geq t_0$) implies constant profitability ($n_i(\hat{x}_t) = n_i(\hat{x}_{t_0}) = k_i(t_0)\,\kappa_i$ for all $t \geq t_0$); this in turn implies that the best user at date t_0 remains best user throughout ($\tilde{i}(t) = \tilde{i}(t_0)$ for all $t \geq t_0$). As a result, the best user at date t_0 should operate all assets at all dates $t \geq t_0$. If i is the initial owner of asset j at date t_0, i should sell j to $\tilde{i}(t_0)$ at t_0 in case $i \neq \tilde{i}(t_0)$.

As noted in Section 2, we adopt the generalized Nash bargaining solution to determine the price, $p_{j,U}(t_0)$, at which i sells j to $\tilde{i}(t_0)$. In order to simplify the notation, we define $i^- \equiv \tilde{i}(t_0)$ when $i \neq \tilde{i}(t_0)$. The sale price of j can be shown to equal[23]

$$p_{j,U}(t_0) = \beta_i\, V_{i^-,j,U}(k_{i^-}(t_0)) \;+\; \beta_{i^-}\, V_{i,j,U}(k_i(t_0)). \tag{16}$$

[22]We return to this somewhat extreme result in the Conclusion.

[23]The incremental value for firm i^- of buying the asset is $V_{i^-,j,U}(k_{i^-}(t_0)) - p_{j,U}(t_0)$. The incremental

The value to firm i at date $t_0 \geq 0$ of separate operations, including the value of the option to trade one or more assets, is

$$U_{i,U}(\mathbf{k}(t_0)) = \sum_{j \in \mathcal{J}} U_{i,j,U}(\mathbf{k}(t_0)) , \quad \text{where} \tag{17}$$

$$U_{i,j,U}(\mathbf{k}(t_0)) \equiv \begin{cases} V_{i,j,U}(k_i(t_0)) & \text{if } j \in \mathcal{J}_i \text{ and } i = \tilde{i}(t_0), \\ -\, p_{j,U}(t_0) + V_{i,j,U}(k_i(t_0)) & \text{if } j \notin \mathcal{J}_i \text{ and } i = \tilde{i}(t_0), \\ p_{j,U}(t_0) & \text{if } j \in \mathcal{J}_i \text{ and } i \neq \tilde{i}(t_0), \\ 0 & \text{if } j \notin \mathcal{J}_i \text{ and } i \neq \tilde{i}(t_0), \end{cases} \tag{18}$$

with $V_{i,j,U}(k_i(t_0))$ and $p_{j,U}(t_0)$ given by (9) and (16), respectively.

Note that, as the two firms internalize the option to trade the assets, the total value of separate operations to the two firms, including the option value of trading assets, equals the value of each asset operated by the best user, summed over all assets

$$W_U(\mathbf{k}(t_0)) \equiv U_{a,U}(\mathbf{k}(t_0)) + U_{b,U}(\mathbf{k}(t_0)) = \sum_{j \in \mathcal{J}} V_{\tilde{i}(t_0),j,U}(k_{\tilde{i}(t_0)}(t_0)) \tag{19}$$

$$= m(\hat{x}_{t_0}) \sum_{j \in \mathcal{J}} \Gamma_j . \tag{20}$$

3.2 The Opposite Polar Case: Merged Operations

The second section assumes that the joint operation of *all* assets $j \in \mathcal{J}$ within a merged firm is the superior pattern of operation at date $t_0 \geq 0$.

The End of Joint Operations: Joint operations within the merged firm need not be permanent.[24] The joint operation of all assets by the two organizations that correspond to the original merging firms may end, to be replaced by the separate operation of every asset by one or the other organization. For the same reasons as in Section 3.1, that organization is the same for all assets, i.e., one organizational form entirely supersedes the other within the merged firm.[25]

value for firm i of selling the asset is $p_{j,U}(t_0) - V_{i,j,U}(k_i(t_0))$. The Nash solution is characterized as

$$\max_{V_{i,j,U}(k_i(t_0)) \leq \; p_{j,U}(t_0) \; \leq V_{i^-,j,U}(k_{i^-}(t_0))} \left(V_{i^-,j,U}(k_{i^-}(t_0)) - p_{j,U}(t_0)\right)^{\beta_{i^-}} \left(p_{j,U}(t_0) - V_{i,j,U}(k_i(t_0))\right)^{\beta_i} . \tag{15}$$

[24]Unlike joint operations, separate operations necessarily are permanent. Should the separate operation of a given asset dominate the joint operation of that asset at a given date, separate operations dominate at all further dates. This is because both knowhow and learning conditions remain unchanged during the phase of separate operations that follows the end of joint operations.

[25]There is therefore no divesture or demerger in our model, in the sense that there are no circumstances in which some assets are operated by one organization and other assets by another organization following the ending of joint operations. We return to this somewhat extreme result in the Conclusion.

The organizational form that supersedes the other is that of the original firm that has been made best user at the date at which joint operations end by its knowhow at that date and its core competencies. We refer to that date as the superseding date, which we denote $\hat{t}$, $\hat{t} \geq t_0$. The value to the best user $\tilde{i}(\hat{t})$ of operating asset j separately at a date $t \geq \hat{t}$ equals $V_{\tilde{i}(\hat{t}),j,U}(k_{\tilde{i}(\hat{t})}(\hat{t}))$ given in (9). Summing over all assets, the value of the assets at the superseding date is $W^*(\hat{t}) \equiv \sum_{j\in\mathcal{J}} V_{\tilde{i}(\hat{t}),j,U}(k_{\tilde{i}(\hat{t})}(\hat{t})) = m(\hat{x}_{\hat{t}}) \sum_{j\in\mathcal{J}} \Gamma_j$.

In order to determine the identity of the best user at the superseding date, it is useful to examine the dynamics of $\tilde{i}(t)$ under joint operations. From (4) and (6), we can show that

$$\tilde{i}(t) \;=\; \begin{cases} \arg\max_i\{\kappa_i\} & \text{if } \hat{x}_t \geq x^* \\ \arg\min_i\{\kappa_i\} & \text{if } \hat{x}_t < x^* \end{cases} , \tag{21}$$

$$\text{where} \quad x^* \;\equiv\; 1 + \frac{k_a(0)\,\kappa_b - k_b(0)\,\kappa_a}{\bar{k}\,(\kappa_a - \kappa_b)} . \tag{22}$$

That $\hat{x}_t$ is non-decreasing in time has the implication that follows: A firm made best user at date t by virtue of its higher core competencies remains best user at all subsequent dates. In contrast, if the firm with higher core competencies is not best user at the date t, the opportunity for learning present under joint operations implies that there will come at time at which that firm becomes best user.

The preceding implies that if the best user at the date t_0 at which the merger takes place has higher core competencies, i.e., if $\tilde{i}(t_0) = \arg\max\{\kappa_i\}$, then the best user at the time of the merger is also the best user at the time of superseding, $\tilde{i}(\hat{t}) = \arg\max\{\kappa_i\}$ for all $\hat{t} \geq t_0$. If instead the best user at the time of the merger has lower core competencies, $\tilde{i}(t_0) = \arg\min\{\kappa_i\}$, then that user is also best user at the time of superseding if superseding occurs before a threshold date t^*. If superseding occurs after t^*, the best user at the time of superseding is the firm with higher core competencies. Formally, if $\tilde{i}(t_0) = \arg\min\{\kappa_i\}$, then $\tilde{i}(\hat{t}) = \arg\min\{\kappa_i\}$ for $t_0 \leq \hat{t} < t^*$ and $\tilde{i}(\hat{t}) = \arg\max\{\kappa_i\}$ for $\hat{t} \geq t^*$. The (random) threshold date t^* is such that $k_a(t^*)\,\kappa_a = k_b(t^*)\,\kappa_b$, i.e., it is the date at which learning conditions reach the threshold state x^*.

The Value of the Merger: The merged firm has access to the aggregate knowhow of the two merging firms, $\bar{k}$. It does not suffer from the problem of double moral hazard, as a single management team makes all resource allocations decisions.

At date t, $t_0 \leq t \leq \hat{t}$, the value of the merged firm is

$$W_M(\bar{k}) \;\equiv\; \max_{e_{a,j}(.),e_{b,j}(.),\hat{t}} \left\{ E_t\left[\int_t^{\hat{t}} \exp^{-r_0(\tau-t)} \sum_{j\in\mathcal{J}}\{R_j(\mathbf{e}(\tau)) - \sum_{i\in\{a;b\}} C_{i,j}(e_{i,j}(\tau))\}\,1_{\{\tau<\bar{t}\}}\,d\tau\right] + E_t\Big[\exp^{-r_0(\hat{t}-t)}\,W^*(\hat{t})\,1_{\{\hat{t}<\bar{t}\}}\Big]\right\} , \tag{23}$$

where $g_j(\mathbf{e}(\tau))$ in $R_j(\mathbf{e}(\tau))$ equals $\left(\delta_a^{\delta_a}\delta_b^{\delta_b}\right)^{-1} e_{a,j}(\tau)^{\delta_{a,j}}\, e_{b,j}(\tau)^{\delta_b}$ and $h(\mathbf{k}(\tau))$ in $C_{i,j}(e_{i,j}(\tau))$

equals $\bar{k}$. There is no double moral hazard problem in that resource contribution are made to maximize the value of the entire merged firm, rather than the stakes owned by the original firms: $e_{i,j}(\tau) \equiv \arg\max_{e_{i,j}(\tau)} \left[W_M(\bar{k})\right]$.

Two features of problem (23) make it tractable. First, the optimization problem regarding the choice of superseding date is time homogeneous: the gain in knowhow $k_i^+(t)$ in (6) being an increasing function of the state variable $\hat{x}_t$, the optimization problem is weakly path dependent in the historical maximum of the geometric Brownian motion x_t over the duration of joint operations.[26] Hence, the optimal time of superseding, $\hat{t}$, is the first time x_t reaches some upper, time-independent threshold level. That is, there exists a constant $\hat{x}$ such that $\hat{t} = \inf\{\, t \mid x_t = \hat{x} \,\}$. We refer to $\hat{x}$ as the superseding state. Second, instantaneous resources contributed to the operation of a given asset affect neither the gain in knowhow $k_i^+(t)$ in (6) nor the revenues from the operation of that same asset at another time or from the operations of any other asset at any time. Optimization over $\hat{t}$ and each $e_{i,j}(\tau)$ for all $i \in \{a; b\}$, $j \in \mathcal{J}$, and $\tau \geq t$ is therefore separable.

We show in the Appendix that

$$W_M(\bar{k}) \;=\; Y_M\, Z_M\ , \tag{24}$$

$$\text{where}\quad Y_M \;\equiv\; \bar{m} \sum_{j\in\mathcal{J}} \Gamma_j\ , \tag{25}$$

$$Z_M \;\equiv\; 1 \;+\; \frac{1}{(1+\xi)} \left[\frac{m^\infty}{\bar{m}} - 1\right] \left(\frac{x_{t_0}}{\hat{x}}\right)^{\xi}\ , \tag{26}$$

$$\bar{m} \;\equiv\; \bar{k}\,(\kappa_a^{\delta_a}\,\kappa_b^{\delta_b}) \tag{27}$$

$$m^\infty \;\equiv\; \lim_{x\to\infty} n_{\tilde{i}(\hat{t})}(x) = \;\bar{k}\;\max_i\{\kappa_i\}\ , \tag{28}$$

$$\text{and}\quad \xi \;\equiv\; \sigma^{-2}[\sigma^2/2 - \mu + \sqrt{(\mu-\sigma^2/2)^2 + 2(r_0+\lambda)\sigma^2}]\ . \tag{29}$$

The instantaneous resource contribution made by the organization that corresponds to the original firm i to asset j at date $\tau \geq t$ is

$$e_{i,j,M}(\tau) \;=\; \delta_i\,\left(\bar{k}\,\kappa_i\right)^{\frac{1-\gamma_j}{\gamma_j}}\;\bar{m}\,\gamma_j^{\frac{1}{1-\gamma_j}}\ . \tag{30}$$

Superseding takes place the first time the state variable x_t reaches

$$\hat{x} \;=\; \frac{1+\xi}{\xi}\left(\frac{m^\infty - n_{\tilde{i}(\hat{t})}(\hat{x}_{t_0})}{m^\infty - \bar{m}}\right). \tag{31}$$

Like $n_i(\hat{x}_t)$ in (12), $\bar{m}$ is an index of instantaneous profitability. Unlike $n_i(\hat{x}_t)$, $\bar{m}$ depends neither upon the identity of the user of the asset, i, nor upon learning conditions, $\hat{x}_t$. This is

[26] Equilibrium strategies are thus Markov, open loop (i.e., state dependent) and perfect state (i.e., with perfect information).

because the asset is operated jointly in a merger, with aggregate knowhow $\bar{k}$ that does not depend on learning conditions.

Note from (12) and (27) that $\bar{m}$ further differs from $n_i(\hat{x}_t)$ in that its implied core competencies $(\kappa_a^{\delta_a}\,\kappa_b^{\delta_b})$ is a geometric average of those of the original two firms a and b. This gives rise to what we refer to as the cost of core competencies differences, CCD for short. All else equal, it is clearly preferable for resources to be contributed solely by the firm with higher core competencies. However, all else is not equal, and knowhow in particular differs under separate and joint operations. Thus, when joint operations within a merged firm are chosen, and resources are contributed to each asset through two organizations co-existing within the same merged firm, the organization with higher core competencies will be somewhat penalized by having to work with a counterpart with lower core competencies. The two organizations are those of the original two firms that became the merged firm.

The value of the merged firm $W_M(\bar{k})$ consists of two components. The first component Y_M denotes the value of the merged firm absent the possibility of superseding. That component closely resembles the values $W^*(\hat{t})$ of separate operations starting at the superseding date, $\hat{t}$, and $W_U(\mathbf{k}(t_0))$ of separate operations starting at the date t_0. It differs from these in that the merged firm draws on aggregate knowhow, $\bar{k}$, but suffers from CCD, $\kappa_a^{\delta_a}\,\kappa_b^{\delta_b}$.

The second component $Z_M \geq 1$ is the option value of superseding. The amount by which that value is greater than 1 is the normalized gain obtained on superseding at date $\hat{t}$, discounted to date t_0. The ratio $(\hat{x}_{t_0}/\hat{x})^{\xi}$ is the probability adjusted discount factor. That ratio represents the present value at t_0 of one unit of currency obtained at the superseding date. The gain obtained on superseding can be factored into two components. The ratio $(m^{\infty} - \bar{m})/\bar{m}$ represents the normalized gain from superseding, under the hypothesis that superseding takes place with full knowhow having been acquired. The ratio $1/(1+\xi)$ reflects the reality that optimal superseding well predates the acquisition of full knowhow: discounting makes waiting for full knowhow excessively costly. Note that there is no corresponding option value under separate operations: once chosen, separate operations remain the superior pattern of operation.

As learning conditions reach the superseding state, $\hat{x}$, with probability 1, superseding occurs with probability 1. The best user at the superseding date is shown to be $\tilde{i}(\hat{t}) = \arg\max_i\{\kappa_i\}$. That is, the best user at the superseding date is the original firm with higher core competencies. The intuition for this result is as follows: joint operations are abandoned for separate operations only when the latter are more profitable. The higher profitability of separate operations cannot possibly be ascribed to higher knowhow, for knowhow is highest under joint operations. Higher profitability therefore must be ascribed to higher core competencies. This result implies that the ending of joint operations within a merged firm does not occur before the original firm with higher core competencies has acquired enough knowhow to become best user, in case it was not best user at the time of the merger.

The superseding state, $\hat{x}$ in (31) increases in $\bar{m}$ and decreases in $n_{\tilde{i}(\hat{t})}(\hat{x}_{t_0})$. The intuition for the former is as follows: the higher the instantaneous profitability of merged operations, the lower the cost of CCD, and the lower therefore the incentive to end joint operations through superseding in order to put an end to such cost. The intuition for the latter is follows: the higher the instantaneous profitability of the use of the assets by the firm with higher core competencies with the knowhow that firm has at the time of the merger, the lower the increase in profitability from knowhow acquisition by that firm during the period of joint operations, the lesser the incentive to continue joint operations and bear the attending cost of CCD.

Although no asset is sold at date t_0 in a merger, each merging firm must be compensated for the assets and other contributions that it makes to the merger. As for separate operations, we adopt the generalized Nash bargaining solution to determine that compensation. Assume that the merging firm i is to own a share ϕ_i of the merged firm, and denote $p_{i,M}(\phi_i)$ the (possibly negative) payment from firm i to the other merging firm i^- at the time of the merger, i.e., $p_{i,M}(\phi_i) = -p_{i^-,M}(\phi_{i^-})$. That payment can be shown to be[27]

$$p_{i,M}(\phi_i) \;=\; \beta_i \left[U_{i^-,U}(\mathbf{k}(t_0)) \;-\; \phi_{i^-}\, W_M(\bar{k})\right] \;+\; \beta_{i^-} \left[\phi_i\, W_M(\bar{k}) \;-\; U_{i,U}(\mathbf{k}(t_0))\right] \;. \qquad (32)$$

The value to firm i at date $t_0 \geq 0$ of a merger is

$$U_{i,M}(\mathbf{k}(t_0)) \;=\; U_{i,U}(\mathbf{k}(t_0)) \;+\; \beta_i \left[W_M(\bar{k}) \;-\; W_U(\mathbf{k}(t_0))\right] \;, \qquad (33)$$

with $U_{i,U}(\mathbf{k}(t_0))$, $W_U(\mathbf{k}(t_0))$, and $W_M(\bar{k})$ given by (17), (20), and (24), respectively.

Note that $W_M(\bar{k}) = U_{a,M}(\mathbf{k}(t_0)) \;+\; U_{b,M}(\mathbf{k}(t_0))$, and that the payment, $p_{i,M}(\phi_i)$, from firm i to firm i^- at the date of the merger, t_0, depends on firm i's share of the merged firm, ϕ_i. Consequently, there exists a sharing of the merged firm that ensures that no cash payment is required at date t_0. We denote such sharing $\tilde{\phi}_i$ and show that

$$\tilde{\phi}_i \;\equiv\; \{\phi_i \mid p_{i,M}(\phi_i) = 0\} \;=\; \left(\frac{U_{i,U}(\mathbf{k}(t_0))}{W_U(\mathbf{k}(t_0))} \;-\; \beta_i\right) \frac{W_U(\mathbf{k}(t_0))}{W_M(\bar{k})} \;+\; \beta_i \;. \qquad (34)$$

3.3 An Intermediate Case: Partially Related Operations within an Alliance

The third section assumes that the joint operations of some (but generally not all) assets jointly by the two firms within an alliance is the superior pattern of operation at date $t_0 \geq 0$.

[27] We assume that the assets would be operated separately by the best user in the absence of an agreement to merge. The incremental value of merging for firm i^- is $\phi_{i^-}\, W_M(\bar{k}) + p_{i,M}(\phi_i) - U_{i^-,U}(\mathbf{k}(t_0))$. It is $\phi_i\, W_M(\bar{k}) - p_{i,M}(\phi) - U_{i,U}(\mathbf{k}(t_0))$ for firm i.

The assets not operated jointly are operated separately by one or the other firm. Thus, $\mathcal{J}_{ab} \subset \mathcal{J}$ and $\mathcal{J}_{ab} \neq \emptyset$. As noted in Section 2.3, we consider contracts that promise each partner a constant share $\phi_{i,j}$, $0 < \phi_{i,j} < 1$, of the revenue from each individual asset operated jointly within the alliance, $R_j(\mathbf{e}(t))$ for $j \in \mathcal{J}_{ab}$, with $\phi_{a,j} + \phi_{b,j} = 1$. We initially take each share, $\phi_{i,j}$, as given. We later derive its optimal value.

Dissolution of the Alliance: Should the two firms choose to put an end to joint operations and thereby dissolve the alliance at a date $\hat{t} \geq t_0$, all assets hitherto operated jointly within the alliance henceforth are operated separately.[28] The value to firm i of operating asset $j \in \mathcal{J}_{ab}$ separately at a date $t \geq \hat{t}$ equals $V_{i,j,U}(k_i(\hat{t}))$ given in (9).

Recall from Section 2.3 that we seek a renegotiation-proof equilibrium. A first necessary condition for such an equilibrium is that the rules that govern dissolution allocate all hitherto jointly operated assets to the best user at dissolution. Thus, in case the alliance should be dissolved at the date $\hat{t}$, dissolution rules should be such as to allocate all assets $j \in \mathcal{J}_{ab}$ to $\tilde{i}(\hat{t})$ defined in (14).

As in Section 3.2, the dynamics of $\tilde{i}(t)$ can help determine the identity of the best user at dissolution. This is done by comparing the state of learning conditions at the time of dissolution $\hat{t}$ with the threshold state x^* defined in (22). It can be done already at the time of alliance formation, t_0. The first necessary condition for renegotiation-proofness can therefore be satisfied by specifying in the alliance contract the identity of the firm to which the assets operated jointly should be allocated at dissolution.

A second necessary condition for the equilibrium to be renegotiation-proof is that the two firms' privately optimal dissolution times coincide. This is because there would otherwise be renegotiation to a common dissolution time. Let $\hat{t}_i$ denote firm i's privately optimal dissolution time. Renegotiation-proofness thus requires that $\hat{t}_a = \hat{t}_b$.[29] To each firm, though, renegotiation-proofness simply means that dissolution takes place at the minimum of the two privately optimal dissolution times.[30]

We consider dissolution rules that make the payoff to each firm an affine function of the value of each asset $j \in \mathcal{J}_{ab}$ to the best user at dissolution, $\tilde{i}(\hat{t})$. Specifically, firm i's payoff at dissolution, $i \in \{a; b\}$, is of the form

$$\varphi_i \; + \sum_{j \in \mathcal{J}_{ab}} \psi_{i,j} \; V_{\tilde{i}(\hat{t}),j,U}(k_{\tilde{i}(\hat{t})}(\hat{t})) \; = \varphi_i \; + \; m(\hat{x}_{\hat{t}}) \sum_{j \in \mathcal{J}_{ab}} \psi_{i,j} \; \Gamma_j, \tag{35}$$

[28] We assume that no partial dissolution is possible. It is therefore impossible to remove some assets from the alliance but keep other assets within.

[29] The equality of the privately optimal dissolution times makes such time jointly optimal as well. There is therefore no possibility of increasing the parties' joint payoff by changing the time of dissolution.

[30] The equilibrium we seek is therefore a Nash Equilibrium in which (i) both firms expect no renegotiation, (ii) each firm chooses its privately optimal dissolution time, (iii) the dissolution rules ensure that the two firms' optimal dissolution times coincide, and (iv) neither firms wishes to renegotiate.

where $\varphi_i \in \mathbf{R}$ and $\psi_{i,j} \in (0,1)$. The constraint that the two payoffs add up to the combined value of the assets to their best users implies $\varphi_a + \varphi_b = 0$ and $\psi_{a,j} + \psi_{b,j} = 1$.

The Value of the Alliance: The maximization problems solved by firms a and b are

$$\begin{cases} V_{a,P}(t) & = & \max_{\substack{e_{a,j}(\cdot),\hat{t}_a \\ j\in\mathcal{J}}} \left\{ E_t \left[\int_t^{\hat{t}} \exp^{-r_0(\tau-t)}\ P_a(\tau)\, 1_{\{\tau<\bar{t}\}} d\tau \right] +\ E_t \left[\exp^{-r_0(\hat{t}-t)}\, V_a^*(\hat{t})\, 1_{\{\hat{t}<\bar{t}\}} \right] \right\}, \\ V_{b,P}(t) & = & \max_{\substack{e_{b,j}(\cdot),\hat{t}_b \\ j\in\mathcal{J}}} \left\{ E_t \left[\int_t^{\hat{t}} \exp^{-r_0(\tau-t)}\ P_b(\tau)\, 1_{\{\tau<\bar{t}\}} d\tau \right] +\ E_t \left[\exp^{-r_0(\hat{t}-t)}\, V_b^*(\hat{t})\, 1_{\{\hat{t}<\bar{t}\}} \right] \right\}. \end{cases} \tag{36}$$

The components of the maximization problems in (36) are as follows. The time of dissolution is $\hat{t} \equiv \min\{\,\hat{t}_a\,;\,\hat{t}_b\,\}$. The instantaneous profitability at date τ of both joint operations and separate operations by firm i is

$$P_i(\tau) \;\equiv\; \sum_{j\in\mathcal{J}_{ab}\cup\mathcal{J}_i} P_{i,j}(\tau)\ , \tag{37}$$

$$\text{where}\quad P_{i,j\in\mathcal{J}_{ab}}(\tau) \;\equiv\; \phi_{i,j}\, R_{j\in\mathcal{J}_{ab}}(\mathbf{e}(\tau)) \;-\; C_{i,j}(e_{i,j}(\tau))\ , \tag{38}$$

$$\text{and}\quad P_{i,j\in\mathcal{J}_i}(\tau) \;\equiv\; R_{j\in\mathcal{J}_i}(\mathbf{e}(\tau)) \;-\; C_{i,j}(e_{i,j}(\tau))\ . \tag{39}$$

Instantaneous profitability, $P_i(\tau)$, is thus the sum of (i) the instantaneous revenues from all assets $j \in \mathcal{J}_{ab}$ operated jointly within the alliance, net of the costs to firm i of the resources contributed to these assets and (ii) the instantaneous revenues from all assets $j \in \mathcal{J}_i$ operated separately by firm i, again net of costs. Finally, $V_i^*(\hat{t})$ is the sum of the payoff to firm i from dissolution at date $\hat{t}$ and the value at that same date of those assets that were and remain operated separately by firm i

$$V_i^*(\hat{t}) \;\equiv\; \varphi_i \;+\; \sum_{j\in\mathcal{J}_{ab}} \psi_{i,j}\ V_{\tilde{i}(\hat{t}),j,U}(k_{\tilde{i}(\hat{t})}(\hat{t})) \;+ \sum_{j\in\mathcal{J}_i} V_{i,j,U}(k_i(\hat{t}))\ , \tag{40}$$

We denote $W_P(t)$ the total value of all operations, both joint and separate

$$W_P(t) \;\equiv\; V_{a,P}(t) \;+\; V_{b,P}(t)\ . \tag{41}$$

An alliance presents both advantages and disadvantages. On the one hand, the two firms combine their respective knowhow and therefore take advantage of aggregate knowhow, $\bar{k}$, in operating the assets within the alliance. Furthermore, despite not being able to combine knowhow in operating the assets without the alliance, each firm can take advantage in operating these assets of the knowhow it has acquired from its partner in the alliance. Throughout the duration of the alliance, the knowhow that each firm can apply to its separate operations increases with $\hat{x}_t$.

On the other hand, each firm $i \in \{a; b\}$ chooses its instantaneous resource contribution, $e_{i,j}(\tau)$, to each jointly operated asset $j \in \mathcal{J}_{ab}$ to maximize only its share of the revenues from the asset, $P_{i,j}(\tau)$, rather than the total profitability of the asset.

Formally, the contrast between joint and separate operations can be expressed as such. Under joint operations, revenues have $g_j(\mathbf{e}(\tau)) = \left(\delta_a^{\delta_a}\delta_b^{\delta_b}\right)^{-1} e_{a,j}(\tau)^{\delta_a}\, e_{b,j}(\tau)^{\delta_b}$ and costs have $h(\mathbf{k}(\tau)) = \bar{k}$. Under separate operations, revenues have $g_j(\mathbf{e}(\tau)) = e_{i,j}(\tau)$ and costs have $h(\mathbf{k}(\tau)) = k_i(t) = k_i(0) \ + \ k_i^+(t)$, with $k_i^+(t)$ given in (6).

As for problem (23), two features of problem (36) make it tractable. First, each firm's optimization problem regarding the choice of dissolution time is time homogeneous. Each partner's privately optimal time of dissolution, $\hat{t}_i$, is the first time x_t reaches some upper time-independent threshold level. That is, there exists a constant $\hat{x}_i$ such that $\hat{t}_i = \inf\{\, t \mid x_t = \hat{x}_i \,\}$. Second, for each firm i's optimization problem, optimization over $\hat{t}_i$ and each $e_{i,j}(\tau)$ for all $i \in \{a; b\}$, $j \in \mathcal{J}$, and $\tau \geq t$ is separable.

We initially assume that the dissolution rules are renegotiation-proof. We show later in this section how they can be made so. Under the assumption of renegotiation-proofness, we show in the Appendix that

$$V_{i,P}(t) \ = \ V_{i,P}^{\infty}(t) + \left[V_i^*(\hat{t}) \ - \ V_{i,P}^{\infty}(\hat{t})\right] \left(\frac{x_t}{\hat{x}}\right)^{\xi} . \tag{42}$$

$$\text{where} \qquad V_{i,P}^{\infty}(t) \ \equiv \ \sum_{j \in \mathcal{J}_{ab} \cup \mathcal{J}_i} V_{i,j}^{\infty}(t) \ , \tag{43}$$

and $V_{i,j}^{\infty}(t)$ depends on whether asset j is (i) operated within the alliance, in which case,

$$V_{i,j \in \mathcal{J}_{ab}}^{\infty}(t) \ \equiv \ \Phi_{i,j}\, h_j\, \bar{m}\, \Gamma_j \ , \tag{44}$$

$$\text{with} \qquad \Phi_{i,j} \ \equiv \ \frac{\phi_{i,j}\,(1 - \gamma_j\ \delta_i)}{1 - \gamma_j\,(\phi_{a,j}\,\delta_a + \phi_{b,j}\,\delta_b)} \ , \tag{45}$$

$$h_j \ \equiv \ \frac{1 - \gamma_j\,(\phi_{a,j}\,\delta_{a,j} + \phi_{b,j}\,\delta_b)}{1 - \gamma_j} \ \left[\phi_{a,j}^{\delta_a}\,\phi_{b,j}^{\delta_{b,j}}\right]^{\frac{\gamma_j}{1-\gamma_j}} \ , \tag{46}$$

or (ii) operated separately with knowhow that gradually increases over time as firm i acquires some of the other firm's knowhow through joint operations of one or more assets other than asset j,

$$V_{i,j \in \mathcal{J}_i}^{\infty}(t) \ \equiv \ n_i\,(\,q(t)\,)\ \Gamma_j \ , \tag{47}$$

$$\text{with} \quad q(t) \ \equiv \ \hat{x}_t \left[1 \ - \ (1 - Q^{-1}) \left(\frac{x_t}{\hat{x}_t}\right)^{\xi}\right]^{-1} \tag{48}$$

$$Q \ \equiv \ \left[\frac{a_1\, r}{s_1}\left(1 + \frac{a_1}{s_1\sqrt{2\,s_1 + a_1^2}}\right) + a_2\left(1 - \frac{a_2}{r\sqrt{2\,r + a_2^2}}\right)\right]^{-1} \frac{\sigma^3}{2(\mu - \sigma^2)} \ , \tag{49}$$

$$s_1 \ \equiv \ r + \mu - \sigma^2 \ , \quad a_1 \equiv a_2 - \sigma \ , \quad a_2 \equiv \frac{\mu - \sigma^2/2}{\sigma} \ , \quad r \equiv r_0 + \lambda \ . \tag{50}$$

The instantaneous resources contributed by firm i at dates $\tau \in (t, \hat{t})$ are

$$e_{i,j\in\mathcal{J}_{ab}}(\tau) = \phi_{i,j} \left[\phi_{a,j}^{\delta_a}\, \phi_{b,j}^{\delta_b}\right]^{\frac{\gamma_j}{1-\gamma_j}} e_{i,j,M}(\tau) \,, \tag{51}$$

$$e_{i,j\in\mathcal{J}_i}(\tau) = n_i(\hat{x}_\tau)^{\frac{1}{\gamma_j}}\, \gamma_j^{\frac{1}{1-\gamma_j}} \,. \tag{52}$$

The alliance is dissolved the first time the state variable x_t reaches

$$\hat{x} = \frac{1+\xi}{\xi}\left[m^\infty - n_{\tilde{i}(\hat{t})}(\hat{x}_{t_0})\right] \left[\frac{\sum_{j\in\mathcal{J}}\Gamma_j \;-\; Q^{-1}\sum_{j\in\mathcal{J}\setminus\mathcal{J}_{ab}}\Gamma_j}{\sum_{j\in\mathcal{J}_{ab}}\left([m^\infty - \bar{m}\; h_j]\,\Gamma_j\right)}\right], \tag{53}$$

where m^∞ is defined in (28). We refer to $\hat{x}$ as the dissolution state.

It is interesting to compare these results with those obtained in the two preceding cases of wholly unrelated operations and merged operations. Consider for example $V^\infty_{i,j\in\mathcal{J}_{ab}}(t)$ in (44) which, when added for the two firms a and b, denotes the value of the joint operation of asset j within an alliance, excluding the value of the option to put an end to such operations through the dissolution of the alliance (this option is reflected in the second term on the RHS of equation (42)). Compare $V^\infty_{a,j\in\mathcal{J}_{ab}}(t) + V^\infty_{b,j\in\mathcal{J}_{ab}}(t)$ with the term $\bar{m}\Gamma_j$ in Y_M in (25), which represents the value of operating asset j within the merged firm, excluding the option to supersede. The difference between the two expressions is the presence of the term h_j in (44). That term is less than 1, and arises from the presence of the problem of double moral hazard in the alliance.[31] The term h_j does not appear in (25) because there is no problem of double moral hazard in a merger. Note that both alliances and mergers suffer from CCD, which is reflected in $\bar{m}$. That alliances may be preferred to mergers, despite the presence of moral hazard as expressed by h_j, is due to the option to include only a subset of assets in the alliance, as compared to the requirement to include all assets in a merger. This option is valuable, for it limits the cost of CCD to those assets included in the alliance. In contrast, all assets suffer from the cost CCD in a merger.

Now consider $V^\infty_{i,j\in\mathcal{J}_i}(t)$ in (47) and compare it to $V_{i,j,U}(k_i(t_0))$ in (9). Both terms represent the value of the separate operation of asset j by firm i. These two terms would be identical for the same starting date t_0 were it not for the fact that the term $q(t)$ in (48) differs from $\hat{x}_{t_0}$ by the presence of the term in square brackets. This second term can be shown to be greater than 1. It represents the value of applying to separate operations the knowhow acquired in the course of joint operations.

A different but complementary perspective is obtained by comparing resource contributions. On the one hand, comparing $e_{i,j\in\mathcal{J}_{ab}}(\tau)$ in (51) with $e_{i,j,M}(\tau)$ in (30) shows the detrimental effect of double moral hazard. On the other hand, comparing $e_{i,j\in\mathcal{J}_i}(\tau)$ in (52)

[31] If both $\phi_{a,j}$ and $\phi_{b,j}$ were to equal 1, implying the absence of double moral hazard as each firm would then be residual claimant to the entire resource contribution it has made to asset j, h_j would equal 1.

with $e_{i,j,U}(\tau)$ in (10) shows that, when some assets are in joint operations within an alliance, learning yields a positive externality on separate operations: $\hat{x}_\tau \geq \hat{x}_{t_0}$ for $\tau \geq t_0$.

The alliance is dissolved with probability 1. The best user at dissolution need not be the firm with higher core competencies: the alliance may be dissolved before that firm has acquired enough knowhow to become best user, in case it was not so at the time of the formation of the alliance.[32] This is because the alliance suffers from the problem of double moral hazard, which may be so costly as to prompt the dissolution of the alliance before the firm with higher core competencies has become best user. The exact identity of the best user at dissolution depends on the various parameter values, which affect the direction of the ordering of $\hat{x}$ versus x^*.

Turning to $\hat{x}$ in (53), note that the greater the loss in value created by the problem of double moral hazard (the smaller h_j in the denominator), the smaller $\hat{x}$: moral hazard decreases the time to dissolution. Also, the larger the benefits of knowhow acquisition within the alliance for the assets without (the larger Q in the numerator), the larger $\hat{x}$: the acquisition of knowhow increases the time to dissolution.

Note that it is not possible to draw any general conclusion as to which of dissolution or superseding occurs first. Comparing $\hat{x}$ in (53) with its counterpart for superseding in (31) shows that while h_j alone would decrease the time to dissolution below that to superseding, Q alone would do the opposite.

We show in the Appendix that a renegotiation-proof contract that maximizes the value of total operations with some assets operated jointly within an alliance, $W_P(t)$, consists of (i) a rule $\phi_{i,j}$ for sharing the revenues from each asset operated jointly within the alliance $j \in \mathcal{J}_{ab}$, with

$$\phi_{i,j} = (\sqrt{1 + N_{i,j}} - 1)/N_{i,j} \, , \qquad \text{where} \qquad N_{i,j} \equiv \frac{(1 - 2\delta_i)}{\delta_i(1 - \gamma_j + \gamma_j \delta_i)} \, , \tag{54}$$

if $\delta_a \neq \delta_b$, and $\phi_{i,j} = 1/2$ if $\delta_a = \delta_b$, and (ii) a rule $\psi_{i,j}$ for allocating the value of the hitherto jointly operated assets at dissolution, as well as a (possibly negative) transfer φ_i from firm i to its partner, with

$$\begin{aligned} \varphi_i \;=\; & \sum_{j \in \mathcal{J}_{ab}} V_{i,j}^{\infty}(0) \;-\; \sum_{j \in \mathcal{J}_{ab}} \psi_{i,j} \; V_{\tilde{i}(\hat{t}),j,U}(\bar{k}) \\ & + \; \frac{(1+\xi)}{\xi \, \hat{x}} \left[\sum_{j \in \mathcal{J}_{ab}} \psi_{i,j} \, V_{\tilde{i}(\hat{t}),j,U}(\bar{k} - k_{\tilde{i}(\hat{t})}(0)) \;+\; (1 - Q^{-1}) \sum_{j \in \mathcal{J}_i} V_{i,j,U}(\bar{k} - k_i(0)) \right] . \end{aligned} \tag{55}$$

Note that $N_{i.j}$ is decreasing in δ_i and that $\phi_{i,j}$ can be shown to be increasing in $N_{i.j}$. Thus,

[32] Note the contrast with mergers, where superseding does not take place before the firm with higher core competencies has become best user.

the more important firm i's resource contribution to the profitability of asset j operated jointly within the alliance, the larger must be firm i's share of the revenues from asset j.

Note also that φ_i in (55) depends on $\psi_{i,j}$. A multitude of dissolution rules is therefore possible, with the transfer φ_i serving to make the two firms agree on a common dissolution time for any given allocation of value at dissolution, $\psi_{i,j}$.

As for separate operations and for mergers, we adopt the generalized Nash bargaining solution to determine the compensation paid by one form to the other at formation of the alliance at date t_0. The alliance is characterized by the sets $(\mathcal{J}_{ab}, \mathcal{J}_a, \mathcal{J}_b)$ of jointly and separately operated assets. We denote $p_{i,P}$ the (possibly negative) payment from firm i to firm i^- at date t_0, i.e., $p_{i,P} = -p_{i^-,P}$. That payment can be shown to be[33]

$$p_{i,P} \;=\; \beta_{i^-}\left[V_{i,P}(t_0) \;-\; U_{i,U}(\mathbf{k}(t_0))\right] \;+\; \beta_i\left[U_{i^-,U}(\mathbf{k}(t_0)) \;-\; V_{i^-,P}(t_0)\right] \;. \tag{56}$$

The value to firm i at date t_0 of an alliance $(\mathcal{J}_{ab}, \mathcal{J}_a, \mathcal{J}_b)$ is

$$U_{i,P}(t_0) \;=\; U_{i,U}(\mathbf{k}(t_0)) \;+\; \beta_i\left[W_P(t_0) \;-\; W_U(\mathbf{k}(t_0))\right] \;, \tag{57}$$

with $U_{i,U}(\mathbf{k}(t_0))$, $W_U(\mathbf{k}(t_0))$, and $W_P(t_0)$ given by (17), (20) and (41). Note that $U_{a,P}(t_0) + U_{b,P}(t_0) \;=\; W_P(t_0)$.

Optimal Alliance Structure: We now examine the choice of alliance structure upon formation at date t_0. Both firms benefit from choosing the optimal structure $(\mathcal{J}_{ab}, \mathcal{J}_a, \mathcal{J}_b) = \arg\max_{(\mathcal{J}_{ab}, \mathcal{J}_a, \mathcal{J}_b)} W_P(t_0)$.

We have established that the value of the assets at date t_0 with a given alliance structure $(\mathcal{J}_{ab}, \mathcal{J}_a, \mathcal{J}_b)$ is

$$\begin{aligned} W_P(t_0) \;=\; & \sum_{j\in\mathcal{J}_{ab}} (\bar{m}\; h_j\; \Gamma_j) + \sum_{i\in\{a;b\}}\sum_{j\in\mathcal{J}_i} (n_i(q(t_0))\; \Gamma_j) \\ & + \left[\sum_{j\in\mathcal{J}_{ab}} (m(\hat{x}) \;-\; \bar{m}\; h_j)\,\Gamma_j \;+\; \sum_{i\in\{a;b\}}\sum_{j\in\mathcal{J}_i} \left(m(\hat{x}) - m(q(\hat{t}))\right)\Gamma_j\right] \left(\frac{x_{t_0}}{\hat{x}}\right)^{\xi}. \end{aligned} \tag{58}$$

Consider first the assets that are operated separately, $j \in \mathcal{J}_a$ or $j \in \mathcal{J}_b$. Given that an asset can be traded, $j \in \mathcal{J}_i$ at date t_0 if and only if $n_i(q(t_0)) = m(q(t_0))$. As neither $n_i(q(t_0))$ nor $m(q(t_0))$ depends on j, all assets operated separately are operated by the same user. We denote the best user of the assets operated separately $\tilde{i}(q(t_0)) \equiv \arg\max_i\{n_i(q(t_0))\}$. We thus have $\mathcal{J}_{\tilde{i}(q(t_0))} = \mathcal{J} \setminus \mathcal{J}_{ab}$.

[33] We assume that the assets would be operated separately by the best user in the absence of an agreement to form an alliance. The incremental value of forming the alliance for firm i^- is $V_{i^-,P}(t_0) + p_{i,P} - U_{i^-,U}(\mathbf{k}(t_0))$. It is $V_{i,P}(t_0) - p_{i,P} - U_{i,U}(\mathbf{k}(t_0))$ for firm i.

It is useful to introduce the operator:

$$S(x) \;\equiv\; \frac{m(x)}{\bar{m}}\;. \tag{59}$$

The operator $S(x)$ measures the profitability of separate operations relative to merged.

Now consider the assets that are operated jointly. Consider asset $j \in \mathcal{J}$. Either (i) $S(q(t_0)) > h_j$, or (ii) $S(q(t_0)) \leq h_j$. It is immediate from (58) that $j \notin \mathcal{J}_{ab}$ if $S(q(t_0)) > h_j$. Conversely, $j \in \mathcal{J}_{ab}$ if $S(q(t_0)) \leq h_j$. Thus, $\mathcal{J}_{ab} = \{j \mid S(q(t_0)) \leq h_j\}$. In words, an asset j is operated jointly within the alliance, rather than separately without, when the benefits of access to aggregate knowhow, net of the costs of CCD and double moral hazard, dominate the benefits that accrue to separate operations from the knowhow acquired through joint operations.

The preceding reasoning assumes that there exists at least one asset j for which the inequality $S(q(t_0)) \leq h_j$ is true. If that should not be the case, then the assumption that an alliance is formed implies that it is the asset j entailing the least cost of moral hazard that should be operated jointly within the alliance.[34] No other asset would be operated jointly. Whether an alliance is desirable under these and other circumstances is analyzed in Section 3.4.

Simplifying (58), we can write the optimal alliance structure, $(\mathcal{J}_{ab}, \mathcal{J}_a, \mathcal{J}_b)$, at date $t_0 \geq 0$ as

$$\begin{cases} \mathcal{J}_{ab} \;=\; \{j \in \mathcal{J} \mid S(q(t_0)) \;\leq\; h_j\;\} \;\cup\; \arg\min_{j\in\mathcal{J}}\{(S(q(t_0)) - h_j)\, w_j\}\;; \\ \mathcal{J}_a \;=\; \mathcal{J} \setminus \mathcal{J}_{ab} \quad \text{if} \quad a = \tilde{i}(q(t_0)) \quad \text{or} \quad \mathcal{J}_a \;=\; \emptyset \quad \text{if} \quad a \neq \tilde{i}(q(t_0))\;; \\ \mathcal{J}_b \;=\; \mathcal{J} \setminus (\mathcal{J}_{ab} \cup \mathcal{J}_a)\;. \end{cases} \tag{60}$$

The value of the alliance to the two firms is

$$\tilde{W}_P(t_0) \;\equiv\; \max_{(\mathcal{J}_{ab},\mathcal{J}_a,\mathcal{J}_b)} W_P(t_0) \;=\; Y_P\, Z_P\;, \tag{61}$$

$$\text{where} \quad Y_P \;\equiv\; \bar{m} \sum_{j\in\mathcal{J}} \Gamma_j \left(S(q(t_0)) - \sum_{j\in\mathcal{J}_{ab}} (S(q(t_0)) - h_j)\, w_j \right)\;, \tag{62}$$

$$Z_P \;\equiv\; 1 \;+\; \frac{\sum_{j\in\mathcal{J}_{ab}} ([m^\infty - \bar{m}\; h_j]\;\; \Gamma_j)}{(1+\xi)\, Y_P} \left(\frac{x_{t_0}}{\hat{x}}\right)^{\xi}\;, \tag{63}$$

$$w_j \;\equiv\; \frac{\Gamma_j}{\sum_{j\in\mathcal{J}} \Gamma_j}\;, \tag{64}$$

with $\hat{x}$ is given in (53) and m^∞ is defined in (28).

[34] If an alliance should be formed despite the fact that there exists no asset j for which the inequality $S(q(t_0)) \leq h_j$ is true, the asset operated jointly within the alliance would in a sense be 'sacrificed' for the purpose of making the acquisition of knowhow possible.

3.4 Ex-ante Optimal Pattern of Operation

We now turn to the comparison of the three patterns of operation discussed in the preceding sections: wholly unrelated, merged, or partially related operations. We wish to determine the optimal pattern of operation at date t_0. For that purpose, we need to compare the three values obtained in Sections 3.1, 3.2, and 3.3. We define

$$M_{\mathcal{J}}(t_0) \quad \equiv \quad \max\left\{W_U(\mathbf{k}(t_0))\,;\,W_M(\bar{k})\,;\,\tilde{W}_P(t_0)\right\}\ . \tag{65}$$

Recall that $W_U(\mathbf{k}(t_0))$, $W_M(\bar{k})$, and $\tilde{W}_P(t_0)$ are the date t_0 values of the entire set of assets to wholly unrelated, merged, and partially related firms, respectively. The three values are defined in equations (20), (24), and (61), respectively.

We first note that there are no circumstances under which the joint operation of some or all assets is preceded by a phase of separate operation of these same assets. Such an occurrence would require $M_{\mathcal{J}}(t_0) = W_U(\mathbf{k}(t_0))$ at the date t_0, and $M_{\mathcal{J}}(t_1) \neq W_U(\mathbf{k}(t_1))$ at a later date t_1 that follows a period of separate operations, $t_1 - t_0$. But neither a merger nor an alliance is made more desirable by separate operations. This is because both knowhow and learning conditions remain unchanged under separate operations: $\mathbf{k}(t_1) = \mathbf{k}(t_0)$ and $x_{t_1} = x_{t_0}$. The preceding implies that only when a merger or an alliance is formed at date t_0 will there be a subsequent change in the pattern of operation, as joint operations are abandoned for separate operations. In contrast, when separate operations are the optimal pattern of operation at date t_0, they remain optimal throughout.

We further observe that, regardless of the nature of the optimal pattern of operation at date t_0, there are no circumstances under which operations should be delayed. This is because not operating is never worthwhile, as the values $V_{i,j,U}(k_i(t))$, $W_{M,j}(\bar{k})$, and $V_{i,P}(t)$ in (9), (24), and (42), respectively, are always positive, and both knowhow and learning conditions remain unchanged in the absence of joint operations.[35] Operations, whether wholly unrelated, merged, or partially related, therefore start at date 0 and $t_0 = 0$.

Allowing for the possibility to choose across patterns of operation implies the following result for alliances

Proposition 1 *When partially related operations within an alliance are the optimal pattern of operation at date* 0*, only a single asset is operated jointly within the alliance. That asset is that which suffers the least from the cost of double moral hazard:*

$$\mathcal{J}_{ab} \quad = \quad \arg\min_{j\in\mathcal{J}}\{(S(Q) - h_j)\,w_j\}\ . \tag{66}$$

[35] The constancy of learning conditions under separate operations implies that there is no option value to waiting for an improvement in learning conditions. See McDonald and Siegel (1986) for an analysis of the value of the option of waiting to invest.

The intuition is as follows: Consider the case where there is an asset j such that $S(Q) \leq h_j$. This implies that $S(Q) \leq 1$ for all assets $j \in \mathcal{J}$. This is because $S(Q)$ does not depend on j, and $h_j \leq 1\ \forall j$. So, the presence of an asset j that is more profitably operated within the alliance implies that all assets are more profitably operated within the alliance. But in such case, the merger clearly dominates the alliance, for it avoids double moral hazard without involving any greater extent of CCD.[36] This implies that, in case an alliance actually is chosen over a merger or separate operations at date 0, only a single asset is operated jointly within the alliance. As discussed in Section 3.3, that asset is 'sacrificed' for the purpose of making the acquisition of knowhow possible (see footnote 34).

Clearly, the sacrifice made in operating an asset jointly within an alliance should be as small as possible. This means that the asset that is sacrificed should be one that suffers little from double moral hazard. The term h_j measures the extent of double moral hazard: the larger h_j, the lesser the problem of double moral hazard.

Unlike resource contributions, knowhow and core competencies are not affected by double moral hazard. This implies that the sacrifice made in jointly operating an asset depends not only on the extent to which that asset suffers from double moral hazard, but also on the importance of resource contribution relative to knowhow and core competencies in determining the value of the asset. The weight w_j is a normalized measure of the perpetuity factor Γ_j that reflects that relative importance. Expression (66) therefore indicates that the asset that is sacrificed should be one that suffers little from moral hazard (small $S(Q) - h_j$) *and* for which moral hazard is of relatively little importance as compared to knowhow and core competencies (small w_j).

We divide $M_{\mathcal{J}}(0)$ in (65) by $W_M(\bar{k})$, the value of merged operations, and denote that ratio $M'_{\mathcal{J}}(0)$: $M'_{\mathcal{J}}(0) \equiv M_{\mathcal{J}}(0)/W_M(\bar{k})$. We wish to express the values of wholly unrelated and partially related operations relative to those of merged operations. We can then write

Proposition 2 *At the initial date* 0, *the two firms a and b choose (i) wholly unrelated operations if $M'_{\mathcal{J}}(0) = S(1)/Z_M$, (ii) merged operations if $M'_{\mathcal{J}}(0) = 1$, or (iii) partially related operations if $M'_{\mathcal{J}}(0) = Y\, Z_P/Z_M$. Here,*

$$M'_{\mathcal{J}}(0) \quad = \quad \max\left\{S(1)/Z_M\,;\,1\,;Y\,Z_P/Z_M\right\} \quad , \tag{67}$$

where

$$S(x) \;=\; \left[1 - \frac{(1\,-\,K)}{x}\right]\Omega \;, \qquad Y \;\equiv\; S(Q) \,-\, w_{j\in\mathcal{J}_{ab}}(S(Q) \,-\, h_{j\in\mathcal{J}_{ab}})\;, \tag{68}$$

$$Z_M \;\equiv\; 1 \,+\, \frac{z}{(1+\xi)\,\hat{x}_M^{\xi}}\;, \qquad Z_P \;\equiv\; 1 \,+\, \frac{w_{j\in\mathcal{J}_{ab}}\; z_{j\in\mathcal{J}_{ab}}}{(1+\xi)\,Y\,\hat{x}_P^{\xi}}\;, \tag{69}$$

$$z_j \;\equiv\; \Omega\,\hat{\rho}' \,-\, h_j\;, \quad z \;\equiv\; \Omega\,\hat{\rho}' \,-\, 1\;, \quad z' \;\equiv\; \Omega\,\hat{\rho}'\,[1 - K\,\hat{\rho}]\;. \tag{70}$$

[36] In both cases, all assets are operated jointly.

K and Ω are the relative knowhow and relative core competencies of the best user at date 0:

$$K \;\equiv\; \frac{k_{\tilde{i}(0)}(0)}{\bar{k}} \,, \quad \Omega \;\equiv\; \frac{\kappa_{\tilde{i}(0)}}{\kappa_a^{\delta_a}\,\kappa_b^{\delta_b}} \,. \tag{71}$$

Superseding within a merged firm occurs the first time x_t reaches

$$\hat{x}_M \;=\; \frac{(1+\xi)\,z'}{\xi\,z} \,, \tag{72}$$

whereas the dissolution of an alliance occurs the first time x_t reaches

$$\hat{x}_P \;=\; \frac{(1+\xi)\,z'}{\xi\,w_{j\in\mathcal{J}_{ab}}\;z_{j\in\mathcal{J}_{ab}}}\left[w_{j\in\mathcal{J}_{ab}} \,+\, (1-Q^{-1})\right] \,. \tag{73}$$

$\hat{\rho}$ and $\hat{\rho}'$ are adjustment factors to K and Ω, in case the identity of the best user changes in the course of joint operations:

$$(\,\hat{\rho}\,;\,\hat{\rho}'\,) \;\equiv\; \begin{cases} (\,1\;\;;\,1\,) & \textit{if}\quad \tilde{i}(0)=\tilde{i}(\hat{t}). \\ \left(\frac{\min_i\{k_i(0)\}}{\max_i\{k_i(0)\}}\;;\;\frac{\max_i\{\kappa_i\}}{\min_i\{\kappa_i\}}\right) & \textit{if}\quad \tilde{i}(0)\neq\tilde{i}(\hat{t}). \end{cases} \tag{74}$$

Proposition 2 reproduces many of the results obtained in the preceding three sections, but rewrites these in such a way as to make the derivation of comparative statics result in Section 4 more natural. Proposition 2 can be interpreted as such. The pattern of operation chosen at date 0 is, naturally, that which imparts the most value to the complete set of assets. Value is measured relative to the value of merged operations, $W_M(\bar{k})$. The (relative) value of separate operations, $S(1)$ can be decomposed into two components: $S(1) \;=\; K\,\Omega$.[37] This decomposition is an attempt at isolating the effects of knowhow and core competencies on the choice between separate and merged operations. A merger benefits from aggregate knowhow ($K \leq 1$ by definition of $\bar{k}$), but may suffer from CCD ($\Omega \geq 1$ when $\tilde{i}(0) = \arg\max\{\kappa_i\}$). Note that double moral hazard does not enter the comparison, as double moral hazard affects neither separate nor merged operations.

The (relative) value of partially related operations, $Y\,Z_P$, consists of two components. The first component, $Y \equiv Y_P/Y_M$, represents the benefit to the assets operated separately of the knowhow acquired through the operations of the single asset operated jointly. This is suggested by the term Q in $S(Q)$: recall that $Q > 1$ is the term that increases $q(t)$ above $\hat{x}_t$ in (48). The subtracted term $w_{j\in\mathcal{J}_{ab}}\,(S(Q) \,-\, h_{j\in\mathcal{J}_{ab}})$ indicates that the aforementioned benefit does not apply to the sacrificed asset, which instead suffers from the problem of double moral hazard.[38] The weight $w_{j\in\mathcal{J}_{ab}}$ reflects the importance of that asset.

[37] Recall that x_0=1.

[38] The difference $S(Q) \,-\, h_{j\in\mathcal{J}_{ab}}$ is positive as $S(Q) > h_j\;\forall j$ when an alliance is chosen over competing organizational forms at date 0.

The second component, $Z_P \geq 1$, captures the option value of dissolving the alliance within which separate operations are conducted and reverting to separate operations. That component is decreasing in Y: the greater the benefit to separate operations of the knowhow acquired through joint operations, the more is lost on dissolution, and the less valuable therefore the option to dissolve. It is increasing in $w_j\, z_j$, where z_j measures the gain from abandoning the joint operation of asset j for the separate operation of that asset, in terms of avoiding both CCD and double moral hazard: the greater that gain, the more valuable the option to dissolve.[39] Finally, it is decreasing in the dissolution state, $\hat{x}_P$: the further away dissolution, the lesser the value of the option to dissolve.

It is interesting to compare Z_P with $Z_M \geq 1$. As with Z_P and $\hat{x}_P$, Z_M is decreasing in the superseding state $\hat{x}_M$: the further away superseding, the lesser the value of the option to supersede. As with Z_P and z_j, Z_M is increasing in z, where z measures the gain from abandoning merged operations for separate operations: the greater that gain, the more valuable the option to supersede. Note the presence of h_j in z_j but not in z: only in the case of the dissolution of an alliance does a gain arise from putting an end to double moral hazard. Finally, note the absence of a term equivalent to Y in Z_M: all assets are operated jointly under merged operations; there are no separate operations to benefit from joint operations.

Both the similarity and the difference between Z_M and Z_P reappear in the superseding and dissolution states, $\hat{x}_M$ and $\hat{x}_P$, respectively. The state $\hat{x}_M$ increases in the ratio z'/z, whereas $\hat{x}_P$ increases in $z'/(w_j z_j)$, where z' includes the likely loss in knowhow that results from the abandonment of joint operations.

The choice of organizational form is represented graphically in Figure 1 along two dimensions: the (relative) value of separate operations, $S(1)/Z_M$, and that of partially related operations, $Y\, Z_P/Z_M$. A merger is chosen when the two firms have similar core competencies: there is little cost of CCD in a merger in such case, no double moral hazard, and access to combined knowhow for all assets. When one firm has markedly higher core competencies than the other, that firm operates the assets separately if it also has markedly higher knowhow: there is little to be gained and much to be lost from joint operations in such case. Finally, an alliance is chosen when the firm with markedly higher core competencies has markedly lower knowhow: the alliance then serves to confine the cost of CCD to the single asset operated jointly, yet makes accessible to the assets operated separately the knowhow acquired from joint operations within the alliance.

We noted in the Introduction that knowhow and its acquisition are a distinctive feature of our analysis. Just how important were they to our results, and could these results have been obtained in the absence of knowhow and its acquisition? We consider the role of knowhow

[39] The term $\hat{\rho}'$ in z_j adjusts for a possible change in the identity of the best user, from the lower to the higher core competencies firm.

in more detail in Section 4.4, but we note at this stage that there would only be separate operations if there were no knowhow, and there would be no alliances if knowhow could not be acquired. Where there is nothing to be learned, there is no need to heed others' advice.[40] Where there is nothing that can be learned, heeding others' advice is worthwhile only if it brings immediate benefits (mergers); it is not worthwhile as a means to greater learning (alliances).

4 Analysis

We now derive a number of testable implications from the results obtained in Section 3.

4.1 How Likely is a Given Pattern of Operation?

We first wish to examine the extent to which one pattern of operation, say separate operations, will be chosen over one or both other patterns. For that purpose, we examine the changes in $M'_{\mathcal{J}}(0)$ in response to changes in the characteristics of a firm, an asset, or learning or external economic conditions. Such changes can be viewed as representing changes in the likelihood of observing a given pattern of operation.

Consider the set $\mathcal{S}(Z)$ of parameter values which is such that the pattern of operation Z dominates, where $Z \in \{S(1)/Z_M\,;\,1\,;Y\,Z_P/Z_M\}$. That is,

$$\mathcal{S}(Z) \equiv \{\, ((k_i(0)); \bar{k}; (\kappa_i); (\delta_i); (\gamma_j); \mu; \sigma; r_0; \lambda) \mid M'_{\mathcal{J}}(0) = Z\,\}. \tag{75}$$

If, following a change in a given parameter, the set $\mathcal{S}(Z)$ increases in the sense of nesting the initial set of parameter values, the likelihood of observing the pattern of operation Z increases in the characteristic represented by the changed parameter. For the econometrician, differences in relative likelihood take the form of differences in the frequency of observation. When the characteristics of interest are easily identified, differences in characteristics can be related to differences in the frequency of observation.

Note that knowhow and core competency affect the values $S(1)$, Z_M, and $Y\,Z_P$ primarily through $K \equiv k_{\tilde{i}(0)}(0)/\bar{k}$ and $\Omega \equiv \kappa_{\tilde{i}(0)}/(\kappa_a^{\delta_a}\,\kappa_b^{\delta_b})$. It is thus relative rather than absolute values of knowhow and core competencies that determine the choice of organizational form. This distinction is important for the purpose of generating testable implications, because relative values of knowhow and core competencies are much easier to assess than are absolute values.

We examine changes in $M'_{\mathcal{J}}(0)$ starting from a situation in which all three patterns of operation have equal value, i.e. $S(1) = Z_M$, $S(1) = Y\,Z_P$, and $Z_M = Y\,Z_P$. This is because

[40] 'Advice' can be given across firms (alliances) or within (mergers before superseding).

$M'_{\mathcal{J}}(0) \equiv \max\{S(1)/Z_M\,;\,1\,;Y\,Z_P/Z_M\}$ is unlikely to change in response to small changes in the parameter of interest when one pattern of operation clearly dominates the other two forms.

We first show

$$\frac{\partial}{\partial \Xi}\left[\frac{A}{B}\right]\Bigg|_{A/B=1} \geq 0, \quad \text{for } \frac{A}{B} \in \left\{\frac{S(1)}{Z_M}\,;\,\frac{Y Z_P}{Z_M}\,;\,\frac{S(1)}{Y Z_P}\right\} \quad \text{and } \Xi \in \{K;\Omega\}\ . \tag{76}$$

The preceding implies that an increase in $\Xi \in \{K;\Omega\}$ increases the set $\mathcal{S}(S(1)/Z_M)$ and decreases the set $\mathcal{S}(1)$. This in turn implies

Result 1 (Competitive Advantage) *Higher (a) relative knowhow, K, or (b) relative core competencies, Ω, on the part of the best user (i) increases the likelihood of separate operations and (ii) decreases the likelihood of merged operations.*

The intuition is simple. An increase in the knowhow or the core competencies of the best user at date 0 increases the value to that user of operating alone. It decreases the value of joint operations within a merged firm, as there is less to be gained from combining knowhow and more to be lost, or less to be gained from CCD.[41] As for partially related operations within an alliance, the value of such operations increases relative to merged operations, but decreases relative to separate operations. The net effect is indeterminate.

We now note that the discount rate, r_0, and the failure rate, λ, which together make up what may be called the impatience factor, $r_0 + \lambda$, affect $Y\,Z_P$, and Z_M, but they do not affect $S(1)$. We show

$$\frac{\partial}{\partial \Xi}\left[\frac{A}{B}\right]\Bigg|_{A/B=1} \geq 0, \quad \text{for } \frac{A}{B} \in \left\{\frac{S(1)}{Z_M}\,;\,\frac{Y Z_P}{Z_M}\,;\,\frac{S(1)}{Y Z_P}\right\} \quad \text{and } \Xi \in \{r_0;\lambda\}\ . \tag{77}$$

We thus have

Result 2 (Impatience) *A higher discount rate, r_0, or failure rate, λ, (i) increases the likelihood of separate operations and (ii) decreases the likelihood of merged operations.*

Again, the intuition is quite simple. Joint operations, whether within a merged firm or an alliance, can be viewed as an investment in acquiring knowhow. Such investment is made to increase the profitability of the separate operations that will follow superseding or dissolution. A higher impatience factor decreases the attractiveness of investing in knowhow acquisition. This unambiguously decreases the value of merged operations and increases that of separate operations. Again, the net effect on partially related operations within an alliance is indeterminate.

[41] The best user at date 0 suffers from CCD when that user has higher core competencies. It benefits when it has lower core competencies.

As do the discount and failure rates, r_0 and λ, the learning uncertainty factors, μ and σ, affect $Y\,Z_P$ and Z_M, but not $S(1)$. We show

$$\frac{\partial}{\partial \Xi}\left[\frac{A}{B}\right]\bigg|_{A/B=1} \leq 0, \quad \text{for } \frac{A}{B} \in \left\{ \frac{S(1)}{Z_M}\,;\,\frac{YZ_P}{Z_M}\,;\,\frac{S(1)}{YZ_P} \right\} \quad \text{and } \Xi \in \{\mu;\sigma\}\ . \tag{78}$$

We thus have

Result 3 (Learning Uncertainty) *A higher drift, μ, or volatility, σ, of learning conditions, (i) decreases the likelihood of separate operations and (ii) increases the likelihood of merged operations.*

The intuition recalls that of Result 2. Learning conditions depend on the maximum of the geometric Brownian motion x_t. The higher the drift and volatility of x_t, the higher its maximum, and the more favorable therefore learning conditions. This increases the attractiveness of investing in acquiring knowhow, thereby increasing the value of merged operations and decreasing that of separate operations. Yet again, the net effect on partially related operations within an alliance is indeterminate.

The extent of double moral hazard, represented by $h_{j\in\mathcal{J}_{ab}}$, affects $Y\,Z_P$ but neither $S(1)$ nor Z_M. We show

$$\frac{\partial}{\partial h_j}\left[\frac{S(1)}{Z_M}\right]\bigg|_{S(1)/Z_M=1} = 0\,,\quad \frac{\partial}{\partial h_j}\left[\frac{YZ_P}{Z_M}\right]\bigg|_{YZ_P/Z_M=1} \geq 0\,,\ \text{and}\quad \frac{\partial}{\partial h_j}\left[\frac{S(1)}{YZ_P}\right]\bigg|_{S(1)/(YZ_P)=1} \leq 0\,. \tag{79}$$

We thus have

Result 4 (Moral Hazard) *A more severe problem of double moral hazard (a decrease in $h_{j\in\mathcal{J}_{ab}}$) decreases the likelihood of alliances.*

The intuition is very simple. Only joint operations within an alliance suffer from the problem of double moral hazard. The more severe that problem, the less valuable such operations. Separate and merged operations remain unaffected.

4.2 The Time to Superseding or to Dissolution

Joint operation is a temporary pattern of operation. The time to superseding or to dissolution is the time elapsed between the date 0 at which a merger occurs or an alliance is formed and the date at which superseding takes place or the alliance is dissolved. The latter dates are the time at which learning conditions reach the state $\hat{x}_M$ for the merger and $\hat{x}_P$ for the alliance.

Result 5 *The times to superseding and to dissolution have the comparative statics that follow:*

(Competitive Advantage) *An increase in the relative knowhow at date* 0 *of the best user at superseding or dissolution,* K*, or in the relative core competencies of that user,* Ω*, decreases the time to superseding or to dissolution* ($\partial\hat{x}/\partial\Xi < 0$*, for* $\hat{x} \in \{\hat{x}_M; \hat{x}_P\}$ *and* $\Xi \in \{K; \Omega\}$*).*

(Impatience) *An increase in the discount rate,* r_0*, or the failure rate,* λ*, decreases the time to superseding or to dissolution*($\partial\hat{x}/\partial\Xi < 0$*, for* $\hat{x} \in \{\hat{x}_M; \hat{x}_P\}$ *and* $\Xi \in \{r_0; \lambda\}$*).*

(Learning Uncertainty) *An increase in the drift,* μ*, or the volatility,* σ*, of learning conditions decreases the time to superseding or to dissolution* ($\partial\hat{x}/\partial\Xi < 0$*, for* $\hat{x} \in \{\hat{x}_M; \hat{x}_P\}$ *and* $\Xi \in \{\mu; \sigma\}$*).*

(Moral Hazard) *A more severe problem of double moral hazard (a decrease in* $h_{j\in\mathcal{J}_{ab}}$*) decreases the time to dissolution* ($\partial\hat{x}_P/\partial h_{j\in\mathcal{J}_{ab}} > 0$*). It does not affect that to superseding.*

The intuition for the preceding results is in many ways quite similar to that for the results in Section 4.1. That section showed that higher relative knowhow and core competencies on the part of the best user at date 0 decrease the attractiveness of joint operations relative to separate operations. The present section shows that, besides decreasing the attractiveness of joint operations, higher relative knowhow and core competencies decrease the duration of joint operations when these should be chosen. This is to be expected, as the decreased attractiveness of joint relative to separate operations implies that the former are sooner to be abandoned for the latter.

Recall from Section 4.1 the interpretation of joint operations as an investment in the acquisition of knowhow. The higher the discount or the failure rate, the lesser the attractiveness of such investment, the shorter the period of time over which such investment is made, and the lower therefore the time to superseding or dissolution. Regarding learning conditions, an improvement in learning conditions increases the rate at which knowhow is acquired, thereby hastening the time at which the best user has acquired the complementary knowhow it seeks in order to operate the asset alone. Finally, the more severe the problem of double moral hazard that is unique to the alliance, the greater the cost of joint operations within the alliance, the greater therefore the incentive to put an end to such joint operations.

For the econometrician, differences in time to superseding or dissolution may take the form of differences in the frequency of superseding or dissolution. The econometrician can examine the probability that superseding or dissolution occurs before a given date $t > 0$. In our setting, this expected superseding or dissolution frequency at date t is

$$edf(t) = N\left[\frac{-\ln[\hat{x}] + (\mu - \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}\right] + \exp\left(\frac{2\ln[\hat{x}]\,(\mu - \frac{\sigma^2}{2})}{\sigma^2}\right) N\left[\frac{-\ln[\hat{x}] - (\mu - \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}\right] \,, \quad (80)$$

where $N(.)$ denotes the cumulative normal distribution.[42] Note that the expected superseding or dissolution frequency decreases in $\hat{x}$, $\hat{x} = \hat{x}_M$ or $\hat{x}_P$.

4.3 The Effect of Size

We now examine how the desirability of one or another pattern of operation is affected by the number of assets, J, in the set of assets, $\mathcal{J}$.

We note that an increase in the number of assets leaves the desirability of both separate and merged operations unchanged: in Proposition 2, both $S(1)$ and Z_M are independent of $\mathcal{J}$. In contrast, we show

Result 6 *An increase in the number of assets, J, increases both (i) the likelihood and (ii) the duration of the alliance.*

The intuition is relatively straightforward. An increase in the number of assets has two effects. The first effect is to increase the number of separately operated assets that can prevail themselves of the increase in knowhow made possible by the joint operation of the 'sacrificed' asset within the alliance. The second effect is to decrease the cost of such 'sacrifice:' a larger number of assets makes possible the selection for 'sacrificial' purposes of a less valuable asset that suffers less from double moral hazard. The lower cost of and the greater benefit to 'sacrificing' an asset combine to make the alliance both more valuable and longer lived.

The preceding result has the possibly counterintuitive implication that alliances are more likely to be entered into by large firms than by small firms. Large firms have more separately operated assets that can profit from the knowhow acquired in the alliance. They also have a wider pool of assets from which to choose the single asset that is to be 'sacrificed.'

4.4 The Importance of Knowhow and Core Competencies

Much of our analysis and many of our results have revolved around knowhow and its acquisition. This is not surprising: knowhow is viewed as a central, if not the central concern of corporations by much of the management literature today. Reflecting that emphasis on knowhow (or knowledge), many corporations have created the position of Chief Knowledge Officer (CKO). In the present section, we wish to explore the importance of knowhow and its acquisition, by asking what would happen if there were no knowhow or if the knowhow there is were impossible to acquire. We show

Result 7 *If there is no knowhow, there are only separate operations. If knowhow is impossible to acquire, there are no alliances.*

[42] See Cox and Miller (1984).

The second result dramatically simplifies Proposition 2, which then states that separate operations are chosen at 0 if $K\Omega \geq 1$. Merged operations are otherwise chosen.

The intuition is simple. If there is no knowhow, only core competencies matter, and the firm with higher core competencies should operate all assets alone. If the acquisition of knowhow is impossible, there is obviously no reason to sacrifice an asset for the purpose of acquiring knowhow, i.e., there is no reason to enter into an alliance.

Besides knowhow, core competencies have loomed large in our analysis. In a sense, this is not surprising, for core competencies have been viewed by management theorists as the main, if not the unique source of competitive advantage to a firm.[43] We explore the importance of core competencies by asking what would happen if core competencies were identical for the two firms. We show

Result 8 *If there no core competencies,or two firms have identical core competencies, then there are only mergers.*

Again, the intuition is simple. The only cost of a merger is that of CCD, which arises from the difference in core competencies. There is no such cost where there are no core competencies, or where core competencies are identical. There are therefore only benefits and no cost to a merger.

5 Empirical Evidence and Testable Implications

We provide some empirical evidence in support of, and derive some testable implications from the comparative statics results of Section 4.

The phenomenon of merger waves has recently received much attention in the economics and finance literature. By and large, merger waves appear to be associated with technological and regulatory shocks (Gort, 1969; Andrade, Mitchell, and Stafford, 2001). Can we reconcile our theory with the existence and the characteristics of merger waves?

By impacting the environment in which firms operate, a shock impacts the value to a firm of its core competencies and its knowhow. These can be made more or less valuable, in absolute terms or in terms relative to those of other firms. For example, a shock may render obsolete any advantage a firm may have had over other firms in its industry, thereby leveling core competencies and knowhow. Conversely, a shock may validate the technological choices made by some firms but invalidate those made by others, thereby conferring an advantage to the former firms over the latter. The losing or gaining of an advantage may affect core

[43]See Prahalad and Hamel (1990) for example.

competencies alone, knowhow alone, or both core competencies and knowhow. We consider a number of cases in turn.

Where a shock levels core competencies and knowhow, Result 8 suggests that firms should engage in mergers: firms seek to replenish their depleted knowhow, and can do so with little concern for CCD as all core competencies have been reduced to approximately equal–and low–levels. This appears to be the pattern among established pharmaceutical firms, which have sought through mergers to replenish drug pipelines dried up by the vastly increased technological and regulatory hurdles to developing new drugs.[44]

Where, in contrast to the preceding case, a shock confers an advantage to some firms over others, both in core competencies and in knowhow, Result 1 suggests that the first set of firms should acquire the assets of the second, or indeed the entire firms. Acquiring firms add value to the assets of the firms acquired using the acquiring firms' own core competencies and knowhow. When an entire firm is acquired, its core competencies and knowhow are effectively lost, to be replaced by those of the acquiring firm. This appears to be the view of mergers and acquisitions put forward by Manne (1965) and Jovanovic and Rousseau (2002a and b) among others.

Result 1 is of little direct help in the case where one or more shocks have simultaneous and contrasting effects on core competencies and knowhow. Nonetheless, it may be possible to draw some conclusion as to the resulting pattern of operations from our understanding of the trade-offs involved in the three patterns we consider. When a first shock decreases the knowhow of one firm relative to another, and a second shock increases the core competencies of the one firm relative to the other, there may be a rationale for having the two firms enter into an alliance whereby the higher knowhow, lower core competencies firm communicates its knowhow to the lower knowhow, higher core competencies firm. The alliance is dissolved when the latter firm has acquired enough knowhow to make the most of the assets in the alliance on its own.

The pharmaceutical industry may be said to have experienced two such shocks. On the one hand, the growing importance of biotechnology has decreased the knowhow of established, chemistry-based pharmaceutical firms relative to that of biotechnology firms. On the other hand, increased regulatory requirements have increased the importance for pharmaceutical firms of managing relations with regulators. Such management can be said to be part of large pharmaceutical firms' core competencies. Alliances between large pharmaceutical firms and biotechnology firms may then be interpreted as an attempt at communicating the latter's knowhow to the former, eventually to capitalize on the former's core competencies. Interestingly, following the completion of the more purely scientific phase of a new drug's development, and before the new drug is shepherded through the process of regulatory ap-

[44]See Jack and Weismann (2006).

proval, the rights to the drug–the asset–often are acquired by the large pharma partner in the alliance.

Another reason for the increased number of alliances between pharmaceutical and biotechnology firms may be the adoption by the partners in biotechnology alliances of contractual features that have made possible a decrease in double moral hazard, in line with result 4. Robinson and Stuart (2002) report the presence in alliance contracts of many of the clauses generally found in venture capital contracts. Mitigating moral hazard is of course a central purpose of venture capital contracts (Kaplan and Strömberg, 2002), and the incorporation in alliance contracts of clauses that make this possible may provide an additional explanation for the recent increase in the number of biotechnology alliances.

We now turn to superseding in mergers. To what extent does such a process occur, and does it conform to our theory and our predictions?

To answer this question, we consider the 1998 merger between Citigroup (Citi) and Travelers Group (Travelers), which created the world's largest financial services firm.[45] What made the merger in some ways unique, besides its unprecedented scale and scope, was the relatively large number of top management positions assumed jointly by co-heads.[46] Indeed, so many positions were shared that CSFB banking analyst Michael Mayo referred to the extensive use of the co-head structure by the merged firm as "the Noah's Ark school of management," with people brought along "two by two."[47] For most shared positions, one head had been with Citi and the other with Travelers. This arrangement extended to the CEO position, which was shared by Citi's John Reed and Travelers's Sandy Weil.[48] At the outset, then, the two forms of organizations corresponding to the original merging firms can be said to have co-existed within the merged firm.

Perhaps unsurprisingly, the co-head structure created a number of problems, what we have referred to as the costs of CCD. In this particular case, these seem to have taken the form for many employees of having to satisfy two bosses with often conflicting demands. As Victor Menezes, himself co-head of the global corporate and investment bank, rather nicely put it, "we can follow any compass, but we have to know where north, south, east, and west are."[49] Weil eventually became sole CEO of the firm, Reed retired, and most of the co-head structure was abandoned. Having become sole CEO, Weil was able to put his imprint on the firm to a much greater extent than he had been able to under the co-

[45]There appears to be no formal empirical study of superseding in mergers.

[46]While the co-head structure had not been uncommon among financial services firms, never before had it been used to such an extent.

[47]See Authers and Corrigan (1998).

[48]The sharing of the CEO position may be considered necessary for Reed and Weil to have agreed to the merger. It is not clear why this should extend below the level of CEO, though.

[49]As reported by Lowenstein. See Stone and Brewster (2002, p. 264).

CEO arrangement. As a result, the merged firm came to resemble in its organization and its culture the original Travelers much more than the original Citi. In our terminology, the organizational form corresponding to Travelers had superseded that corresponding to Citi. Interestingly, superseding occurred only after Weil felt "he no longer needed Reed's knowledge of the bank or Reed's allies within the bank. Weil knew what he wanted to do, where to make cuts, whom to trust."[50] In our terminology, superseding occurred only after one organization had acquired much of the knowhow of the other.

A first testable implication of our analysis, then, is that the greater the stated importance of the skills and knowhow of the two merging firms in a merger, the more likely it is that these firms' organizational forms are made to co-exist within the merged firm. Conditional on such being the case, a second implication is that superseding, or the eventual dominance of one form of organization over the other, occurs only after the superseding organization has acquired much of the knowhow of its superseded counterpart. A third implication is this: where knowhow is not an issue, or where only the knowhow of one firm is deemed important, asset sales or acquisitions are likely to be chosen over alliances and mergers.

In contrast, where knowhow is an issue, a fourth implication is that the more different two firms are, in their businesses and their markets, their organization and their structure, and their culture and their technologies, the more likely are the firms to choose an alliance over a merger. A fifth implication is that alliances motivated by skills and knowhow are more likely to be dissolved than those motivated by alternative explanations such as economies of scale or scope.[51]

6 Conclusion

We believe our analysis has highlighted three important, perhaps unexpected considerations: the paramount role of core competencies, the determining role of the congruence between knowhow and core competencies, and the temporary nature of joint operations.

The paramount role of core competencies is highlighted by the result that mergers are chosen in the absence of a large difference in core competencies–regardless of what the state of knowhow might be. This may be viewed as confirming the primacy accorded core competencies by much of the management and strategy literature.[52]

The congruence between knowhow and core competencies is important because it determines the choice between separate operations and alliances where large differences in core

[50]See Stone and Brewster (2002, p. 253).

[51]It is easy to incorporate scale and scope considerations into our model; there are then cases in which joint operations are permanent.

[52]See for example Barney (2006).

competencies exclude mergers. When knowhow and core competencies are congruent, in the sense that the party that has higher knowhow also has higher core competencies, separate operations are chosen. When knowhow and core competencies are not congruent, partially related operations within an alliance are chosen, generally to provide the party with higher core competencies with the opportunity to acquire part of the knowhow of the party with higher knowhow.

The temporary nature of joint operations–where these are motivated primarily by knowhow considerations–is an immediate consequence of the parties' ability to acquire knowhow. There comes a point in time at which the benefits of combining knowhow and acquiring further knowhow have been so diminished by the knowhow acquisition that has already taken place that these benefits are no longer sufficient to offset the costs of joint operations: moral hazard in alliances and core competencies difference in alliances and in mergers.

Our analysis is of course not without its limitations; it has left out many relevant considerations. In particular, it has assumed that both competition and aggregate knowhow remain unchanged. Such may not be the case: a merger in particular may decrease competition, and R&D may increase aggregate knowhow. Either consideration would affect the values of some or all patterns of operation. Both considerations are, however, 'orthogonal' to our analysis, in the sense that their effect can be reduced to the scaling of the terms on the right-hand side of (67) by multiplicative factors that measure the impact of changed competition and aggregate knowhow on the values of the three patterns of operations.[53]

As noted in footnotes 22 and 25, our analysis has the somewhat extreme result that all assets eventually will be operated by the same user. Thus, in our model, there are no circumstances in which one firm operates some assets and the other firm other assets, no divestures, and no demergers. This result is a consequence of our assumption – made for the purpose of obtaining clear testable implications – that core competencies are firm- but not asset-specific. Making core competencies asset-specific, in the sense that a firm's core competencies may be more applicable to some assets than to other assets, may be one way to extend the model to analyze such issues as divestures or demergers, or the relatively recent developments of focussing and outsourcing.[54] We view this as a topic for further research.

[53]For example, if competition were most reduced following a merger, or R&D conducted most efficiently within a merged firm, the value of merged operations would be scaled by larger factor than would the values of wholly unrelated and partially related operations.

[54]As do divestures and demergers, focussing and outsourcing imply the separate operation of different assets by different firms.

Appendix

Proof of (9) and (10): Discounting when liquidation occurs with intensity λ is analogous to discounting at the sum of the short rate and that same intensity (Lando, 1998). To see this, use $E_t\left[1_{\{\tau<\bar{t}\}}\right] = \exp^{-\lambda(\tau-t)}$ to obtain

$$V_{i,j,U}(k_i(t_0)) = \max_{e_{i,j}(.)}\left\{E_t\left[\int_t^{+\infty}\exp^{-(r_0+\lambda)(\tau-t)}\left\{e_{i,j}(\tau)^{\gamma_j} - \frac{e_{i,j}(\tau)}{(k_i(t_0)\,\kappa_i)^{\frac{1-\gamma_j}{\gamma_j}}}\right\}d\tau\right]\right\}. \quad (81)$$

Solving (81), we obtain $e_{i,j}(\tau)$ in (10). Substituting yields (9). □

Proof of (24) to (31): Problem (23) is equivalent to

$$W_M(\bar{k}) = \max_{\hat{x}}\left\{\sum_{j\in\mathcal{J}} W_{M,j}(\bar{k}\mid\hat{x}) + E_t\left[\exp^{-r_0(\hat{t}-t)}\,W^*(\hat{t})\,1_{\{\hat{t}<\bar{t}\}}\right]\right\} \quad (82)$$

$$\text{where} \quad W_{M,j}(\bar{k}\mid\hat{x}) \equiv E_t\left[\int_t^{\hat{t}}\exp^{-r_0(\tau-t)}\max_{e_{a,j}(\tau),e_{b,j}(\tau)}\left\{P_j(\tau)\,1_{\{\tau<\bar{t}\}}\right\}\right], \quad (83)$$

denotes the value at date t, $t\in(t_0,\hat{t})$, of operating asset j, $j\in\mathcal{J}$, within a merged firm until the time of superseding $\hat{t} = \inf\{\,t\mid x_t = \hat{x}\,\}$.[55] In turn,

$$P_j(\tau) = R_j(\mathbf{e}(\tau)) - \sum_{i\in\{a;b\}} C_{i,j}(e_{i,j}(\tau)) = \left(\frac{e_{a,j}(\tau)^{\delta_a}\,e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}}\right)^{\gamma_j} - \sum_{i\in\{a;b\}}\frac{e_{i,j}(\tau)}{(\bar{k}\,\kappa_i)^{\frac{1-\gamma_j}{\gamma_j}}}. \quad (84)$$

By analogy to the derivation in the Proof of (9) and (10), we have

$$W_{M,j}(\bar{k}\mid\hat{x}) = E_t\left[\int_t^{\hat{t}}\exp^{-(r_0+\lambda)(\tau-t)}\max_{e_{a,j}(\tau),e_{b,j}(\tau)}\{P_j(\tau)\}\right] \quad \text{and} \quad (85)$$

$$E_t\left[\exp^{-r_0(\hat{t}-t)}\,W^*(\hat{t})\,1_{\{\hat{t}<\bar{t}\}}\right] = E_t\left[\exp^{-(r_0+\lambda)(\hat{t}-t)}\,W^*(\hat{t})\right]. \quad (86)$$

Denote $W^{\infty}_{M,j}(\bar{k})$ the value at date t, $t\geq t_0$, of operating asset j, $j\in\mathcal{J}$, assuming superseding never occurs. That is, $W^{\infty}_{M,j}(\bar{k}) \equiv W_{M,j}(\bar{k}\mid\hat{x})\mid_{\hat{x}\to\infty}$. We have

$$W^{\infty}_{M,j}(\bar{k}) = E_t\left[\int_t^{+\infty}\exp^{-(r_0+\lambda)(\tau-t)}\max_{e_{a,j}(\tau),e_{b,j}(\tau)}\{P_{i,j}(\tau)\}\,d\tau\right]. \quad (87)$$

Define $T(\bar{x}) \equiv \inf\{\,T\mid x(T) = \bar{x}\,\}$ to be the first time $x(t)$ reaches an upper barrier $\bar{x}$. Conditional on the information at date t, the density of $T(\bar{x})$ has Laplace transform[56]

$$\mathcal{L}\left(f(T(\bar{x}))\mid r_0+\lambda\right) \equiv \int_t^{\infty}\exp^{-(r_0+\lambda)(T(\bar{x})-t)}\,f(T(\bar{x}))\,dT(x^*) = \left(\frac{x(t)}{\bar{x}}\right)^{\xi}, \quad (88)$$

$$\text{where} \quad \xi \equiv \sigma^{-2}\left[\sigma^2/2-\mu+\sqrt{(\mu-\sigma^2/2)^2+2(r_0+\lambda)\sigma^2}\right]. \quad (89)$$

[55] In writing (82) we have used our model's property that the resource contributions that maximize instantaneous profits are those that maximize value. That is, we have used $\arg\max_{e_{a,j}(\tau),e_{b,j}(\tau)}[P_j(\tau)] = \arg\max_{e_{a,j}(\tau),e_{b,j}(\tau)}\left[W_{M,j}(\bar{k}\mid\hat{x})\right] = \arg\max_{e_{a,j}(\tau),e_{b,j}(\tau)}\left[W_M(\bar{k})\right]$.

[56] The Laplace transform is defined as $\mathcal{L}(f(t)\mid s) \equiv \int_0^{\infty} e^{-st}f(t)dt$.

Note that $\xi > 1$. Intuitively, $\mathcal{L}\left(f(T(\bar{x})) \mid r_0 + \lambda\right)$ is the expected value at date t of 1$ received at the random time $T(\bar{x})$. Using (88) and (89) we can write (85) as

$$W_{M,j}(\bar{k} \mid \hat{x}) \;=\; W^{\infty}_{M,j}(\bar{k})\left[1 \;-\; \left(\frac{x_t}{\hat{x}}\right)^{\xi}\right] \;,\quad \text{and}\quad E_t\Big[\exp^{-(r_0+\lambda)(\hat{t}-t)}\,W^*(\hat{t})\Big] \;=\; W^*(\hat{t})\left(\frac{x_t}{\hat{x}}\right)^{\xi} \tag{90}$$

We can therefore rewrite (82) as

$$W_M(\bar{k}) = \max_{\hat{x}}\left\{\sum_{j\in\mathcal{J}} W^{\infty}_{M,j}(\bar{k})\left[1 \;-\; \left(\frac{x_t}{\hat{x}}\right)^{\xi}\right] + W^*(\hat{t})\left(\frac{x_t}{\hat{x}}\right)^{\xi}\right\}. \tag{91}$$

The optimal dissolution time solves the f.o.c. for $\hat{x}$, $\partial W_M(\bar{k})/\partial\hat{x} = 0$. Now, differentiating (84) and solving for $e_{i,j}(\tau)$ yields

$$e_{i,j}(\tau) = \delta_i\,\gamma_j\,\left(\bar{k}\,\kappa_i\right)^{\frac{1-\gamma_j}{\gamma_j}}\left(\frac{e_{a,j}(\tau)^{\delta_a}\,e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}}\right)^{\gamma_j}. \tag{92}$$

Using (92) for a and b, we can write

$$\frac{e_{a,j}(\tau)^{\delta_a}\,e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}} \;=\; \left(\bar{k}\,\kappa_a^{\delta_a}\,\kappa_b^{\delta_b}\right)^{\frac{1}{\gamma_j}}\,\gamma_j^{\frac{1}{1-\gamma_j}}. \tag{93}$$

Substituting (93) into (92) yields (30). Substituting (92) into (84), we have

$$P_j(\tau) \;=\; R_j(\mathbf{e}(\tau)) - \sum_{i\in\{a;b\}} C_{i,j}(e_{i,j}(\tau)) \;=\; (1-\gamma_j)\left(\frac{e_{a,j}(\tau)^{\delta_a}\,e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}}\right)^{\gamma_j}. \tag{94}$$

Combining (87), (93), and (94), we can rewrite (91) as

$$W_M(\bar{k}) \;=\; \left[\bar{m} \;+\; (m(\hat{x}) \;-\; \bar{m})\left(\frac{x_t}{\hat{x}}\right)^{\xi}\right]\sum_{j\in\mathcal{J}}\Gamma_j\,. \tag{95}$$

Given that $m(\hat{x}) = m^{\infty} - [m^{\infty} - n_{\tilde{i}(\hat{t})}(\hat{x}_{t_0})]/\hat{x}$, where $m^{\infty} \equiv \bar{k}\,\kappa_{\tilde{i}(\hat{t})}$, we decompose $W_M(\bar{k})$ into $W_M(\bar{k}) = Y_M\,Z_M$, where

$$Y_M \;\equiv\; \bar{m}\sum_{j\in\mathcal{J}}\Gamma_j\,,\qquad Z_M \;\equiv\; 1 \;+\; \left[\frac{A \;-\; B/\hat{x}}{Y_M}\right]\left(\frac{x_{t_0}}{\hat{x}}\right)^{\xi}, \tag{96}$$

with $A \equiv [m^{\infty} - \bar{m}]\sum_{j\in\mathcal{J}_{ab}}\Gamma_j$, $B \equiv [m^{\infty} - n_{\tilde{i}(\hat{t})}(\hat{x}_{t_0})]\;\sum_{j\in\mathcal{J}}\Gamma_j$. With $W_M(\bar{k})$ decomposed as such, the f.o.c. for $\hat{x}$ yields $\hat{x} = (1+\xi)B/[\xi\,A]$, which is (31). Substituting into (96) we obtain $Z_M = 1 + \frac{A}{(1+\xi)\,Y_M}\left(\frac{x_{t_0}}{\hat{x}}\right)^{\xi}$. Now, if $\hat{x}$ is finite, then $Z_M \geq 1$, hence $m^{\infty} > \bar{m}$. Therefore $\kappa_{\tilde{i}(\hat{t})} > \kappa_a^{\delta_a}\,\kappa_b^{\delta_b}$. So, $\tilde{i}(\hat{t}) = \arg\max_i\{\kappa_i\}$ and $m^{\infty} = \bar{k}\;\max_i\{\kappa_i\}$. □

Proof of (42) to (55): Problem (36) is equivalent to

$$\begin{cases} V_{a,P}(t) \;=\; \max_{\hat{x}_a}\left\{\sum_{j\in\mathcal{J}_{ab}\cup\mathcal{J}_a} V_{a,j}(t\mid\hat{x}) + E_t\Big[\exp^{-r_0(\hat{t}-t)}\,V_a^*(\hat{t})\,1_{\{\hat{t}<\bar{t}\}}\Big]\right\} \\ V_{b,P}(t) \;=\; \max_{\hat{x}_b}\left\{\sum_{j\in\mathcal{J}_{ab}\cup\mathcal{J}_b} V_{b,j}(t\mid\hat{x}) + E_t\Big[\exp^{-r_0(\hat{t}-t)}\,V_b^*(\hat{t})\,1_{\{\hat{t}<\bar{t}\}}\Big]\right\} \end{cases} \tag{97}$$

$$\text{where}\qquad V_{i,j}(t\mid\hat{x}) \;\equiv\; E_t\left[\int_t^{\hat{t}}\exp^{-r_0(\tau-t)}\max_{e_{i,j}(\tau)}\left\{P_{i,j}(\tau)\,1_{\{\tau<\bar{t}\}}\right\}\right], \tag{98}$$

denotes the value to firm i at date t, $t \in (t_0, \hat{t})$, of operating asset j, $j \in \mathcal{J}_{ab} \cup \mathcal{J}_i$, until the time of dissolution $\hat{t} = \inf\{\, t \mid x_t = \hat{x} \,\}$.[57] By analogy to the derivation in the Proof of (9) and (10),

$$V_{i,j}(t \mid \hat{x}) \;=\; E_t\left[\int_t^{\hat{t}} \exp^{-(r_0+\lambda)(\tau-t)} \max_{e_{i,j}(\tau)} \{P_{i,j}(\tau)\}\right] \quad \text{and} \tag{99}$$

$$E_t\Big[\exp^{-r_0(\hat{t}-t)}\, V_i^*(\hat{t})\, 1_{\{\hat{t}<\bar{t}\}}\Big] \;=\; E_t\Big[\exp^{-(r_0+\lambda)(\hat{t}-t)}\, V_i^*(\hat{t})\Big]\,. \tag{100}$$

Denote $V_{i,j}^{\infty}(t)$ the value to firm i at date t, $t \geq t_0$, of operating asset j, $j \in \mathcal{J}_{ab} \cup \mathcal{J}_i$, assuming dissolution never occurs. That is, $V_{i,j}^{\infty}(t) \equiv V_{i,j}(t \mid \hat{x})\mid_{\hat{x}\to\infty}$. We have

$$V_{i,j\in\mathcal{J}_{ab}}^{\infty}(t) \;\equiv\; E_t\left[\int_t^{+\infty} \exp^{-(r_0+\lambda)(\tau-t)} \max_{e_{i,j}(\tau)} \{P_{i,j}(\tau)\}\, d\tau\right]. \tag{101}$$

As for the rewriting of (85) as (90), we can rewrite (99) as

$$V_{i,j}(t \mid \hat{x}) \;=\; V_{i,j}^{\infty}(t) \;-\; V_{i,j}^{\infty}(\hat{t})\left(\frac{x_t}{\hat{x}}\right)^{\xi}, \quad \text{and} \quad E_t\Big[\exp^{-(r_0+\lambda)(\hat{t}-t)}\, V_i^*(\hat{t})\Big] \;=\; V_i^*(\hat{t})\left(\frac{x_t}{\hat{x}}\right)^{\xi}. \tag{102}$$

We can therefore rewrite (97) as

$$V_{i,P}(t) = \max_{\hat{x}_i}\left\{V_{i,P}^{\infty}(t) + \left[V_i^*(\hat{t}) \;-\; V_{i,P}^{\infty}(\hat{t})\right]\left(\frac{x_t}{\hat{x}}\right)^{\xi}\right\}, \qquad \text{with } \hat{x} \;\equiv\; \min\{\hat{x}_a; \hat{x}_b\}\,. \tag{103}$$

Each firm's privately optimal dissolution time solves the f.o.c. for $\hat{x}_i$, $\partial V_{i,P}(t)/\partial \hat{x}_i = 0$. A necessary condition for the contract to be renegotiation-proof is that the two firm' privately optimal dissolution times coincide: $\hat{t}_a = \hat{t}_b$. We henceforth consider only those contracts that satisfy the equivalent condition $\hat{x}_a = \hat{x}_b$.

We first consider $V_{i,j\in\mathcal{J}_{ab}}^{\infty}(t)$. This is the value to firm i at date t, $t \geq t_0$, of operating within an alliance an asset j, $j \in \mathcal{J}_{ab}$, assuming dissolution never occurs. We have

$$V_{i,j\in\mathcal{J}_{ab}}^{\infty}(t) \;=\; E_t\left[\int_t^{+\infty} \exp^{-(r_0+\lambda)(\tau-t)} \max_{e_{i,j}(\tau)} \{\phi_{i,j}\, R_j(\mathbf{e}(\tau)) \;-\; C_{a,j}(e_{a,j}(\tau))\}\, d\tau\right], \tag{104}$$

$$\text{where}\;\; \phi_{i,j}\, R_j(\mathbf{e}(\tau)) - C_{i,j}(e_{i,j}(\tau)) \;=\; \phi_{i,j}\left(\frac{e_{a,j}(\tau)^{\delta_a}\, e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}}\right)^{\gamma_j} - \frac{e_{i,j}(\tau)}{\left(\bar{k}\,\kappa_i\right)^{\frac{1-\gamma_j}{\gamma_j}}}\,. \tag{105}$$

Differentiating (105) and solving for $e_{i,j}(\tau)$ yields

$$e_{i,j}(\tau) = \phi_{i,j}\,\delta_i\,\gamma_j\,\left(\bar{k}\,\kappa_i\right)^{\frac{1-\gamma_j}{\gamma_j}}\left(\frac{e_{a,j}(\tau)^{\delta_a}\, e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}}\right)^{\gamma_j}. \tag{106}$$

Using (106) for a and b, we can write

$$\frac{e_{a,j}(\tau)^{\delta_a}\, e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}} \;=\; \left(\phi_{a,j}^{\delta_a}\,\phi_{b,j}^{\delta_b}\right)^{\frac{1}{1-\gamma_j}}\left(\bar{k}\,\kappa_a^{\delta_a}\,\kappa_b^{\delta_b}\right)^{\frac{1}{\gamma_j}}\gamma_j^{\frac{1}{1-\gamma_j}}. \tag{107}$$

[57] In writing (97), we have used our model's property that the resource contributions that maximize instantaneous profits are those that maximize value. That is, we have $\arg\max_{e_{i,j}(\tau)}[P_{i,j}(\tau)] = \arg\max_{e_{i,j}(\tau)}[V_{i,j}(t \mid \hat{x})] = \arg\max_{e_{i,j}(\tau)}[V_{i,P}(t)]$.

Substituting (107) into (106) yields (51). Substituting (106) into (105), we have

$$\phi_{i,j}\, R_j(\mathbf{e}(\tau)) - C_{i,j}(e_{i,j}(\tau)) \;=\; \phi_{i,j}\;(1\,-\,\gamma_j\,\delta_i)\,\left(\frac{e_{a,j}(\tau)^{\delta_a}\; e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}}\right)^{\gamma_j} \;. \tag{108}$$

Combining (104), (107), and (108) yields (44). We now consider $V^{\infty}_{i,j\in\mathcal{J}_i}(t)$. This is the value to firm i at date t, $t \geq t_0$, of operating separately an asset j, $j \in \mathcal{J}_i$, assuming dissolution never occurs. We have

$$V^{\infty}_{i,j\in\mathcal{J}_i}(t) \;=\; E_t\left[\int_t^{+\infty} \exp^{-(r_0+\lambda)(\tau-t)} \max_{e_{i,j}(\tau)}\left\{ e_{i,j}(\tau)^{\gamma_j} \;-\; \frac{e_{i,j}(\tau)}{(k_i(\tau)\,\kappa_i\,)^{\frac{1-\gamma_j}{\gamma_j}}}\right\} d\tau\right] \;. \tag{109}$$

Maximizing, we obtain $e_{i,j}(\tau) = (k_i(\tau)\,\kappa_i\,)^{\frac{1}{\gamma_j}}\,\gamma_j^{\frac{1}{1-\gamma_j}}$. Substituting, we have

$$V^{\infty}_{i,j\in\mathcal{J}_i}(t) \;=\; E_t\left[\int_t^{+\infty} \exp^{-(r_0+\lambda)(\tau-t)}\; k_i(\tau)\,\kappa_i\; d\tau\right]\;(1-\gamma_j)\,\gamma_j^{\frac{\gamma_j}{1-\gamma_j}} \;. \tag{110}$$

Denote $\hat{x}(t,\tau) \equiv \max_{t\leq u\leq\tau} x_u$, $r \equiv r_0 + \lambda$, and

$$\Pi(x_t, \hat{x}(0,t) \mid k_i(.), \kappa) \;\equiv\; E_t\left[\int_t^{\infty} e^{-r(\tau-t)}\, k_i(\hat{x}(0,\tau))\,\kappa\, d\tau\right] \;. \tag{111}$$

Denote $T(\bar{x}) \equiv \inf\{\, T \mid x(T) = \bar{x}\,\}$ the first time $x(t)$ reaches an upper barrier $\bar{x}$. Given that at dates τ within the interval $(\,t, T(\hat{x}(0,t))\,)$ knowhow $k_i(\hat{x}(0,\tau))$ is constant and equal to $k_i(\hat{x}(0,t))$, we can use (88) to write

$$\Pi(x_t, \hat{x}(0,t) \mid k_i(.), \kappa) \;=\; \frac{k_i(\hat{x}(0,t))\,\kappa}{r}\left[1 - \left(\frac{x(t)}{\hat{x}(0,t)}\right)^{\xi}\right] + \Pi(\hat{x}(0,t), \hat{x}(0,t) \mid k_i(.), \kappa)\left(\frac{x(t)}{\hat{x}(0,t)}\right)^{\xi} \tag{112}$$

Denote $Z(\hat{x}(0,t), \eta)$ the expected value at a date t, with $x(t) = \hat{x}(0,t)$, of an infinite stream of instantaneous income flow $1\$ \times \hat{x}(0,\tau)^{\eta}$ to be received at all subsequent dates τ, $\tau \in (t;\infty)$.

$$Z(\hat{x}(0,t), \eta) \;\equiv\; E_t\left[\int_t^{\infty} e^{-r(\tau-t)}\; \hat{x}(0,\tau)^{\eta}\; d\tau\right] \;. \tag{113}$$

Given that $x(t) = \hat{x}(0,t)$, we have $k_i(\hat{x}(0,\tau)) = \bar{k} \,-\, \left(\bar{k} \,-\, k_i(0)\right)/\hat{x}(0,\tau)$ and can therefore write

$$\Pi(\hat{x}(0,t), \hat{x}(0,t) \mid k_i(.), \kappa) \;=\; \left[\bar{k}\, Z(\hat{x}(0,t), 0) \,-\, \left(\bar{k} - k_i(0)\right)\, Z(\hat{x}(0,t), -1)\right]\,\kappa \;. \tag{114}$$

Denote $z(\tau - t \mid \hat{x}(0,t), \eta) \equiv E_t\left[e^{-r(\tau-t)}\; \hat{x}(0,\tau)^{\eta}\right]$, the expected value at a date t, with $x(t) = \hat{x}(0,t)$, of $1\$ \times \hat{x}(0,\tau)^{\eta}$ to be received at a subsequent date τ. We thus have $Z(\hat{x}(0,t), \eta) = \int_t^{\infty} z(\tau - t \mid \hat{x}(0,t), \eta)\; d\tau$. We effectively value $z(.)$ as a look-back power option, and $Z(.)$ as the sum of an infinite, converging series of such options. Denote $X(t,\tau) \equiv ln(x_\tau/x(t))$ and $\hat{X}(t,\tau) \equiv ln(\hat{x}(t,\tau)/x(t))$. The probability density function of $\hat{X}(t,\tau) = M$ (See Harrison, 1985, p. 15), is

$$\begin{aligned}\hat{f}(M) \;=\;& \frac{1}{\sigma\sqrt{(\tau-t)}}\,\phi\left[\frac{M - m(\tau-t)}{\sigma\sqrt{\tau-t}}\right] \,+\, \exp\left(\frac{2mM}{\sigma^2}\right)\frac{1}{\sigma\sqrt{(\tau-t)}}\,\phi\left[\frac{M + m(\tau-t)}{\sigma\sqrt{\tau-t}}\right] \\ & -\, \frac{2m}{\sigma^2}\,\exp\left(\frac{2mM}{\sigma^2}\right)\,\Phi\left[\frac{-M - m(\tau-t)}{\sigma\sqrt{\tau-t}}\right].\end{aligned} \tag{115}$$

Given that $\hat{x}(0,t) = x(t)$, we have $\hat{x}(0,\tau) = \hat{x}(t,\tau)$ and $\hat{x}(t,\tau) \geq x(t)$. Changing variable,

$$z(\tau - t \mid \hat{x}(0,t),\eta) \quad = \quad \hat{x}(0,t)^{\;\eta}\, e^{-r(\tau-t)} \int_0^{+\infty} exp[\eta \hat{X}(t,\tau)]\ \hat{f}(\hat{X}(t,\tau))\ d\hat{X}(t,\tau)\ . \qquad (116)$$

Integrating by parts, we have

$$z(\tau - t \mid \hat{x}(0,t),\eta) \quad = \quad \hat{x}(0,t)^{\;\eta} \left((1+d_1)\, e^{-s_1(\tau-t)}\, \Phi\left[a_1\sqrt{\tau-t}\right] \;+\; (1-d_1)\, e^{-r(\tau-t)} \Phi\left[-a_2\sqrt{\tau-t}\right]\right) (117)$$

where $s_1 \equiv r - \eta(\mu + (\eta-1)\,\sigma^2/2)$, $d_1 \equiv \frac{\eta\,\sigma^2}{2(\mu+(\eta-1)\,\sigma^2/2)}$, $a_1 \equiv \frac{\mu+(2\eta-1)\sigma^2/2}{\sigma}$, $a_2 \equiv \frac{\mu-\sigma^2/2}{\sigma}$. To compute $Z(\hat{x}(0,t),\eta)$, we use the Laplace transform of the error function, $\mathrm{erf}(t) \equiv \frac{2}{\sqrt{\pi}}\int_0^t e^{-u^2}du$ (e.g., Schiff, 1999, p. 213), $\mathcal{L}\left(\mathrm{erf}\left(a\sqrt{t}\right) \mid s\right) = a/[s\sqrt{s+a^2}]$, for $a > 0$ and $t > 0$, to obtain $\int_0^\infty e^{-st}\Phi\left[a\sqrt{t}\right]dt = 1/2s + |a|\,\mathrm{sg}[a]/[2s\sqrt{2\,s+a^2}]$. Given that $\mathrm{sg}[a]\,|a| = a$, for all $a \in \mathbf{R}$, we have

$$Z(\hat{x}(0,t),\eta) \quad = \quad \frac{\hat{x}(0,t)^{\;\eta}}{2}\left[\frac{1+d_1}{s_1}\left(1+\frac{a_1}{s_1\sqrt{2\,s_1+a_1^2}}\right)+\frac{1-d_1}{r}\left(1-\frac{a_2}{r\sqrt{2\,r+a_2^2}}\right)\right] \quad (118)$$

In particular, for $\eta = -1$ we have $s_1 \equiv r + \mu - \sigma^2$, $d_1 \equiv \frac{-\sigma^2/2}{\mu-\sigma^2}$, $a_2 \equiv \frac{\mu-\sigma^2/2}{\sigma}$ and $a_1 \equiv a_2 - \sigma$. Using the fact that then $1 + d_1 = -2a_1/(d_1\sigma)$ and $1 - d_1 = -2a_2/(d_1\sigma)$, we express $Q \equiv 1/[\hat{x}(0,t)\,Z(\hat{x}(0,t),-1)\,r]$ in the simpler fashion (49). Also, clearly, $Z(\hat{x}(0,t),0) = 1/r$. Substituting in (114) and in turn in (112), we obtain

$$\Pi(x_t,\hat{x}(0,t) \mid k_i(.),\kappa) \quad = \quad \left\{\bar{k} - \frac{\left(\bar{k}-k_i(0)\right)}{\hat{x}(0,t)}\left[1 - (1-\frac{1}{Q})\left(\frac{x(t)}{\hat{x}(0,t)}\right)^{\xi}\right]\right\}\frac{\kappa}{r} \qquad (119)$$

With (111) and (119), we can rewrite (110) as (47). With the expressions above for $V^{\infty}_{i,j\in\mathcal{J}_i}(t)$ and $V^{\infty}_{i,j\in\mathcal{J}_{ab}}(t)$, the f.o.c. for $\hat{x}_i$ yields

$$\hat{x}_i \quad = \quad \left(\frac{1+\xi}{\xi}\right)\frac{\sum_{j\in\mathcal{J}_{ab}}\psi_{i,j}\,V_{\tilde{i}(\hat{t}_i),j,U}(\bar{k}-k_{\tilde{i}(\hat{t}_i)}(0)) \;+\; (1-Q^{-1})\sum_{j\in\mathcal{J}_i}V_{i,j,U}(\bar{k}-k_i(0))}{\varphi_i \;+\; \sum_{j\in\mathcal{J}_{ab}}\psi_{i,j}\;V_{\tilde{i}(\hat{t}_i),j,U}(\bar{k}) \;-\; \sum_{j\in\mathcal{J}_{ab}}V^{\infty}_{i,j}(0)}\ . \ (120)$$

The renegotiation-proofness condition $\hat{x}_a = \hat{x}_b$, is then equivalent to (55). For contracts satisfying (55), $\hat{x}$ can be expressed as

$$\hat{x} \quad = \quad \left(\frac{1+\xi}{\xi}\right)\frac{\sum_{j\in\mathcal{J}_{ab}}V_{\tilde{i}(\hat{t}),j,U}(\bar{k}-k_{\tilde{i}(\hat{t})}(0)) \;+\; (1-Q^{-1})\sum_{j\in\mathcal{J}_a\cup\mathcal{J}_b}V_{i,j,U}(\bar{k}-k_i(0))}{\sum_{j\in\mathcal{J}_{ab}}V_{\tilde{i}(\hat{t}),j,U}(\bar{k}) \;-\; \sum_{j\in\mathcal{J}_{ab}}(V^{\infty}_{a,j}(0)+V^{\infty}_{b,j}(0))}\ . \quad (121)$$

which can be rewritten as (53). We now derive the optimal sharing rule, $\phi_{i,j}$. Using the envelope theorem,we have $\arg\max_{\phi_{i,j}}\ W_P(t) = \arg\max_{\phi_{i,j}}\{R_J(\mathbf{e}(\tau) - \sum_{i\in\{a;b\}}C_{i,j}(e_{i,j}(\tau))\} = \arg\max_{\phi_{i,j}}\{h_j\}$. Using (46), we then have

$$\frac{\partial\,h_j}{\partial\,\phi_{i,j}} \quad = \quad \left\{\frac{(1-\phi_{a,j}\,\delta_a\,\gamma_j - \phi_{b,j}\,\delta_b\,\gamma_j)}{1-\gamma_j}\left(\frac{\delta_i}{\phi_{i,j}} - \frac{\delta_{i^-}}{\phi_{i^-,j}}\right) - (\delta_i - \delta_{i^-})\right\}\frac{\gamma_j\left[\phi_{a,j}^{\delta_a}\,\phi_{b,j}^{\delta_b}\right]^{\frac{\gamma_j}{1-\gamma_j}}}{(1-\gamma_j)}\ . \quad (122)$$

Solving for $\phi_{i,j}$ and $\phi_{i^-,j}$, we obtain (54). We then have $\partial\left[R_J(\mathbf{e}(\tau) - \sum_{i\in\{a;b\}}C_{i,j}(e_{i,j}(\tau))\right]/\partial\,\psi_{i,j} = 0$, so $\partial\,W_P(t)/\partial\,\psi_{i,j} = 0$. Therefore, a contract ϕ_i and $(\varphi_{i,j},\psi_{i,j})$ maximizes $W_J(x(t))$ and is renegotiation-proof at all dates $t \geq t_0$ if it satisfies (54) and (55). □

Proof of (61): Note that at date $\hat{t}$, $x_{\hat{t}} = \hat{x}_{\hat{t}} = \hat{x}$, so $q(\hat{t}) = \hat{x}\,Q$. We can then decompose $\tilde{W}_P(t_0)$ into $\tilde{W}_P(t_0) = Y_P\,Z_P$, where

$$Y_P \equiv \sum_{j\in\mathcal{J}_{ab}} [\bar{m}\ h_j\ \Gamma_j] + \sum_{j\in\mathcal{J}\setminus\mathcal{J}_{ab}} [m(q(t_0))\,\Gamma_j]\ , \tag{123}$$

$$Z_P \equiv 1 + \left[\frac{\sum_{j\in\mathcal{J}_{ab}}([m(\hat{x}) - \bar{m}\ h_j]\ \Gamma_j) + \sum_{j\in\mathcal{J}\setminus\mathcal{J}_{ab}}([m(\hat{x}) - m(\hat{x}\,Q)]\,\Gamma_j)}{Y_P}\right]\left(\frac{x_{t_0}}{\hat{x}}\right)^{\xi}. \tag{124}$$

Using (59), we write Y_P in (123) as (62). Given that $m(\hat{x}) = m^{\infty} - [m^{\infty} - n_{\tilde{i}(\hat{t})}(\hat{x}_{t_0})]/\hat{x}$, where $m^{\infty} = \bar{k}\,\kappa_{\tilde{i}(\hat{t})}$, we expand the numerator in the squared brackets term of Z_P to obtain

$$Z_P = 1 + \left[\frac{A - B/\hat{x}}{Y_P}\right]\left(\frac{x_{t_0}}{\hat{x}}\right)^{\xi}. \tag{125}$$

where $A \equiv \sum_{j\in\mathcal{J}_{ab}}([m^{\infty} - \bar{m}\ h_j]\ \Gamma_j)$ and $B \equiv [m^{\infty} - n_{\tilde{i}(\hat{t})}(1)]\left(\sum_{j\in\mathcal{J}_{ab}}\Gamma_j + (1-Q^{-1})\sum_{j\in\mathcal{J}\setminus\mathcal{J}_{ab}}\Gamma_j\right)$. Note from (53) that $\hat{x} = (1+\xi)B/[\xi\ A]$. Substituting in (125) we can write $Z_P = 1 + \frac{A}{(1+\xi)\,Y_P}\left(\frac{x_{t_0}}{\hat{x}}\right)^{\xi}$, which is (63). □

Proof of Proposition 1: Recall that $\mathcal{J}_{ab} = \{j\in\mathcal{J}\mid S(Q)\le h_j\} \cup \arg\min_{j\in\mathcal{J}}\{w_j(S(Q)-h_j)\}$. There are two possible cases: Case 1– If $1 \le S(Q)$, then given that $h_j < 1$ for all $j\in\mathcal{J}$, we have $\{j\in\mathcal{J}\mid S(Q)\le h_j\} = \emptyset$. So there is only one asset in an alliance. Case 2 – If $1 > S(Q)$, then given that $h_j < 1$ for all $j\in\mathcal{J}$, we have $Y_P < Y_M$ and thus $Y_P Z_P < Y_M Z_M$. The merger dominates the alliance. □

Proof of Proposition 2: Note that at date 0, $x_0 = \hat{x}_0 = 1$, so $q(t_0) = Q$. Define $K \equiv k_{\tilde{i}(0)}(0)/\bar{k}$, $\Omega \equiv \kappa_{\tilde{i}(0)}/[\kappa_a^{\delta_a}\,\kappa_b^{\delta_b}]$, $\hat{K} \equiv k_{\tilde{i}(\hat{t})}(0)/\bar{k}$, $\hat{\Omega} \equiv \kappa_{\tilde{i}(\hat{t})}/[\kappa_a^{\delta_a}\,\kappa_b^{\delta_b}]$. Note that $\hat{K} = K$ and $\hat{\Omega} = \Omega$ if $\tilde{i}(\hat{t}) = \tilde{i}(0)$, whereas $\hat{K} = K\,[k_{\tilde{i}(\hat{t})}(0)/k_{\tilde{i}(0)}(0)]$ and $\hat{\Omega} = \Omega\,[\kappa_{\tilde{i}(\hat{t})}/\kappa_{\tilde{i}(0)}]$ if $\tilde{i}(\hat{t}) \ne \tilde{i}(0)$. If $\tilde{i}(\hat{t}) \ne \tilde{i}(0)$, we have $\tilde{i}(\hat{t}) = \arg\max_i\{\kappa_i\}$, and consequently $k_{\tilde{i}(0)}(0) = \max_i\{k_i(0)\}$. So $\hat{K} = K\,\hat{\rho}$ with $\hat{\rho}$ (and $\hat{\rho}'$) defined in (74). Note that if $\tilde{i}(\hat{t}) \ne \tilde{i}(0)$, then $\hat{\rho} \le 1$, $\hat{\rho}' \ge 1$, and $\hat{\rho}\,\hat{\rho}' \le 1$. Define $z_j \equiv (m^{\infty}/\bar{m}) - h_j$, $z \equiv (m^{\infty}/\bar{m}) - 1$, and $z' \equiv (m^{\infty} - n_{\tilde{i}(\hat{t})}(1))/\bar{m}$. We have $m^{\infty}/\bar{m} = \Omega\,\hat{\rho}'$ and $n_{\tilde{i}(\hat{t})}(0)/\bar{m} = K\,\Omega\,\hat{\rho}\,\hat{\rho}'$. Then, A and B in (96) can be written as $A = z\,\bar{m}\sum_{j\in\mathcal{J}}\Gamma_j$ and $B = z'\,\bar{m}\sum_{j\in\mathcal{J}}\Gamma_j$. Similarly, A and B in (125) can be written as $A = \sum_{j\in\mathcal{J}_{ab}}(z_j\,\bar{m}\,\Gamma_j)$ and $B = \sum_{j\in\mathcal{J}_{ab}}(z'\,\bar{m}\,\Gamma_j) + (1-Q^{-1})\sum_{j\in\mathcal{J}\setminus\mathcal{J}_{ab}}(z'\,\bar{m}\,\Gamma_j)$. Now define $Y \equiv Y_P/Y_M$ and use (61) to write $\tilde{W}_P(0)$ as $\tilde{W}_P(0) = Y\,Y_M\,Z_P$, where Y and Z_P are given by (68) and (69). □

Proof of (76): Given that $\frac{\partial}{\partial\Xi}\left[\frac{A}{B}\right]\Big|_{A=B} = \frac{1}{A}\left[\frac{\partial A}{\partial\Xi} - \frac{\partial B}{\partial\Xi}\right]$, we compute $\frac{\partial A}{\partial\Xi}$ for $A \in \{S(1); Z_M; Y Z_P\}$ and $\Xi \in \{K;\Omega\}$. From the expressions in Proposition 2, we have $\partial S(1)/\partial\Xi = S(1)/\Xi > 0$ for $\Xi \in \{K;\Omega\}$, $\partial S(1)/\partial r_0 = 0$, $\partial S(1)/\partial\lambda = 0$, $\partial S(1)/\partial\mu = 0$, $\partial S(1)/\partial\sigma = 0$, $\partial S(1)/\partial h_j = 0$. Now turning to $Y\,Z_P$, we have

$$Y\,Z_P = [\sum_{j\in\mathcal{J}_{ab}} w_j\,(h_j + \frac{z_j}{(1+\xi)\,\hat{x}_P^{\xi}})] + [\sum_{j\in\mathcal{J}\setminus\mathcal{J}_{ab}} w_j\,S(Q)]\ . \tag{126}$$

Define $W \equiv (1 - [\sum_{j\in\mathcal{J}_{ab}} w_j]) = [\sum_{j\in\mathcal{J}\setminus\mathcal{J}_{ab}} w_j])$. We have

$$\frac{\partial \hat{x}_P}{\partial \Xi} = \frac{-1}{[\sum_{j\in\mathcal{J}_{ab}} w_j\, z_j]} \left(\theta_1 \frac{S(1)}{\Xi} + \theta_0(\Xi) \right) , \quad \text{for } \Xi \in \{K;\Omega\} , \tag{127}$$

$$\text{where} \quad \theta_1 \equiv \frac{(1+\xi)}{\xi} \left(1 - W\, Q^{-1}\right) \hat{\rho}\hat{\rho}' , \quad \theta_0(\Omega) \equiv \hat{\rho}' \left[\hat{x}_P\, (1 - W) - \frac{(1+\xi)}{\xi} \left(1 - W\, Q^{-1}\right)\right] \tag{128}$$

and $\theta_0(K) = 0$. Computing, we have

$$\frac{\partial [Y\, Z_P]}{\partial \Xi} = \Theta_1 \frac{S(1)}{\Xi} + \Theta_0(\Xi) , \quad \text{for } \Xi \in \{K;\Omega\} , \tag{129}$$

$$\text{where} \quad \Theta_1 \equiv \left(1 - W\, Q^{-1}\right) \frac{\hat{\rho}\hat{\rho}'}{\hat{x}_P^{\xi+1}} + W\, Q^{-1} , \qquad \Theta_0(K) \equiv 0 , \tag{130}$$

$$\Theta_0(\Omega) \equiv \left(1 - W\, Q^{-1}\right) \left[1 - \frac{\hat{\rho}'}{\hat{x}_P^{\xi+1}}\right] - (1 - W) \left[1 - \frac{\hat{\rho}'}{\hat{x}_P^{\xi}}\right] . \tag{131}$$

Similarly,

$$\frac{\partial \hat{x}_M}{\partial \Xi} = \frac{-1}{z} \left(\theta_1' \frac{S(1)}{\Xi} + \theta_0'(\Xi) \right) , \quad \text{for } \Xi \in \{K;\Omega\} , \tag{132}$$

$$\text{where} \quad \theta_1' \equiv \frac{(1+\xi)}{\xi} \hat{\rho}\hat{\rho}' , \quad \theta_0'(K) \equiv 0 , \quad \theta_0'(\Omega) \equiv \hat{\rho}' \left[\hat{x}_M - \frac{(1+\xi)}{\xi}\right] . \tag{133}$$

We also have

$$\frac{\partial [Z_M]}{\partial \Xi} = \Theta_1' \frac{S(1)}{\Xi} + \Theta_0'(\Xi) , \quad \text{for } \Xi \in \{K;\Omega\} , \tag{134}$$

$$\text{where} \quad \Theta_1' \equiv \frac{\hat{\rho}\hat{\rho}'}{\hat{x}_M^{\xi+1}} , \qquad \Theta_0'(K) \equiv 0 , \qquad \Theta_0'(\Omega) \equiv \frac{\hat{\rho}'}{\hat{x}_M^{\xi+1}} [\hat{x}_M - 1] . \tag{135}$$

As $W\, Q^{-1} \in (0;1)$ and $\frac{\hat{\rho}\hat{\rho}'}{\hat{x}_P^{\xi+1}} \in (0;1)$, we have $\Theta_1 \in (0;1)$ and $\Theta_1' \in (0;1)$. We also have $\Theta_1 = \Theta_1' + WQ^{-1}(1-\Theta_1')$, hence $\Theta_1 > \Theta_1'$, when $Y Z_P = Z_M$. Therefore $1-\Theta_1 > 0$, $1-\Theta_1' > 0$ and $\Theta_1 - \Theta_1' > 0$. So (76) holds for $\Xi = K$. Furthermore, we have $(1 - \Theta_1)K - \Theta_0(\Omega) > 0$, when $S(1) = Y Z_P$. We have $(1 - \Theta_1')K - \Theta_0'(\Omega) > 0$, when $S(1) = Z_M$. We have $(\Theta_1 - \Theta_1')K + (\Theta_0(\Omega) - \Theta_0'(\Omega)) > 0$, when $Z_M = Y Z_P$. So (76) holds for $\Xi = \Omega$.

Note that $\theta_1 > 0$ and $\theta_1 K + \theta_0(\Omega) > 0$. Also, $\theta_1' > 0$ and $\theta_1' K + \theta_0'(\Omega) > 0$. So $\frac{\partial \hat{x}_P}{\partial \Xi} < 0$ and $\frac{\partial \hat{x}_M}{\partial \Xi} < 0$, for $\Xi \in \{K;\Omega\}$. □

Proof of (77), (78), and (79): We have $\partial S(1)]/\partial \Xi = 0$ for $\Xi \in \{-r_0; -\lambda; \mu; \sigma; h_j\}$. In $Y\, Z_P$, only ξ, $\hat{x}_P$, and Q depend on at least one of r_0, λ, μ and σ. In Z_M, only ξ and $\hat{x}_M$ depend on at least one of r_0, λ, μ and σ. ξ is the positive root of $\xi^2\sigma^2/2 + \xi(\mu - \sigma^2/2) - r = 0$ and $\xi > 1$. Let $D \equiv [\mu - \sigma^2/2]^2 + 2\sigma^2 r$. Then, using the fact that $(\xi\sigma^2 + \mu - \sigma^2/2) = D^{1/2}$, we have

$$\frac{\partial \xi}{\partial r} = \frac{1}{D^{1/2}} > 0 , \quad \frac{\partial \xi}{\partial \mu} = -\frac{\xi}{D^{1/2}} < 0 , \text{ and } \quad \frac{\partial \xi}{\partial (\sigma^2/2)} = -\frac{\xi\,(\xi - 1)}{D^{1/2}} < 0 . \tag{136}$$

– From the Proof of (42) to (55), we have $Q = 1/[\hat{x}(0,t)\, Z(\hat{x}(0,t), -1)\, r]$, where $r \equiv r_0 + \lambda$.

$$Q = \left[\int_t^{\infty} r\, e^{-r(\tau-t)} \left(\frac{\hat{x}(0,t)}{E_t\,[\hat{x}(0,\tau)]}\right) d\tau\right]^{-1} . \tag{137}$$

– Given that $d[re^{-r}]/dr > 0$ for $r \in [0,1)$, we have $\partial Q/\partial r_0 < 0$, $\partial Q/\partial \lambda < 0$. Given that $\partial E_t[\hat{x}(0,\tau)]/\partial \mu > 0$ and $\partial E_t[\hat{x}(0,\tau)]/\partial \sigma > 0$, we have $\partial Q/\partial \mu > 0$ and $\partial Q/\partial \sigma > 0$.
– From the expressions for $\hat{x}_P$, $\hat{x}_M$, and (136), it follows that $\partial\hat{x}/\partial r_0 < 0$, $\partial\hat{x}/\partial\lambda < 0$, $\partial\hat{x}/\partial\mu > 0$, and $\partial\hat{x}/\partial(\sigma^2/2) > 0$, for $\hat{x} \in \{\hat{x}_P; \hat{x}_M\}$. Given that $\xi > 1$, we also have $\partial[(1+\xi)^{-1}\hat{x}^{-\xi}]/\partial r_0 < 0$, $\partial[(1+\xi)^{-1}\hat{x}^{-\xi}]/\partial\lambda < 0$, $\partial[(1+\xi)^{-1}\hat{x}^{-\xi}]/\partial\mu > 0$, and $\partial[(1+\xi)^{-1}\hat{x}^{-\xi}]/\partial(\sigma^2/2) > 0$, for $\hat{x} \in \{\hat{x}_P; \hat{x}_M\}$.
– Hence $\partial[Y\,Z_P]/\partial r_0 < 0$, $\partial[Y\,Z_P]/\partial\lambda < 0$, $\partial[Y\,Z_P]/\partial\mu > 0$, and $\partial[Y\,Z_P]/\partial(\sigma^2/2) > 0$. Also, $\partial Z_M]/\partial r_0 < 0$, $\partial Z_M]/\partial\lambda < 0$, $\partial Z_M/\partial\mu > 0$, and $\partial Z_M/\partial(\sigma^2/2) > 0$.
– Given that $\frac{\partial}{\partial\Xi}\left[\frac{A}{B}\right]\Big|_{A=B} = \frac{1}{A}\left[\frac{\partial A}{\partial\Xi} - \frac{\partial B}{\partial\Xi}\right]$, (77) and (78) hold.
Finally, $\partial\hat{x}_P/\partial h_j = w_j\,\hat{x}_P > 0$ and $\partial\hat{x}_M/\partial h_j = 0$. So, $\partial[Y\,Z_P]/\partial h_j = y(1 - 1/\hat{x}_P^{\xi}) \in (0;1)$ and $\partial Z_M/\partial h_j = 0$. So, (79) holds. □

Proof of Result 5: $\partial\hat{x}/\partial K < 0$, $\partial\hat{x}/\partial\Omega < 0$, $\partial\hat{x}/\partial r_0 < 0$, $\partial\hat{x}/\partial\lambda < 0$, $\partial\hat{x}/\partial\mu > 0$, and $\partial\hat{x}/\partial(\sigma^2/2) > 0$, for $\hat{x} \in \{\hat{x}_P; \hat{x}_M\}$, and $\partial\hat{x}_P/\partial h_j > 0$ and $\partial\hat{x}_M/\partial h_j = 0$ were established in the proofs of Results 1, 2, 3, and 4. □

Proof of Result 6: An increase in the set $\mathcal{J}$ increases $\sum_{j\in\mathcal{J}}\Gamma_j$ and decreases $w_{j\in\mathcal{J}_{ab}}$. Now, in establishing Proposition 1, we have seen that a necessary condition for the alliance to be optimal is that $S(Q) > h_j$ for all $j \in \mathcal{J}$. Therefore, an increase in the number assets increases Y in (68) as well as Z_P in (69). It also increases $\hat{x}_P$ in (73). □

Proof of Result 7: A situation in which no knowhow acquisition is possible is represented by $\mu/\sigma \to 0$ and $\sigma \to 0$. Then, $\lim_{t\to\infty} k^+(t) = 0$ and $\hat{x} \to \infty$, for $\hat{x} \in \{\hat{x}_P; \hat{x}_M\}$. Consequently, $Z_M \to 1$ and $Z_P \to 1$. Furthermore, $Q \to 1$, so $S(Q) \to S(1)$. Assume an alliance is formed. Then, we necessarily have $S(Q) > h_{j\in\mathcal{J}_{ab}}$. So $Y < 1$. Hence $YZ_P/Z_M < 1$ and a merger is preferable, in contradiction to the assumption that an alliance has been formed. □

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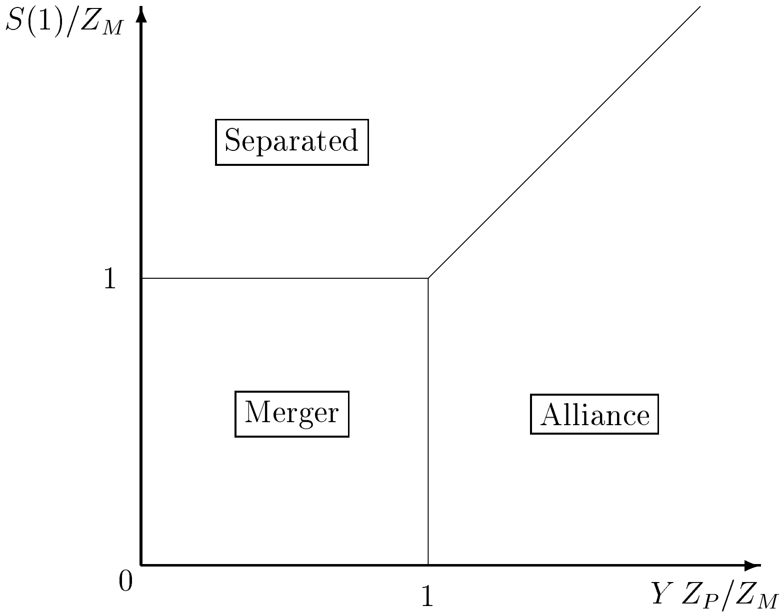

Figure 1: Choice of Organizational Form.

Background Calculations

These are intended to simplify the task of the referee, and are not part of the paper.

Proof of (42) to (55): Details of the calculations are provided following the sequence in the proof provided in the Appendix.

Details of (106): The f.o.c. for $e_{i,j}(\tau)$ directly yields

$$\frac{\phi_{i,j}\,\delta_i\,\gamma_j}{e_{i,j}(\tau)}\left(\frac{e_{a,j}(\tau)^{\delta_a}\,e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}}\right)^{\gamma_j} - \frac{1}{(\bar{k}\,\kappa_i)^{\frac{1-\gamma_j}{\gamma_j}}} = 0\,, \tag{138}$$

hence (106).

Details of (107): Replacing,

$$\left(\frac{e_{a,j}(\tau)^{\delta_a}\,e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}}\right) = \frac{\left(\gamma_j\,\bar{k}^{\frac{1-\gamma_j}{\gamma_j}}\right)^{(\delta_a+\delta_b)}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}}\left(\phi_{a,j}\,\delta_a\,(\kappa_a)^{\frac{1-\gamma_j}{\gamma_j}}\right)^{\delta_a}\times \left(\phi_{b,j}\,\delta_b\,(\kappa_b)^{\frac{1-\gamma_j}{\gamma_j}}\right)^{\delta_b}\left(\frac{e_{a,j}(\tau)^{\delta_a}\,e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}}\right)^{\gamma_j(\delta_a+\delta_b)}, \tag{139}$$

$$\left(\frac{e_{a,j}(\tau)^{\delta_a}\,e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}}\right)^{1-\gamma_j} = \gamma_j\,\bar{k}^{\frac{1-\gamma_j}{\gamma_j}}\left(\phi_{a,j}\,(\kappa_a)^{\frac{1-\gamma_j}{\gamma_j}}\right)^{\delta_a}\left(\phi_{b,j}(\kappa_b)^{\frac{1-\gamma_j}{\gamma_j}}\right)^{\delta_b} \tag{140}$$

$$\frac{e_{a,j}(\tau)^{\delta_a}\,e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}} = \left(\phi_{a,j}^{\delta_a}\,\phi_{b,j}^{\delta_b}\right)^{\frac{1}{1-\gamma_j}}\left(\bar{k}\,(\kappa_a)^{\delta_a}\,(\kappa_b)^{\delta_b}\right)^{\frac{1}{\gamma_j}}\gamma_j^{\frac{1}{1-\gamma_j}}\,. \tag{141}$$

Details of (108): With (106) we have

$$\phi_{i,j}\,R_j(\mathbf{e}(\tau)) - C_{i,j}(e_{i,j}(\tau)) = [\phi_{i,j} - \phi_{i,j}\,\gamma_j\,\delta_i]\left(\frac{e_{a,j}(\tau)^{\delta_a}\,e_{b,j}(\tau)^{\delta_b}}{\delta_a^{\delta_a}\,\delta_b^{\delta_b}}\right)^{\gamma_j}. \tag{142}$$

Details of (115): The distribution of $\hat{X}(t,\tau)$ is

$$F(\hat{X}(t,\tau)\le M) = \Phi\left[\frac{M-m(\tau-t)}{\sigma\sqrt{\tau-t}}\right] - \exp\left(\frac{2mM}{\sigma^2}\right)\Phi\left[\frac{-M-m(\tau-t)}{\sigma\sqrt{\tau-t}}\right], \tag{143}$$

where $m\equiv\mu-\sigma^2/2$ (See Harrison, 1985, p. 15). The probability density function of $\hat{X}(t,\tau)$ equal to M, $\hat{f}(M)\equiv d(F(\hat{X}(t,\tau)\le M))/dM$, is therefore

$$\hat{f}(M) = \frac{1}{\sigma\sqrt{(\tau-t)}}\,\phi\left[\frac{M-m(\tau-t)}{\sigma\sqrt{\tau-t}}\right] + \exp\left(\frac{2mM}{\sigma^2}\right)\frac{1}{\sigma\sqrt{(\tau-t)}}\,\phi\left[\frac{M+m(\tau-t)}{\sigma\sqrt{\tau-t}}\right] - \frac{2m}{\sigma^2}\,\exp\left(\frac{2mM}{\sigma^2}\right)\,\Phi\left[\frac{-M-m(\tau-t)}{\sigma\sqrt{\tau-t}}\right]. \tag{144}$$

Details of (117): The expression to be integrated,

$$\exp[\eta\, M]\, \hat{f}(M) \;=\; \frac{\exp[\eta M]}{\sigma\sqrt{(\tau-t)}}\,\phi\left[\frac{M-m(\tau-t)}{\sigma\sqrt{\tau-t}}\right] + \exp\left(\frac{2m+\eta\sigma^2}{\sigma^2}M\right)\frac{1}{\sigma\sqrt{(\tau-t)}}\,\phi\left[\frac{M+m(\tau-t)}{\sigma\sqrt{\tau-t}}\right] - \frac{2m}{\sigma^2}\,\exp\left(\frac{2m+\eta\sigma^2}{\sigma^2}M\right)\,\Phi\left[\frac{-M-m(\tau-t)}{\sigma\sqrt{\tau-t}}\right]. \quad (145)$$

We will rewrite the first term on the RHS of (145) noting

$$\frac{-\left[M-m(\tau-t)\right]^2}{2\sigma^2(\tau-t)} + \eta\, M \;=\; \frac{-\left[M-(m+\eta\,\sigma^2)(\tau-t)\right]^2}{2\sigma^2(\tau-t)} + \eta(m+\eta\,\sigma^2/2)(\tau-t)\,, \quad (146)$$

Then, express the third term on the RHS of (145), integrating by parts

$$-\frac{2m}{\sigma^2}\,\exp\left(\frac{2m+\eta\sigma^2}{\sigma^2}M\right)\,\Phi\left[\frac{-M-m(\tau-t)}{\sigma\sqrt{\tau-t}}\right] = \frac{d}{dM}\left(-\frac{2m}{2m+\eta\sigma^2}\,\exp\left(\frac{2m+\eta\sigma^2}{\sigma^2}M\right)\,\Phi\left[\frac{-M-m(\tau-t)}{\sigma\sqrt{\tau-t}}\right]\right) - \frac{2m}{2m+\eta\sigma^2}\,\exp\left(\frac{2m+\eta\sigma^2}{\sigma^2}M\right)\,\frac{1}{\sigma\sqrt{(\tau-t)}}\,\phi\left[\frac{-M-m(\tau-t)]}{\sigma\sqrt{\tau-t}}\right]. \quad (147)$$

Using (146) and (147), we rewrite (145) as

$$\exp[\eta\, M]\, \hat{f}(M) \;=\; \frac{\exp[\eta(m+\eta\,\sigma^2/2)(\tau-t)]}{\sigma\sqrt{(\tau-t)}}\,\phi\left[\frac{M-(m+\eta\,\sigma^2)](\tau-t)}{\sigma\sqrt{\tau-t}}\right] + \frac{\eta\,\sigma^2}{2m+\eta\sigma^2}\,\exp\left(\frac{2m+\eta\sigma^2}{\sigma^2}M\right)\,\frac{1}{\sigma\sqrt{(\tau-t)}}\,\phi\left[\frac{M+m(\tau-t)}{\sigma\sqrt{\tau-t}}\right] - \frac{d}{dM}\left(\frac{2m}{2m+\eta\sigma^2}\,\exp\left(\frac{2m+\eta\sigma^2}{\sigma^2}M\right)\,\Phi\left[\frac{-M-m(\tau-t)}{\sigma\sqrt{\tau-t}}\right]\right). \quad (148)$$

We now rewrite the second term on the RHS of (148) noting

$$\frac{-\left[M+m(\tau-t)\right]^2}{2\sigma^2(\tau-t)} + \frac{2m+\eta\sigma^2}{\sigma^2}M \;=\; \frac{-\left[M-(m+\eta\,\sigma^2)(\tau-t)\right]^2}{2\sigma^2(\tau-t)} + \eta(m+\eta\,\sigma^2/2)(\tau-t) \quad (149)$$

This yields

$$\exp[\eta\, M]\, \hat{f}(M) \;=\; \left(1+\frac{\eta\,\sigma^2}{2m+\eta\sigma^2}\right)\frac{\exp[\eta(m+\eta\,\sigma^2/2)(\tau-t)]}{\sigma\sqrt{(\tau-t)}}\,\phi\left[\frac{M-(m+\eta\,\sigma^2)](\tau-t)}{\sigma\sqrt{\tau-t}}\right] - \frac{d}{dM}\left(\left(1-\frac{\eta\,\sigma^2}{2m+\eta\sigma^2}\right)\,\exp\left(\frac{2m+\eta\sigma^2}{\sigma^2}M\right)\,\Phi\left[\frac{-M-m(\tau-t)}{\sigma\sqrt{\tau-t}}\right]\right), \quad (150)$$

which is

$$\exp[\eta\, M]\, \hat{f}(M) \;=\; \left(1+\frac{\eta\,\sigma^2}{2m+\eta\sigma^2}\right)\frac{\exp[\eta(m+\eta\,\sigma^2/2)(\tau-t)]}{\sigma\sqrt{(\tau-t)}}\,\phi\left[\frac{M-(m+\eta\,\sigma^2)](\tau-t)}{\sigma\sqrt{\tau-t}}\right] - \frac{d}{dM}\left(\left(1-\frac{\eta\,\sigma^2}{2m+\eta\sigma^2}\right)\,\exp\left(\frac{2m+\eta\sigma^2}{\sigma^2}M\right)\,\Phi\left[\frac{-M-m(\tau-t)}{\sigma\sqrt{\tau-t}}\right]\right). \quad (151)$$

We then obtain,

$$z(\tau - t \mid \hat{x}(0,t), \eta) = \hat{x}(0,t)^{\,\eta}\, e^{-r(\tau-t)} \Big\{ e^{\eta(m+\eta\,\sigma^2/2)(\tau-t)} \left(1 + \frac{\eta\,\sigma^2}{2m+\eta\sigma^2}\right) \Phi\left[\frac{(m+\eta\,\sigma^2)}{\sigma}\sqrt{\tau-t}\right] + \left(1 - \frac{\eta\,\sigma^2}{2m+\eta\sigma^2}\right) \Phi\left[\frac{-m}{\sigma}\sqrt{\tau-t}\right]\Big\}, \quad (152)$$

which gives (117).

Details of (??):

$$\int_0^\infty e^{-st}\Phi\left[a\sqrt{t}\right] dt = \int_0^\infty e^{-st}\left[\frac{1}{2} + \mathrm{sg}[a]\int_0^{|a|\sqrt{t}} \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{u^2}{2}\right] du\right] dt \quad (153)$$

$$= \frac{1}{2s} + \frac{\mathrm{sg}[a]}{2}\int_0^\infty e^{-st}\,\mathrm{erf}\left[\frac{|a|}{\sqrt{2}}\sqrt{t}\right] dt \quad (154)$$

$$= \frac{1}{2s} + \frac{\mathrm{sg}[a]}{2}\mathcal{L}\left(\mathrm{erf}\left(\frac{|a|}{\sqrt{2}}\sqrt{t}\right) \mid s\right) = \frac{1}{2s} + \frac{\mathrm{sg}[a]}{2}\,\frac{|a|}{s\sqrt{2\,s+a^2}}\,. \quad (155)$$

Details of (119): Replacing, in (114) and then in (112), we obtain

$$\Pi(x_t, \hat{x}(0,t) \mid k_i(.), \kappa) = \Big\{\left[\bar{k} - \frac{(\bar{k}-k_i(0))}{\hat{x}(0,t)}\right]\left[1 - \left(\frac{x(t)}{\hat{x}(0,t)}\right)^{\xi}\right] + \left[\bar{k} - \frac{(\bar{k}-k_i(0))}{\hat{x}(0,t)\,Q}\right]\left(\frac{x(t)}{\hat{x}(0,t)}\right)^{\xi}\Big\}\,\frac{\kappa}{r} \quad (156)$$

$$= \left\{\bar{k} - \frac{(\bar{k}-k_i(0))}{\hat{x}(0,t)}\left[1 - (1-\frac{1}{Q})\left(\frac{x(t)}{\hat{x}(0,t)}\right)^{\xi}\right]\right\}\frac{\kappa}{r} \quad (157)$$

Details of (120): Given that $\frac{\partial}{\partial\,\hat{x}_i}\left[\left(\frac{x_t}{\hat{x}_i}\right)^{\xi}\right] = -\xi\left(\frac{x_t}{\hat{x}_i}\right)^{\xi}/\hat{x}_i$,

$$\frac{\partial\,V_{i,P}(t)}{\partial\,\hat{x}_i} = \frac{1}{\hat{x}_i^2}\left(\frac{x_t}{\hat{x}_i}\right)^{\xi}\left[\xi\,\hat{x}_i\,V_{i,P}^{\infty}(\hat{t}_i) - \xi\,\hat{x}_i\,V_i^*(\hat{t}_i) + \hat{x}_i^2\,\frac{\partial\,V_i^*(\hat{t}_i)}{\partial\,\hat{x}_i} - \hat{x}_i^2\,\frac{\partial\,V_{i,P}^{\infty}(\hat{t}_i)}{\partial\,\hat{x}_i}\right]. \quad (158)$$

We develop the terms of the third factor on the RHS of (158):

• Note that $V_{i,j\in\mathcal{J}_{ab}}^{\infty}(t) = V_{i,j\in\mathcal{J}_{ab}}^{\infty}(0)$, for all t. Also, given that $q(\hat{t}_i) = \hat{x}_i\,Q$, we have that $n_h(q(\hat{t}_i)) = \left[\bar{k} - \left(\bar{k} - k_h(0)\right)/(\hat{x}_i\,Q)\right]\kappa_h$, for all $h \in \{a; b\}$. So $V_{i,j\in\mathcal{J}_i}^{\infty}(t) = V_{i,j,U}(\bar{k}) - V_{i,j,U}(\bar{k} - k_i(0))/(\hat{x}_i\,Q)$. Therefore

$$\xi\,\hat{x}_i\,V_{i,P}^{\infty}(\hat{t}_i) = \xi\,\hat{x}_i\sum_{j\in\mathcal{J}_{ab}} V_{i,j}^{\infty}(0) + \xi\,\hat{x}_i\sum_{j\in\mathcal{J}_i} V_{i,j,U}(\bar{k}) - \frac{\xi}{Q}\sum_{j\in\mathcal{J}_i} V_{i,j,U}(\bar{k} - k_i(0))\,. \quad (159)$$

• From (40),

$$-\xi\,\hat{x}_i\,V_i^*(\hat{t}_i) = -\,\xi\,\hat{x}_i\,\varphi_i - \xi\,\hat{x}_i\sum_{j\in\mathcal{J}_{ab}}\psi_{i,j}\;V_{\tilde{i}(\hat{t}_i),j,U}(k_{\tilde{i}(\hat{t}_i)}(\hat{t}_i)) - \xi\,\hat{x}_i\sum_{j\in\mathcal{J}_i} V_{i,j,U}(k_i(\hat{t}_i))\,. \quad (160)$$

Given that $n_h(\hat{x}_i) = \left[\bar{k} - \left(\bar{k} - k_h(0)\right)/\hat{x}_i\right]\kappa_h$, for all $h \in \{a;b\}$, we have $V_{i,j,U}(k_i(\hat{t}_i)) = V_{i,j,U}(\bar{k}) - V_{i,j,U}(\bar{k} - k_i(0))/\hat{x}_i$. Therefore

$$-\xi\,\hat{x}_i\,V_i^*(\hat{t}_i) = -\xi\,\hat{x}_i\,\varphi_i - \xi\,\hat{x}_i \sum_{j\in\mathcal{J}_{ab}} \psi_{i,j}\; V_{\tilde{i}(\hat{t}_i),j,U}(\bar{k}) + \xi \sum_{j\in\mathcal{J}_{ab}} \psi_{i,j}\; V_{\tilde{i}(\hat{t}_i),j,U}(\bar{k} - k_{\tilde{i}(\hat{t}_i)}(0))$$
$$-\xi\,\hat{x}_i \sum_{j\in\mathcal{J}_i} V_{i,j,U}(\bar{k}) + \xi \sum_{j\in\mathcal{J}_i} V_{i,j,U}(\bar{k} - k_i(0))\;. \tag{161}$$

• Given that, for all $h \in \{a;b\}$, $k_h(\hat{t}_i) = \bar{k} - \left(\bar{k} - k_h(0)\right)/\hat{x}_i$, we have $\frac{\partial}{\partial\hat{x}_i}\left[k_h(\hat{t}_i)\right] = \left[\bar{k} - k_h(0)\right]/\hat{x}_i^2$. It follows that $\hat{x}_i^2\,\frac{\partial V_{h,j,U}(k_h(\hat{t}_i))}{\partial\hat{x}_i} = [\bar{k} - k_h(0)]\,\kappa_h\,\Gamma_j$, hence $\hat{x}_i^2\,\frac{\partial V_{h,j,U}(k_h(\hat{t}_i))}{\partial\hat{x}_i} = V_{h,j,U}(\bar{k} - k_h(0))$. Therefore

$$\hat{x}_i^2\,\frac{\partial V_i^*(\hat{t}_i)}{\partial\hat{x}_i} = \sum_{j\in\mathcal{J}_{ab}} \psi_{i,j}\;\hat{x}_i^2\,\frac{\partial V_{\tilde{i}(\hat{t}_i),j,U}(k_{\tilde{i}(\hat{t}_i)}(\hat{t}_i))}{\partial\hat{x}_i} + \sum_{j\in\mathcal{J}_i} \hat{x}_i^2\,\frac{\partial V_{i,j,U}(k_i(\hat{t}_i))}{\partial\hat{x}_i}\;, \tag{162}$$
$$= \sum_{j\in\mathcal{J}_{ab}} \psi_{i,j}\;V_{\tilde{i}(\hat{t}_i),j,U}(\bar{k} - k_{\tilde{i}(\hat{t}_i)}(0)) + \sum_{j\in\mathcal{J}_i} V_{i,j,U}(\bar{k} - k_i(0))\;. \tag{163}$$

• Given that $q(\hat{t}_i) = \hat{x}_i\,Q$, we have that $n_h(q(\hat{t}_i)) = \left[\bar{k} - \left(\bar{k} - k_h(0)\right)/(\hat{x}_i\,Q)\right]\kappa_h$, for all $h \in \{a;b\}$. It follows that $\hat{x}_i^2\,\frac{\partial}{\partial\hat{x}_i}\left[n_h(q(\hat{t}_i))\right] = Q^{-1}[\bar{k} - k_h(0)]\kappa_h$, hence $\hat{x}_i^2\,\frac{\partial V^\infty_{i,j\in\mathcal{J}_i}(\hat{t}_i)}{\partial\hat{x}_i} = Q^{-1}[\bar{k} - k_i(0)]\,\kappa_i\,\Gamma_j$, so $\hat{x}_i^2\,\frac{\partial V^\infty_{i,j\in\mathcal{J}_i}(\hat{t}_i)}{\partial\hat{x}_i} = Q^{-1}[V_{i,j,U}(\bar{k} - k_i(0))]$. Considering that $\partial\,V^\infty_{i,j\in\mathcal{J}_{ab}}(t)/\partial\,\hat{x}_t = 0$, we have

$$-\hat{x}_i^2\,\frac{\partial V^\infty_{i,P}(\hat{t}_i)}{\partial\hat{x}_i} = -\sum_{j\in\mathcal{J}_i} \hat{x}_i^2\,\frac{\partial V^\infty_{i,j}(\hat{t}_i)}{\partial\hat{x}_i} = -Q^{-1}\sum_{j\in\mathcal{J}_i} V_{i,j,U}(\bar{k} - k_i(0))\;. \tag{164}$$

Consequently, using (159), (161) (163) and (164), (158) becomes

$$\frac{\partial V_{i,P}(t)}{\partial\hat{x}_i} = \frac{1}{\hat{x}_i}\left(\frac{x_t}{\hat{x}_i}\right)^{\xi}\left[-\xi\,\hat{x}_i\left(\varphi_i + \sum_{j\in\mathcal{J}_{ab}} \psi_{i,j}\;V_{\tilde{i}(\hat{t}_i),j,U}(\bar{k}) - \sum_{j\in\mathcal{J}_{ab}} V^\infty_{i,j}(0)\right)\right.$$
$$\left. + (1+\xi)\left(\sum_{j\in\mathcal{J}_{ab}} \psi_{i,j}\,V_{\tilde{i}(\hat{t}_i),j,U}(\bar{k} - k_{\tilde{i}(\hat{t}_i)}(0)) + (1 - Q^{-1})\sum_{j\in\mathcal{J}_i} V_{i,j,U}(\bar{k} - k_i(0))\right)\right]. \tag{165}$$

Rearranging gives (120).

$$\hat{x}_i = \left(\frac{1+\xi}{\xi}\right)\frac{\sum_{j\in\mathcal{J}_{ab}} \psi_{i,j}\,V_{\tilde{i}(\hat{t}_i),j,U}(\bar{k} - k_{\tilde{i}(\hat{t}_i)}(0)) + (1 - Q^{-1})\sum_{j\in\mathcal{J}_i} V_{i,j,U}(\bar{k} - k_i(0))}{\varphi_i + \sum_{j\in\mathcal{J}_{ab}} \psi_{i,j}\;V_{\tilde{i}(\hat{t}_i),j,U}(\bar{k}) - \sum_{j\in\mathcal{J}_{ab}} V^\infty_{i,j}(0)}\;. \tag{166}$$

Details of (55): Let $\Pi_i \equiv \sum_{j\in\mathcal{J}_{ab}} V^\infty_{i,j}(0) - \varphi_i$, $\Pi \equiv \Pi_a + \Pi_b$, $h_i \equiv \sum_{j\in\mathcal{J}_{ab}} \psi_{i,j}\,V_{\tilde{i}(\hat{t}_i),j,U}(\bar{k} - k_{\tilde{i}(\hat{t}_i)}(0)) + (1 - Q^{-1})\sum_{j\in\mathcal{J}_i} V_{i,j,U}(\bar{k} - k_i(0))$, $h \equiv h_a + h_b$, $\bar{h}_i \equiv \sum_{j\in\mathcal{J}_{ab}} \psi_{i,j}\;V_{\tilde{i}(\hat{t}_i),j,U}(\bar{k})$ and $\bar{h} = \bar{h}_a + \bar{h}_b = \bar{h}$. Using (120), we have $\hat{x}_i = \hat{x}_j$ if and only if

$$\frac{h_i}{\bar{h}_i - \Pi_i} = \frac{h - h_i}{(\bar{h} - \bar{h}_i) - (\Pi - \Pi_i)}\;. \tag{167}$$

Expanding, we have

$$h_i(\bar{h} - \bar{h}_i) - h_i(\Pi - \Pi_i) = h(\bar{h}_i - \Pi_i) - h_i(\bar{h}_i - \Pi_i) \quad (168)$$

$$h_i(\bar{h} - \Pi) = h(\bar{h}_i - \Pi_i) \quad (169)$$

$$-\Pi_i = h_i \frac{(\bar{h} - \Pi)}{h} - \bar{h}_i \quad (170)$$

which is (55).

Details of (122): Begin by noting that

$$\frac{\partial \left[\phi_{a,j}^{\delta_a} \phi_{b,j}^{\delta_b}\right]}{\partial \phi_{i,j}} = \left(\frac{\delta_i}{\phi_{i,j}} - \frac{\delta_{i^-}}{\phi_{i^-,j}}\right) \left[\phi_{a,j}^{\delta_a} \phi_{b,j}^{\delta_b}\right] \quad (171)$$

$$\frac{\partial \left[1 - \phi_{a,j}\,\delta_a\,\gamma_j - \phi_{b,j}\,\delta_b\,\gamma_j\right]}{\partial \phi_{i,j}} = -(\delta_i - \delta_{i^-})\,\gamma_j\ . \quad (172)$$

Then,

$$\frac{\partial h_j}{\partial \phi_{i,j}} = \left\{(1 - \phi_{a,j}\,\delta_a\,\gamma_j - \phi_{b,j}\,\delta_b\,\gamma_j)\,\frac{\gamma_j}{1-\gamma_j} \left[\phi_{a,j}^{\delta_a} \phi_{b,j}^{\delta_b}\right]^{\frac{\gamma_j}{1-\gamma_j}-1} \frac{\partial \left[\phi_{a,j}^{\delta_a} \phi_{b,j}^{\delta_b}\right]}{\partial \phi_{i,j}} + \frac{\partial \left[1 - \phi_{a,j}\,\delta_a\,\gamma_j - \phi_{b,j}\,\delta_b\,\gamma_j\right]}{\partial \phi_{i,j}} \left[\phi_{a,j}^{\delta_a} \phi_{b,j}^{\delta_b}\right]^{\frac{\gamma_j}{1-\gamma_j}}\right\} \frac{1}{(1-\gamma_j)} \quad (173)$$

$$= \left\{\frac{(1 - \phi_{a,j}\,\delta_a\,\gamma_j - \phi_{b,j}\,\delta_b\,\gamma_j)}{1-\gamma_j} \left(\frac{\delta_i}{\phi_{i,j}} - \frac{\delta_{i^-}}{\phi_{i^-,j}}\right) - (\delta_i - \delta_{i^-})\right\} \frac{\gamma_j \left[\phi_{a,j}^{\delta_a} \phi_{b,j}^{\delta_b}\right]^{\frac{\gamma_j}{1-\gamma_j}}}{(1-\gamma_j)}\ . \quad (174)$$

Details of (54):

$$(1 - \gamma_j\,(\phi_{a,j}\,\delta_a + \phi_{b,j}\,\delta_b)) \left(\frac{\delta_i}{\phi_{i,j}} - \frac{\delta_{i^-}}{\phi_{i^-,j}}\right) = (1-\gamma_j)(\delta_i - \delta_{i^-}) \quad (175)$$

$$(1 - \gamma_j(\phi_{i,j}\delta_i + (1-\phi_{i,j})(1-\delta_i))) \left(\frac{\delta_i - \phi_{i,j}}{\phi_{i,j}(1-\phi_{i,j})}\right) = (1-\gamma_j)(2\delta_i - 1) \quad (176)$$

$$(1 - \gamma_j(\phi_{i,j}\delta_i + (1-\phi_{i,j})(1-\delta_i)))\,(\delta_i - \phi_{i,j}) = (1-\gamma_j)(2\delta_i - 1)\phi_{i,j}(1-\phi_{i,j}) \quad (177)$$

$$(1 - \gamma_j\phi_{i,j}\delta_i - \gamma_j(1-\delta_i) + \gamma_j\phi_{i,j}(1-\delta_i))\,(\delta_i - \phi_{i,j}) = -(1-\gamma_j)\phi_{i,j}^2(2\delta_i - 1) + (1-\gamma_j)\phi_{i,j}(2\delta_i - 1) \quad (178)$$

$$(-\gamma_j\phi_{i,j}(2\delta_i - 1) + (1-\gamma_j) + \gamma_j\delta_i)\,(\delta_i - \phi_{i,j}) = -(1-\gamma_j)\phi_{i,j}^2(2\delta_i - 1) + (1-\gamma_j)\phi_{i,j}2\delta_i - (1-\gamma_j)\phi_{i,j} \quad (179)$$

$$-\gamma_j\phi_{i,j}(2\delta_i - 1)\delta_i + (1-\gamma_j)\delta_i + \gamma_j\delta_i^2 + \gamma_j\phi_{i,j}^2(2\delta_i - 1) - (1-\gamma_j)\phi_{i,j} - \gamma_j\phi_{i,j}\delta_i = -(1-\gamma_j)\phi_{i,j}^2(2\delta_i - 1) + (1-\gamma_j)\phi_{i,j}2\delta_i - (1-\gamma_j)\phi_{i,j} \quad (180)$$

$$\phi_{i,j}^2(2\delta_i - 1) - \gamma_j\phi_{i,j}(2\delta_i - 1)\delta_i + (1-\gamma_j)\delta_i + \gamma_j\delta_i^2 - \gamma_j\phi_{i,j}\delta_i = (1-\gamma_j)\phi_{i,j}2\delta_i \quad (181)$$

$$\phi_{i,j}^2(2\delta_i - 1) - \phi_{i,j}\delta_i\left[\gamma_j(2\delta_i - 1) + \gamma_j + 2(1-\gamma_j)\right] + (1-\gamma_j)\delta_i + \gamma_j\delta_i^2 = 0 \quad (182)$$

$$\phi_{i,j}^2(2\delta_i - 1) - 2\phi_{i,j}\delta_i\left[\gamma_j\delta_i + 1 - \gamma_j\right] + \delta_i[1 - \gamma_j + \gamma_j\delta_i] = 0 \quad (183)$$

$$\phi_{i,j}^2(2\delta_i - 1) - (2\phi_{i,j} - 1)\,\delta_i\left[1 - \gamma_j + \gamma_j\delta_i\right] = 0 \quad (184)$$

We therefore obtain that the optimal sharing rule solves

$$N_i\,\phi_{i,j}^2 \;+\; (2\phi_{i,j}\;-1)\,\delta_i \;=\; 0\;, \tag{185}$$

$$\text{where} \qquad N_i \;\equiv\; \frac{(1-2\delta_i)}{\delta_i(1-\gamma_j+\gamma_j\delta_i)}\;. \tag{186}$$

Hence,

$$\phi_{i,j} \;=\; \begin{cases} (\sqrt{1\,+\,N_i}\;-\;1)/N_i & \text{if } \delta_a \;\neq\; \delta_b\;, \\ 1/2 & \text{if } \delta_a \;=\; \delta_b\;. \end{cases} \tag{187}$$

Details of (127) and (129): For $\Xi \in \{K;\Omega\}$, we have

$$\begin{aligned} \frac{\partial[Y\,Z_P]}{\partial\Xi} \;&=\; \frac{1}{(1+\xi)\,\hat{x}_P^{\xi}}\,[\sum_{j\in\mathcal{J}_{ab}} w_j\,\frac{\partial z_j}{\partial\Xi}\,] + [\sum_{j\in\mathcal{J}\setminus\mathcal{J}_{ab}} w_j\,\frac{\partial S(Q)}{\partial\Xi}] \\ &\quad + \frac{[\sum_{j\in\mathcal{J}_{ab}} w_j\,z_j]}{(1+\xi)\,\hat{x}_P^{\xi}}\,\frac{(-\xi)}{\hat{x}_P}\,\frac{\partial\hat{x}_P}{\partial\Xi}\;. \end{aligned} \tag{188}$$

We have $\frac{\partial z_j}{\partial\Xi}$ and $\frac{\partial S(Q)}{\partial\Xi}$ are independent of j. Then, denoting $C = [\sum_{j\in\mathcal{J}_{ab}} w_j\,z_j]$ and $D = [\sum_{j\in\mathcal{J}_{ab}} w_j\,z'] + (1-Q^{-1})\,[\sum_{j\in\mathcal{J}\setminus\mathcal{J}_{ab}} w_j\,z']$, we have $\hat{x}_P = (1+\xi)D/[\xi\,C]$ and

$$\frac{\partial\hat{x}_P}{\partial\Xi} \;=\; \hat{x}_P\left(\frac{\partial D}{\partial\Xi}\,\frac{1}{D}\;-\;\frac{\partial C}{\partial\Xi}\,\frac{1}{C}\right)\;, \tag{189}$$

$$\;=\; \frac{\hat{x}_P}{C}\left(\frac{\partial D}{\partial\Xi}\,\frac{C}{D}\;-\;\frac{\partial C}{\partial\Xi}\right)\;, \tag{190}$$

$$\;=\; \frac{1}{C}\left(\frac{\partial D}{\partial\Xi}\,\frac{(1+\xi)}{\xi}\;-\;\hat{x}_P\,\frac{\partial C}{\partial\Xi}\right)\;. \tag{191}$$

Hence

$$\frac{\partial C}{\partial\Xi} \;=\; [\sum_{j\in\mathcal{J}_{ab}} w_j\,\frac{\partial z_j}{\partial\Xi}] \;=\; [\sum_{j\in\mathcal{J}_{ab}} w_j]\,\frac{\partial z_j}{\partial\Xi}\;, \tag{192}$$

$$\frac{\partial D}{\partial\Xi} \;=\; \left(1-Q^{-1}\,(1-[\sum_{j\in\mathcal{J}_{ab}} w_j])\right)\,\frac{\partial z'}{\partial\Xi}\;. \tag{193}$$

Therefore,

$$\frac{[\sum_{j\in\mathcal{J}_{ab}} w_j\,z_j]}{(1+\xi)\,\hat{x}_P^{\xi}}\,\frac{(-\xi)}{\hat{x}_P}\,\frac{\partial\hat{x}_P}{\partial\Xi} \;=\; \frac{1}{\hat{x}_P^{\xi}}\,\frac{\xi}{(1+\xi)}\,[\sum_{j\in\mathcal{J}_{ab}} w_j]\,\frac{\partial z_j}{\partial\Xi}\;-\;\frac{1}{\hat{x}_P^{\xi+1}}\,\frac{\partial D}{\partial\Xi}\;. \tag{194}$$

Consequently,

$$\frac{[\sum_{j\in\mathcal{J}_{ab}} w_j\,\frac{\partial z_j}{\partial\Xi}]}{(1+\xi)\,\hat{x}_P^{\xi}} + \frac{[\sum_{j\in\mathcal{J}_{ab}} w_j\,z_j]}{(1+\xi)\,\hat{x}_P^{\xi}}\,\frac{(-\xi)}{\hat{x}_P}\,\frac{\partial\hat{x}_P}{\partial\Xi} \;=\; \frac{[\sum_{j\in\mathcal{J}_{ab}} w_j]\,\frac{\partial z_j}{\partial\Xi}}{\hat{x}_P^{\xi}}\;-\;\frac{1}{\hat{x}_P^{\xi+1}}\,\frac{\partial D}{\partial\Xi}\;. \tag{195}$$

So, denoting $W \equiv (1 - [\sum_{j\in\mathcal{J}_{ab}} w_j]) = [\sum_{j\in\mathcal{J}\setminus\mathcal{J}_{ab}} w_j]$,

$$\frac{\partial \hat{x}_P}{\partial \Xi} = \frac{1}{[\sum_{j\in\mathcal{J}_{ab}} w_j\, z_j]}\left[\frac{(1+\xi)}{\xi}\left(1 - Q^{-1}\,W\right)\frac{\partial z'}{\partial \Xi} - \hat{x}_P\,(1-W)\,\frac{\partial z_j}{\partial \Xi}\right] , \tag{196}$$

$$\frac{\partial [Y\,Z_P]}{\partial \Xi} = \frac{(1-W)}{\hat{x}_P^{\xi}}\,\frac{\partial z_j}{\partial \Xi} - \frac{\left(1-Q^{-1}\,W\right)}{\hat{x}_P^{\xi+1}}\,\frac{\partial z'}{\partial \Xi} + W\,\frac{\partial S(Q)}{\partial \Xi} . \tag{197}$$

- For $\Xi = K$, we have $\partial z_j/\partial K = 0$, $\partial z'/\partial K = -S(1)\hat{\rho}\hat{\rho}'/K$, $\partial S(Q)/\partial K = Q^{-1}S(1)/K$. Then,

$$\frac{\partial \hat{x}_P}{\partial \Xi} = \frac{-(1+\xi)}{\xi}\,\frac{\left(1-W\,Q^{-1}\right)\,\hat{\rho}\hat{\rho}'}{[\sum_{j\in\mathcal{J}_{ab}} w_j\, z_j]}\,\frac{S(1)}{K} , \tag{198}$$

$$\frac{\partial [Y\,Z_P]}{\partial \Xi} = \left[\left(1-W\,Q^{-1}\right)\,\frac{\hat{\rho}\hat{\rho}'}{\hat{x}_P^{\xi+1}} + W\,Q^{-1}\right]\frac{S(1)}{K} . \tag{199}$$

which is (127) and (129).

- For $\Xi = \Omega$, we have $\partial z_j/\partial \Omega = \hat{\rho}'$, $\partial z'/\partial \Omega = \hat{\rho}'[1-\hat{\rho}K]$, $\partial S(Q)/\partial \Omega = [1-(1-K)Q^{-1}]$. Then,

$$\frac{\partial \hat{x}_P}{\partial \Xi} = \frac{1}{[\sum_{j\in\mathcal{J}_{ab}} w_j\, z_j]}\left[\frac{(1+\xi)}{\xi}\left(1-W\,Q^{-1}\right)\hat{\rho}'[1-\hat{\rho}K] - \hat{x}_P\,(1-W\,)\,\hat{\rho}'\right] , \tag{200}$$

$$= \frac{-(1+\xi)}{\xi}\,\frac{\left(1-W\,Q^{-1}\right)\,\hat{\rho}\hat{\rho}'}{[\sum_{j\in\mathcal{J}_{ab}} w_j\, z_j]}\,\frac{S(1)}{\Omega} - \frac{\hat{\rho}'\left[\hat{x}_P\,(1-W\,) - \frac{(1+\xi)}{\xi}\left(1-W\,Q^{-1}\right)\right]}{[\sum_{j\in\mathcal{J}_{ab}} w_j\, z_j]} , \tag{201}$$

$$\frac{\partial [Y\,Z_P]}{\partial \Omega} = \frac{(1-W)}{\hat{x}_P^{\xi}}\,\hat{\rho}' - \frac{\left(1-W\,Q^{-1}\right)}{\hat{x}_P^{\xi+1}}\,\hat{\rho}'[1-\hat{\rho}K] + W\,[1-(1-K)Q^{-1}] , \tag{202}$$

$$= \frac{(1-W)}{\hat{x}_P^{\xi}}\,\hat{\rho}' + \left(1-W\,Q^{-1}\right)\frac{\hat{\rho}\hat{\rho}'}{\hat{x}_P^{\xi+1}}\,\frac{S(1)}{\Omega} + W\,Q^{-1}\frac{S(1)}{\Omega} - \frac{\left(1-W\,Q^{-1}\right)}{\hat{x}_P^{\xi+1}}\,\hat{\rho}' + W\,[1-Q^{-1}] , \tag{203}$$

$$= \left[\left(1-W\,Q^{-1}\right)\frac{\hat{\rho}\hat{\rho}'}{\hat{x}_P^{\xi+1}} + W\,Q^{-1}\right]\frac{S(1)}{\Omega} + \frac{(1-W)}{\hat{x}_P^{\xi}}\,\hat{\rho}' - \frac{\left(1-W\,Q^{-1}\right)}{\hat{x}_P^{\xi+1}}\,\hat{\rho}' + (1-W\,Q^{-1}) - (1-W) , \tag{204}$$

$$= \left[\left(1-W\,Q^{-1}\right)\frac{\hat{\rho}\hat{\rho}'}{\hat{x}_P^{\xi+1}} + W\,Q^{-1}\right]\frac{S(1)}{\Omega} + \left(1-W\,Q^{-1}\right)\left[1 - \frac{\hat{\rho}'}{\hat{x}_P^{\xi+1}}\right] - (1-W)\left[1 - \frac{\hat{\rho}'}{\hat{x}_P^{\xi}}\right] , \tag{205}$$

which is (127) and (129).

Details of (132) and (134): For $\Xi \in \{K;\Omega\}$, we have

$$\frac{\partial Z_M}{\partial \Xi} = \frac{1}{(1+\xi)\,\hat{x}_P^{\xi}}\,\frac{\partial z}{\partial \Xi} + \frac{z}{(1+\xi)\,\hat{x}_M^{\xi}}\,\frac{(-\xi)}{\hat{x}_M}\,\frac{\partial \hat{x}_P}{\partial \Xi} . \tag{206}$$

Now,

$$\frac{\partial \hat{x}_M}{\partial \Xi} = \hat{x}_M \left(\frac{\partial z'}{\partial \Xi} \frac{1}{z'} - \frac{\partial z}{\partial \Xi} \frac{1}{z} \right) , \tag{207}$$

$$= \frac{\hat{x}_M}{z} \left(\frac{\partial z'}{\partial \Xi} \frac{z}{z'} - \frac{\partial z}{\partial \Xi} \right) , \tag{208}$$

$$= \frac{1}{z} \left(\frac{\partial z'}{\partial \Xi} \frac{(1+\xi)}{\xi} - \hat{x}_M \frac{\partial z}{\partial \Xi} \right) . \tag{209}$$

Consequently,

$$\frac{\partial Z_M}{\partial \Xi} = \frac{1}{\hat{x}_M^{\xi}} \frac{\partial z}{\partial \Xi} - \frac{1}{\hat{x}_M^{\xi+1}} \frac{\partial z'}{\partial \Xi} . \tag{210}$$

• For $\Xi = K$, we have $\partial z / \partial K = 0$ and $\partial z' / \partial K = -S(1) \hat{\rho} \hat{\rho}' / K$. Then,

$$\frac{\partial \hat{x}_M}{\partial \Xi} = \frac{-(1+\xi)}{\xi} \frac{\hat{\rho}\hat{\rho}'}{z} \frac{S(1)}{K} , \tag{211}$$

$$\frac{\partial Z_M}{\partial \Xi} = \frac{\hat{\rho}\hat{\rho}'}{\hat{x}_M^{\xi+1}} \frac{S(1)}{K} . \tag{212}$$

which is (132) and (134).

• For $\Xi = \Omega$, we have $\partial z / \partial \Omega = \hat{\rho}'$ and $\partial z' / \partial \Omega = \hat{\rho}' [1 - \hat{\rho} K]$. Then,

$$\frac{\partial \hat{x}_M}{\partial \Xi} = \frac{1}{z} \left[\frac{(1+\xi)}{\xi} \hat{\rho}' [1 - \hat{\rho} K] - \hat{x}_M \hat{\rho}' \right] , \tag{213}$$

$$= \frac{-(1+\xi)}{\xi} \frac{\hat{\rho}\hat{\rho}'}{z} \frac{S(1)}{\Omega} - \frac{\hat{\rho}' \left[\hat{x}_M - \frac{(1+\xi)}{\xi} \right]}{z} , \tag{214}$$

$$\frac{\partial Z_M}{\partial \Omega} = \frac{1}{\hat{x}_P^{\xi}} \hat{\rho}' - \frac{1}{\hat{x}_P^{\xi+1}} \hat{\rho}' [1 - \hat{\rho} K] , \tag{215}$$

$$= \frac{\hat{\rho}\hat{\rho}'}{\hat{x}_P^{\xi+1}} \frac{S(1)}{\Omega} + \frac{\hat{\rho}'}{\hat{x}_P^{\xi+1}} [\hat{x}_P - 1] , \tag{216}$$

which is (132) and (134).